VISION-BASED UAV GUIDANCE AND ESTIMATION FOR AERIAL TARGET TRACKING

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Abstract

Target tracking of an Unmanned Aerial Vehicle (UAV) is a challenging problem when the UAV uses only a single vision sensor because the vision sensor cannot measure range between the target and the UAV. The problem becomes more difficult when the target flies in airspace. To deal with this problem, a measurement model of the vision sensor based on a specific image processing technique is proposed. By applying the proposed model, the unknown range problem of the vision sensor can be transformed into the unknown parameter problem which is related to the size of the target. Nonlinear adaptive observer is adopted to estimate the unknown parameter and the states of the target. Guidance scheme for the target tracking using the estimation result is also proposed. Numerical simulations are performed to demonstrate the effectiveness of the proposed method.

1 Introduction

There have been an amount of studies on UAVs because UAVs can effectively substitute for the manned aircraft due to their high potential and low risk [1]. Surveillance and reconnaissance are the conventional missions of UAVs, and to perform these missions successfully, guidance and control of UAVs are required especially for autonomous flight. Target tracking is a representative application among various UAV applications for the surveillance and reconnaissance.

For the target tracking, UAVs should have sensor system which can provide information of the target. Recently, vision sensors are widely used for the sensor system due to their passive and non-cooperative characteristics [1, 2]. However, a single vision sensor has an observability problem; it projects a 3-dimensional object onto a 2-dimensional image plane, and thus it cannot provide the range information of the object. In general, this problem can be solved by adopting an additional sensor which can measure the range. If a single vision sensor system solves the observability problem, then it can reduce overall cost of UAVs and exploit the characteristics of the vision sensor.

There are several relevant studies on state estimation problem using the measurements of the single vision sensor [3-11]. For a bearing measurement sensor, the maximization of determinant or trace of Fisher information matrix has been studied in the viewpoint of the optimal trajectory generation [3-5]. However, these approaches do not include the stability analysis and are hard to implement because of high computational load. For a stationary target, the observability problem can be solved by moving the position of the sensor [6]. Tracking a target moving on the ground can be solved by utilizing the altitude of the UAV [7]. On the other hand, for the aerial target tracking problem, the altitude information cannot be utilized to estimate the states of the target. Thus, several studies adopted a specific image processing technique of the vision sensor, and transformed the unknown range problem into the unknown parameter problem [8-11] using a joint state and parameter estimator. Vela et al. proposed a specific image processing technique by introducing a subtended angle for range regulation [8]. Based on the image processing technique, the aerial target tracking problem can be treated using a robust adaptive observer and the intelligent excitation concept [9-11]. However, the robust adaptive observer is only applied to the systems whose relative degree is 1, and therefore it cannot be implemented to the higher relative degree systems [9-10].

Note that the estimation of the target's states and parameter using the measurements of the vision sensor is related to the problem of the higher relative degree systems. In this study, a nonlinear adaptive observer is adopted [12-14]. A guidance scheme is proposed for a simplified nonholonomic UAV model [15, 16]. The guidance scheme is based on the "perfect velocity tracking" concept [16] and backstepping approach.

The contributions of this study are summarized as follows:

1) A state and parameter estimation problem for a 3-dimensional aerial target tracking of a UAV is formulated by using a measurement model of a single vision sensor.

2) A nonlinear adaptive observer is implemented for the estimation problem.

3) A 3-dimensional guidance scheme is proposed for a nonholonomic UAV model.

This paper is organized as follows: Section 2 describes the aerial target tracking problem of a single-vision-based UAV. Section 3 deals with the unknown parameter estimation problem of the vision sensor. Section 4 proposes a guidance scheme for the target tracking. Numerical simulations are performed in Section 5 to demonstrate the effectiveness of the proposed method, and conclusions are made in Section 6.

2 Problem Statements

For the aerial target tracking of a UAV, the UAV should estimate the states which include the position and velocity vectors of the target. The estimation is usually combined with the sensor system of the UAV. Using the estimated states of the target, the UAV can be guided to track the target.

2.1 Single Vision Sensor and Parameter Estimation

Vision sensors are widely used for a UAV system because of their passive and non-cooperative characteristics. Vision sensors project a 3-dimensional object onto a 2-dimensional image plane, and thus a single vision sensor cannot measure the distance between the target and the UAV. Thus, estimation problem should be solved to obtain the states of the target.

