NONLINEAR AEROELASTIC ANALYSIS OF SWEPT WINGS IN COMPRESSIBLE FLOW VIA THE HARMONIC BALANCE METHOD

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Abstract

Nonlinear aeroelastic analysis of swept rectangular wings with two degree of freedom in a compressible flow was performed. Previously driven expressions for aerodynamic loads acting on an element of this aeroelastic system in an incompressible flow and frequency domain were used in this research. A compressibility correction factor is applied to these expressions to modify them for compressible flow. Then they were presented in the time domain using a suitable transform. Considering cubic nonlinearity, amplitude and frequency of limit cycle oscillations were obtained for different cases via harmonic balance method that obtained results are in a close agreement with those of numerical solution of the derived equations.

1 Introduction

Aeroelasticity is a multi-disciplinary field of study dealing with the interaction of inertia, structural and aerodynamic forces. Calculation of instability boundary in aeroelastic systems is one of the main problems that aeroelasticians are faced and many methods have been employed to determine aeroelastic instability boundary. While combination of CFD and FE methods are one of the practical and nearly exact solutions of this problem, application of analytical structural wing model and also aerodynamic models for the certain condition of air flow such as unsteady compressible flow are still valuable and useful. Methods of aeroelastic analysis based on these analytical models are less time consuming and saving largely in expenses for providing proper equipments such as powerful computers for running aeroelastic codes using CFD and FE subroutines.

Governing aeroelastic equations for a two degree-of-freedom (dof) airfoil in an unsteady incompressible flow are derived by Theodorsen [1] in the frequency domain ignoring aeroelastic nonlinearities. Also, the governing aeroelastic equations of a 2-dof airfoil in an incompressible flow are presented by Fung [2] in the time domain. These equations are solved via a numerical solution by Lee and Le Blanc [3]. Lee et al. [4] used a standard fourth-order Runge-Kutta scheme to integrate the system of equations of a 2-dof airfoil containing cubic nonlinearity for given initial conditions in order to solve equations of motion in the time domain. In the issue of aeroelastic analysis of a 2-dof wing, an experimental and linear analytical study of the flutter of swept back cantilever wing in the frequency domain were presented by Barmby et al. [5]. Recently, Ghadiri and Razi [6] investigated the Limit cycle oscillations of unswept rectangular cantilever wings containing cubic nonlinearity in an incompressible flow. They also verify their formulation with the experimental data. In this work, governing aeroelastic equations of 2-dof swept rectangular wing with cubic nonlinearity are derived in time domain based on a relationship between Wagner and Theodorsen function and also applying the strip theory and assumed mode method. Finally, considering cubic nonlinearity, Limit cycle oscillations of this aeroelastic system are investigated using two different method of solution for derived equations.

2 Governing Aeroelastic Equations

The Lagrange equations can be used to obtain the governing aeroelastic equations of a rectangular wing in an incompressible flow. Consider a 2-dof wing oscillating in pitch and plunge. The plunge deflection is denoted by h, positive downward direction, and the pitch angle α, positive nose up, respectively. Assuming the rectangular wing as a uniform cantilever beam, its first mode shapes obtained from Barmby et al. [5] can be used for plunge and pitch degree of freedom . The well known Lagrange equations along with assumed mode method can be applied for a 2-dof swept rectangular wing (see Fig. 1) oscillating in pitch and plunge degree of freedom to derive the aeroelastic governing equations. In order to obtain generalized forces corresponding to plunge and pitch displacements, strip theory is used for an analytical aerodynamic model presented by Barmby et al [5] for the swept wing section motion in frequency domain which can be interpreted as the motion of an element of the wing in an incompressible flow. Some terms of this analytical model are modified for compressible flow using the following Prandtl-Glauert compressibility correction factor:

$$\beta = \frac{1}{\sqrt{1-M_\infty^{\ 2}}} \tag{1}$$

where M_∞ is free stream Mach number. Using Fourier summation and Duhamel superposition formula and after considerable algebra, the governing aeroelastic equations of the swept rectangular cantilever wing can be derived in the time domain as follows:

$$\begin{aligned} &c_0\xi_1'' + c_1\alpha_1'' + c_2\xi_1' + c_3\alpha_1' + c_4\xi_1 + c_5\alpha_1 + \\ &c_6 w_1 + c_7 w_2 + c_8 w_3 + c_9 w_4 + c_{10} G(\xi_1) \ = f(\tau) \end{aligned} \tag{2}$$

$$\begin{aligned} &d_0\xi_1'' + d_1\alpha_1'' + d_2\alpha_1' + d_3\alpha_1 + d_4\xi_1' + d_5\xi_1 + \\ &d_6 w_1 + d_7 w_2 + d_8 w_3 + d_9 w_4 + d_{10} M(\alpha_1) = g(\tau) \end{aligned} \tag{3}$$

where ξ_1 and α_1 are time dependent dimensionless plunge and pitch displacement [6], the prime sign denotes differentiation with respect to the dimensionless time τ, coefficients c_0, c_1, …, c_{10} and d_0, d_1, …, d_{10} are concerned with nondimensional parameters of the wing, constants of the Wagner's function, free stream mach number, sweep angle of the wing and also assumed mode shapes and expressions $f(\tau)$ and $g(\tau)$ are dependent on initial conditions, nondimensional time and constants of the Wagner's function. Terms w_1,…,w_4 are the well known integral variable introduced by Lee and Leblanc [3] and finally $G(\xi_1)$ and $M(\alpha_1)$ are the function representing concentrated structural nonlinearities. In the present work, cubic nonlinearity is considered. The main difference between this equation and previously derived equations for airfoil and unswept wing is in the coefficient of these derived equations.

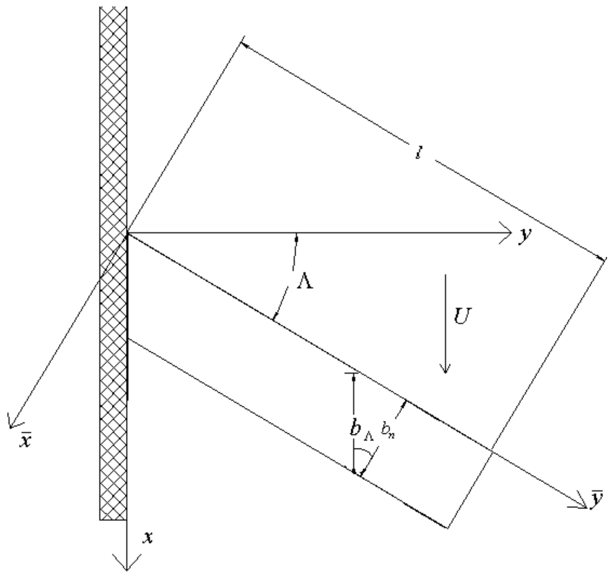

Fig 1. Sketch of a swept rectangular wing

3 Harmonic Balance Method

According to the features of LCO, after decaying transient effects or effects of initial condition, amplitude and frequency of oscillations are constant with respect to time. Therefore, this feature can be used to solve Eqs. (2) and (3) which are the equations of motion in the time domain. In this method, the motion of airfoil can be approximated as follows:

$$\alpha(\tau) = a_1 \sin(\omega\tau) + \sum_{i=3,5,7,9,\ldots} a_i \sin(i\omega\tau) + b_i \cos(i\omega\tau) \tag{4}$$

$$\xi(\tau) = \sum_{j=1,3,5,9,\ldots\ldots} e_j \sin(j\omega\tau) + f_j \cos(j\omega\tau) \tag{5}$$