Let us consider a measurement model of the single vision sensor as shown in Fig. 1 [15]. In Fig. 1, α, χ_c, and γ_c are the measurements of the sensor, where α is a half of the subtended angle obtained from image processing [8], χ_c and γ_c are azimuth and elevation angles of the target, respectively. Note that the range l is unknown, which should be estimated using the measurements.

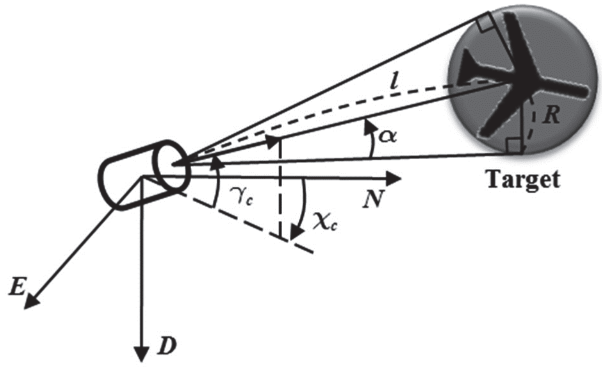

Fig. 1. Measurement model of a single vision sensor.

The range l can be expressed as a function of α and R as:

$$l = \frac{R}{\sin\alpha} \tag{1}$$

where $R \in \mathbb{R}^1$ is also an unknown constant and positive parameter, which is related to the size of the target.

The relative position of the target with respect to the UAV can be obtained in the NED (North-East-Down) coordinate as follows

$$\begin{aligned}\tilde{\mathbf{r}} &\triangleq \mathbf{r}_T - \mathbf{r} \\ &= \frac{R}{\sin\alpha}\left[\cos\gamma_c\cos\chi_c \;\; \cos\gamma_c\sin\chi_c \;\; -\sin\gamma_c\right]^T\end{aligned} \tag{2}$$

where $\mathbf{r}_T$ is the position of the target, and $\mathbf{r}$ is the position of the UAV. The relative velocity and acceleration can be obtained by differentiating Eq. (2) as:

$$\tilde{\mathbf{V}} \triangleq \mathbf{V}_T - \mathbf{V} = R\mathbf{T}\begin{bmatrix}\dot{\alpha} & \dot{\chi}_c & \dot{\gamma}_c\end{bmatrix}^T \tag{3}$$

$$\tilde{\mathbf{a}} \triangleq \mathbf{a}_T - \mathbf{a} = R\left(\mathbf{v} + \mathbf{T}\begin{bmatrix}\ddot{\alpha} & \ddot{\chi}_c & \ddot{\gamma}_c\end{bmatrix}^T\right) \tag{4}$$

where $\mathbf{V}_T$ and $\mathbf{a}_T$ are the velocity and acceleration of the target, respectively, $\mathbf{V}$ and $\mathbf{a}$ are the velocity and acceleration of the UAV, respectively, and

$$\mathbf{T} = \begin{bmatrix} -\dfrac{\cos\gamma_c\cos\chi_c}{\sin\alpha\tan\alpha} & -\dfrac{\cos\gamma_c\sin\chi_c}{\sin\alpha} & -\dfrac{\sin\gamma_c\cos\chi_c}{\sin\alpha} \\ -\dfrac{\cos\gamma_c\sin\chi_c}{\sin\alpha\tan\alpha} & \dfrac{\cos\gamma_c\cos\chi_c}{\sin\alpha} & -\dfrac{\sin\gamma_c\sin\chi_c}{\sin\alpha} \\ \dfrac{\sin\gamma_c}{\sin\alpha\tan\alpha} & 0 & -\dfrac{\cos\gamma_c}{\sin\alpha} \end{bmatrix} \tag{5}$$

$$\mathbf{v} = \begin{bmatrix} \dfrac{\cos\gamma_c\cos\chi_c}{\sin\alpha}\left\{\left(\dfrac{1}{\tan^2\alpha}+\dfrac{1}{\sin^2\alpha}\right)\dot{\alpha}^2 - \dot{\chi}_c^{\,2} - \dot{\gamma}_c^{\,2}\right\} \\ +\dfrac{2}{\sin\alpha}\left\{\dfrac{\sin\gamma_c\cos\chi_c}{\tan\alpha}\dot{\alpha}\dot{\gamma}_c + \sin\gamma_c\sin\chi_c\dot{\chi}_c\dot{\gamma}_c\right\} \\ \dfrac{\cos\gamma_c\sin\chi_c}{\sin\alpha}\left\{\left(\dfrac{1}{\tan^2\alpha}+\dfrac{1}{\sin^2\alpha}\right)\dot{\alpha}^2 - \dot{\chi}_c^{\,2} - \dot{\gamma}_c^{\,2}\right\} \\ +\dfrac{2}{\sin\alpha}\left\{\begin{array}{l}\dfrac{\sin\gamma_c\sin\chi_c}{\tan\alpha}\dot{\alpha}\dot{\gamma}_c - \dfrac{\cos\gamma_c\cos\chi_c}{\tan\alpha}\dot{\alpha}\dot{\chi}_c \\ -\sin\gamma_c\cos\chi_c\dot{\chi}_c\dot{\gamma}_c\end{array}\right\} \\ -\dfrac{\sin\gamma_c}{\sin\alpha}\left\{\left(\dfrac{1}{\tan^2\alpha}+\dfrac{1}{\sin^2\alpha}\right)\dot{\alpha}^2 - \dot{\gamma}_c^{\,2}\right\} \\ +\dfrac{2\cos\gamma_c}{\sin\alpha\tan\alpha}\dot{\alpha}\dot{\gamma}_c \end{bmatrix} \tag{6}$$

For the given R, the relative states can be constructed, and the corresponding states such as position and velocity of the target can be obtained.

Let us define the measurements vector $\mathbf{y} \triangleq \begin{bmatrix}\alpha & \chi_c & \gamma_c\end{bmatrix}^T$, the state vector $\mathbf{x} \triangleq \begin{bmatrix}\alpha & \chi_c & \gamma_c & \dot{\alpha} & \dot{\chi}_c & \dot{\gamma}_c\end{bmatrix}^T$, and the parameter vector $\boldsymbol{\theta} \triangleq 1/R$. Then, the kinematic model in Eqs. (1)-(6) can be represented as follows.

$$\begin{cases}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{f}(\mathbf{x}) + \boldsymbol{\psi}(\mathbf{x},\tilde{\mathbf{a}})\boldsymbol{\theta} \\ \mathbf{y} = \mathbf{C}\mathbf{x}\end{cases} \tag{7}$$

where

$$\mathbf{A} = \begin{bmatrix}\mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3}\end{bmatrix},\ \mathbf{C} = \begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}_{3\times3}\end{bmatrix},$$

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix}\mathbf{0}_{3\times1}^T & -\left(\mathbf{T}^{-1}\mathbf{v}\right)^T\end{bmatrix}^T,$$

$$\boldsymbol{\psi}(\mathbf{x},\tilde{\mathbf{a}}) = \begin{bmatrix}\mathbf{0}_{3\times1}^T & \left(\mathbf{T}^{-1}\tilde{\mathbf{a}}\right)^T\end{bmatrix}^T$$

To obtain the position and velocity of the target, $\mathbf{x}$ and $\boldsymbol{\theta}$ should be estimated. Now, the estimation problem can be solved by designing an appropriate estimator for the system in Eq. (7).

2.2 UAV Guidance

If the target states are given, UAV can be guided to track the target. Let us consider the following velocity equation in the NED coordinate.

$$\dot{\mathbf{r}} = \mathbf{V} = \begin{bmatrix}V\cos\gamma\cos\chi \\ V\cos\gamma\sin\chi \\ -V\sin\gamma\end{bmatrix} \tag{8}$$

where V is the speed, χ is the heading angle, and γ is the flight path angle of the UAV. Let us consider following nonholonomic point-mass UAV model [15].

$$\dot{V} = g\left(u_1 - \sin\gamma\right) \tag{9}$$

$$\dot{\chi} = \frac{g}{V\cos\gamma}u_2 \tag{10}$$

$$\dot{\gamma} = \frac{g}{V}\left(u_3 - \cos\gamma\right) \tag{11}$$

where g is the gravitational acceleration, and the control input $\mathbf{u} \triangleq \begin{bmatrix}u_1 & u_2 & u_3\end{bmatrix}^T$ is defined as the non-dimensional acceleration terms.