Similar to the feature assumed for LCO, coefficients a_i, b_i, e_i, f_i and also LCO frequency ω are constant with respect to dimensionless time τ. Inserting Equations (4) and (5) in Equations (2) and (3) and ignoring the terms which are dependent to initial conditions, a trigonometric system of equations can be derived. Collecting the coefficients of $\sin(i\omega\tau)$ and $\cos(i\omega\tau)$ (i = 1,3,5,…), we obtain a system of algebraic equations with a_1, ω, a_i, b_i, e_i and f_i (i = 3,5,7,…; j = 3,5,7,…) as variables. Maximum values of indices i and j are equal. Order of Harmonic balance method is also equal to this maximum value. The number of equations for first order equations is 4 and as the order of the method is increased, 4 other unknown variable and thus 4 other nonlinear algebraic equations will be added to the previous number of equations. For this reason by increasing the order of Harmonic balance method, the system of algebraic nonlinear equations will be more complex. Solving the resulting system of equations, the LCO frequency and amplitude can be easily obtained.

4 Results and Discussion

Linear aeroelastic analysis of the swept wing is carried out in order to verify the derived formulations. For this reason, experimental data for the flutter speed of the uniform cantilever wing of Barmby et al. [5] are used. The physical characteristics of the tested wings and their nondimensional parameters are presented in [5]. The standard 4th order Runge-Kutta method was applied to obtain numerical solutions for the resulting set of first-order differential equations and as a result the aeroelastic instability boundary of the system.

In the strip theory approximation, the chord wise pressure distribution at any spanwise station is assumed to depend only on the downwash at that station given by the two-dimensional aerodynamic theory and to be independent of the downwash at any other spanwise station and it is the main reason of the difference between experimental and calculated data for the flutter speed. This is the main source of our formulation error. However, by our comparison, it is shown that this error can be ignored. The assumption of two-dimensional flow and also applying only the first mode shape of a cantilever beam can be the other sources of error. Also, the more the Mach number increase upper than 0.6, less exactitude our proposed formulation has. According to comparison made by Anderson [11], Prandtl-Glaurt compressibility correction factor is not reliable in the mach numbers upper than 0.6.

Table 1. Calculated and experimental results for flutter speed

Case	Experimental flutter speed (m/sec)	Calculated flutter speed (m/sec)
30B	120.243	118.1
40A	105.045	100.87
73	86.271	90.46
85-1	143.934	145.00

Using derived formulation [Equations (2) and (3)] and applying proper mathematical model instead of terms $G(\xi_1)$ and $M(\alpha_1)$, it is very easy to treat with the concentrated structural nonlinearity such as cubic. Here, these structural nonlinearities are considered and limit cycle oscillation (LCO) phenomenon (Fig. 2) is observed. LCO pitch and plunge amplitudes and also LCO frequency were investigated by the use of harmonic balance method and standard fourth-order Runge-Kutta method for some different cases. A sample of results for is presented in Figs. 3 for nondimensional speeds from $U^* = 1.005\, U_L^*$ to $U^* = 1.15\, U_L^*$ and for the wing tip ($\eta = 1$).