In this study, it is assumed that an ideal autopilot is designed, and therefore the acceleration terms can be generated. Finally, the UAV guidance problem is defined as an input generation problem which can make UAV track the target.

3 Parameter Estimation

The parameter estimation can be performed by designing a nonlinear adaptive observer. In this section, the nonlinear adaptive observer design and convergence analysis are dealt with.

3.1 Nonlinear Adaptive Observer

Usually, commercial aircraft or military transport aircraft cruise for most of the flight, which are considered as the targets of this study. Hence, the acceleration of the target can be considered as a disturbance of the system in Eq. (7). Or, it can be assumed that $\mathbf{a}_T = \mathbf{0}$ for simplicity. Then, Eq. (7) can be simplified as:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{f}(\mathbf{x}) + \boldsymbol{\psi}(\mathbf{x},\mathbf{a})\boldsymbol{\theta} \\ \mathbf{y} = \mathbf{C}\mathbf{x} \end{cases} \tag{12}$$

Let us define the estimated state vector as $\hat{\mathbf{x}}$ and the estimated parameter as $\hat{\boldsymbol{\theta}}$. Then, the following nonlinear adaptive observer can be designed.

$$\begin{cases} \dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{f}(\hat{\mathbf{x}}) + \boldsymbol{\psi}(\hat{\mathbf{x}},\mathbf{a})\hat{\boldsymbol{\theta}} \\ \qquad -\left(\frac{1}{2}\lambda_1\mathbf{S}^{-1} + \boldsymbol{\gamma}\mathbf{P}\boldsymbol{\gamma}^T\right)\mathbf{C}^T\left(\mathbf{C}\hat{\mathbf{x}} - \mathbf{y}\right) \\ \dot{\hat{\boldsymbol{\theta}}} = -\mathbf{P}\boldsymbol{\gamma}^T\mathbf{C}^T\left(\mathbf{C}\hat{\mathbf{x}} - \mathbf{y}\right) \\ \dot{\boldsymbol{\gamma}} = \left(\mathbf{A} - \frac{1}{2}\lambda_1\mathbf{S}^{-1}\mathbf{C}^T\mathbf{C}\right)\boldsymbol{\gamma} + \boldsymbol{\psi}(\hat{\mathbf{x}},\mathbf{a}) \\ \dot{\mathbf{P}} = \lambda_2\mathbf{P} - 2\mathbf{P}\boldsymbol{\gamma}^T\mathbf{C}^T\mathbf{C}\boldsymbol{\gamma}\mathbf{P} \end{cases} \tag{13}$$

where $\boldsymbol{\gamma}$ and $\mathbf{P}$ are time-varying vector and matrix related to the system of higher order relative degree, and λ_1 and λ_2 are positive constants. $\mathbf{f}(\hat{\mathbf{x}})$ and $\boldsymbol{\psi}(\hat{\mathbf{x}},\mathbf{a})$ are function and matrix using the estimated state vector $\hat{\mathbf{x}}$. Initial values of $\boldsymbol{\gamma}$ and $\mathbf{P}$ are set as $\boldsymbol{\gamma}(0) = \mathbf{0}_{3\times1}$ and $\mathbf{P}(0) = \mathbf{0}_{1\times1}$, respectively.

The proposed nonlinear adaptive observer requires the following assumptions [14].

Assumption 1. The state $\mathbf{x}$, the control $\mathbf{a}$, and the unknown parameter $\boldsymbol{\theta}$ are bounded.

Assumption 2. The matrix $\boldsymbol{\psi}(\mathbf{x},\mathbf{a})$ is continuous with respect to $\mathbf{x}$ and $\mathbf{a}$.

Assumption 3. Functions $\mathbf{f}(\mathbf{x})$ and $\boldsymbol{\psi}(\mathbf{x},\mathbf{a})$ are Lipschitz with respect to $\mathbf{x}$.

Assumption 4. (Persistent Excitation) The nonlinear adaptive observer in Eq. (13) satisfies the following relation for the satisfactory parameter estimation.

$$\rho_1\mathbf{I}_{1\times1} \le \int_t^{t+T} \boldsymbol{\gamma}^T(\tau)\mathbf{C}^T\mathbf{C}\boldsymbol{\gamma}(\tau)d\tau \le \rho_2\mathbf{I}_{1\times1}$$

where $\rho_2 > \rho_1 > 0$, T is an arbitrary positive value, and time t is an arbitrary non-negative value.

Because $\alpha \in (0, \pi/2)$ when the measurements of the vision sensor are available and the accelerations of the target and the UAV are bounded, Assumption 1 is permissible. Assumptions 2 and 3 are also permissible if $\mathbf{a}$ is bounded because $\det(\mathbf{T}) = \cos\gamma_c / \left(\sin^3\alpha\tan\alpha\right)$ $\neq 0$ and corresponding partial derivatives are also bounded. Assumption 4 is a necessary condition for the estimation of the unknown parameter $\boldsymbol{\theta}$. If Assumption 4 is true, then $\mathbf{x}$ and $\boldsymbol{\theta}$ in Eq. (12) are observable.

3.2 Convergence Analysis

Let us define relative state vectors $\tilde{\mathbf{x}} \triangleq \hat{\mathbf{x}} - \mathbf{x}$, $\tilde{\boldsymbol{\theta}} \triangleq \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$, $\tilde{\mathbf{f}} \triangleq \mathbf{f}(\hat{\mathbf{x}}) - \mathbf{f}(\mathbf{x})$, $\tilde{\boldsymbol{\psi}} \triangleq \boldsymbol{\psi}(\hat{\mathbf{x}},\mathbf{a}) - \boldsymbol{\psi}(\mathbf{x},\mathbf{a})$, and a parameter-filtered state vector $\boldsymbol{\eta} \triangleq \tilde{\mathbf{x}} - \boldsymbol{\gamma}\tilde{\boldsymbol{\theta}}$ [12-14]. . Consider the following Lyapunov candidate functions.

$$E_1 \triangleq \boldsymbol{\eta}^T\mathbf{S}\boldsymbol{\eta} \tag{14}$$

$$E_2 \triangleq \tilde{\boldsymbol{\theta}}^T\mathbf{P}^{-1}\tilde{\boldsymbol{\theta}} \tag{15}$$

where $\mathbf{S}$ is a symmetric positive definite matrix that satisfies the following equation.

$$\mathbf{A}^T\mathbf{S} + \mathbf{S}\mathbf{A} - \lambda_1\mathbf{C}^T\mathbf{C} = -\lambda_1\mathbf{S} \tag{16}$$

Note from Eq. (16) that $\mathbf{A} - \frac{1}{2}\lambda_1\mathbf{S}^{-1}\mathbf{C}^T\mathbf{C}$ is Hurwitz. Since $\boldsymbol{\gamma}$ is bounded for a bounded $\boldsymbol{\psi}(\hat{\mathbf{x}},\mathbf{a})$, the following inequality satisfies.

$$\|\tilde{\mathbf{x}}\| \le \|\boldsymbol{\eta}\| + \|\boldsymbol{\gamma}\|\|\tilde{\boldsymbol{\theta}}\| \tag{17}$$

Then, the following relations are obtained by using Eq. (17) and Assumption 3,

$$\left\|\tilde{\mathbf{f}}\right\| \le a_1 \left\|\tilde{\mathbf{x}}\right\| \le a_1 \left\|\boldsymbol{\eta}\right\| + a_2 \left\|\tilde{\boldsymbol{\theta}}\right\| \tag{18}$$

$$\left\|\tilde{\boldsymbol{\psi}}\boldsymbol{\theta}\right\| \le a_3 \left\|\tilde{\mathbf{x}}\right\| \le a_3 \left\|\boldsymbol{\eta}\right\| + a_4 \left\|\tilde{\boldsymbol{\theta}}\right\| \tag{19}$$

where a_1, a_2, a_3, and a_4 are positive constants. The matrix $\mathbf{P}$ is also bounded under Assumption 4 [14]. Therefore, the following inequality can be obtained by differentiating Eq. (14) with respect to time and substituting Eqs. (12)-(13) into the resulting equation.