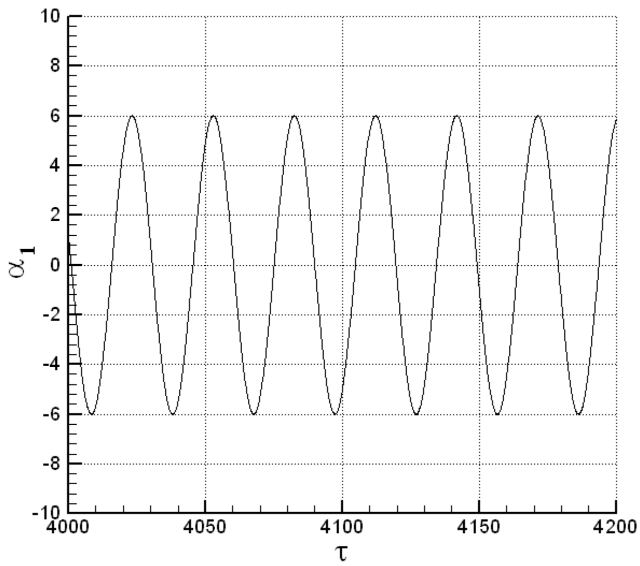

Fig 2. Limit cycle oscillations phenomena

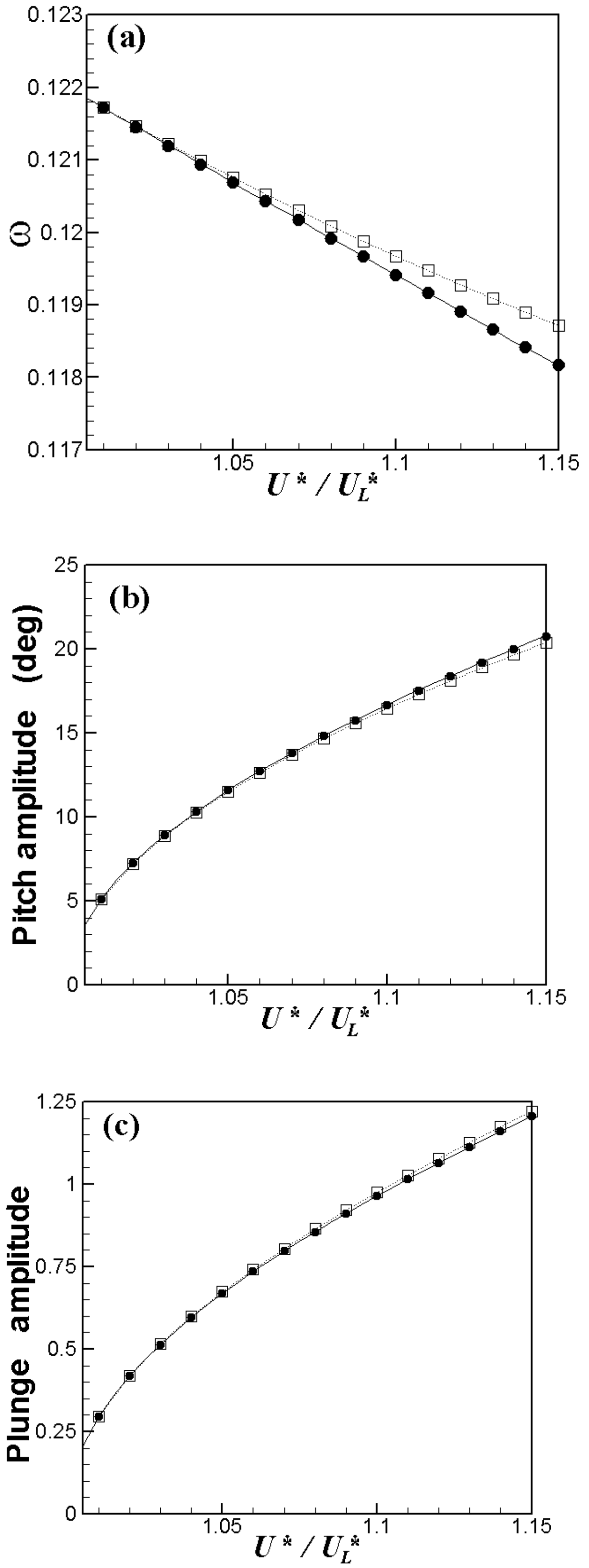

Fig. 3 Wing tip dynamical response: (a) frequency; (b) amplitude of pitch motion; (c) amplitude of plunge motion: ————, numerical result; ···□···, HB1; ●, HB3.

In these figures, the results for the first- and third-order harmonic balance methods were compared with numerical results at the wing tip. The comparison shows good agreement between the HB method and the numerical results, and also it indicates that the accuracy of harmonic balance method increases by increasing the order of this method.

5 Conclusion

In this paper, linear and nonlinear aeroelastic analysis of swept rectangular wings with two degree of freedom in compressible flow was performed. For linear analysis, analytical method approach were adopted that its results are in good agreement with experimental data. In nonlinear analysis, considering cubic nonlinearity, amplitude and frequency of limit cycle oscillations were obtained for different cases. Two applied methods, numerical and harmonic balance, for obtaining LCO amplitude and frequency are in a good agreement with each other. On the other hand, the accuracy of HB method increases by increasing of its order, as compared to the numerical solution.

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