$$\begin{aligned} \dot{E}_1 &= \dot{\boldsymbol{\eta}}^T \mathbf{S}\boldsymbol{\eta} + \boldsymbol{\eta}^T \mathbf{S}\dot{\boldsymbol{\eta}} \\ &= \left(\dot{\tilde{\mathbf{x}}} - \dot{\boldsymbol{\gamma}}\tilde{\boldsymbol{\theta}} - \boldsymbol{\gamma}\dot{\tilde{\boldsymbol{\theta}}}\right)^T \mathbf{S}\boldsymbol{\eta} + \boldsymbol{\eta}^T \mathbf{S}\left(\dot{\tilde{\mathbf{x}}} - \dot{\boldsymbol{\gamma}}\tilde{\boldsymbol{\theta}} - \boldsymbol{\gamma}\dot{\tilde{\boldsymbol{\theta}}}\right) \\ &= -\lambda_1 \boldsymbol{\eta}^T \mathbf{S}\boldsymbol{\eta} + 2\boldsymbol{\eta}^T \mathbf{S}\tilde{\mathbf{f}} + 2\boldsymbol{\eta}^T \mathbf{S}\tilde{\boldsymbol{\psi}}\boldsymbol{\theta} \\ &\le -\lambda_1 \boldsymbol{\eta}^T \mathbf{S}\boldsymbol{\eta} + 2\left\|\boldsymbol{\eta}\right\|\left\|\mathbf{S}\right\|\left\|\tilde{\mathbf{f}}\right\| + 2\left\|\boldsymbol{\eta}\right\|\left\|\mathbf{S}\right\|\left\|\tilde{\boldsymbol{\psi}}\boldsymbol{\theta}\right\| \\ &\le -\lambda_1 \boldsymbol{\eta}^T \mathbf{S}\boldsymbol{\eta} + 2(a_1 + a_3)\left\|\boldsymbol{\eta}\right\|\left\|\mathbf{S}\right\|\left\|\boldsymbol{\eta}\right\| \\ &\quad + 2(a_2 + a_4)\left\|\boldsymbol{\eta}\right\|\left\|\mathbf{S}\right\|\left\|\tilde{\boldsymbol{\theta}}\right\| \\ &\le -\lambda_1 E_1 + b_1 E_1 + b_2 \sqrt{E_1}\sqrt{E_2} \end{aligned} \tag{20}$$

Similar to Eq. (20), the following inequality can be obtained.

$$\begin{aligned} \dot{E}_2 &= -\lambda_2 \tilde{\boldsymbol{\theta}}^T \mathbf{P}^{-1} \tilde{\boldsymbol{\theta}} - 2\boldsymbol{\eta}^T \mathbf{C}^T \mathbf{C}\boldsymbol{\gamma}\tilde{\boldsymbol{\theta}} \\ &\le -\lambda_2 E_2 + b_3 \sqrt{E_1}\sqrt{E_2} \end{aligned} \tag{21}$$

Now, let us consider another Lyapunov candidate function consisting of Eqs. (14) and (15).

$$E_3 \triangleq E_1 + E_2 \tag{22}$$

The time derivative of Eq. (22) can be obtained as:

$$\begin{aligned} \dot{E}_3 &= \dot{E}_1 + \dot{E}_2 \\ &\le -\lambda_1 E_1 + c_1 E_1 + c_2 \sqrt{E_1}\sqrt{E_2} - \lambda_2 E_2 \\ &\le -\left(2\sqrt{\lambda_2(\lambda_1 - c_1)} - c_2\right)\sqrt{E_1}\sqrt{E_2} \end{aligned} \tag{23}$$

By selecting the gains λ_1 and λ_2 such that $\lambda_1 > c_1$ and $2\sqrt{\lambda_2(\lambda_1 - c_1)} > c_2$, it can be concluded that the errors on $\boldsymbol{\eta}$ and $\boldsymbol{\theta}$ converge to zeros according to Lyapunov stability theorem [17].

4 Guidance for Aerial Target Tracking

Using the results of the nonlinear adaptive observer proposed in Section 3, $\hat{\mathbf{r}}_T$ and $\hat{\mathbf{V}}_T$ can be obtained for Eqs. (2)-(3). In this section, the UAV guidance law is designed.

4.1 Guidance Command Generation

To track the aerial target, the UAV is required to maintain a certain distance with respect to the target. Let us consider a reference position of the UAV located from the target by a constant offset $\Delta \triangleq \left[\Delta_1\ \Delta_2\ \Delta_3\right]^T$. Then, the reference position of the UAV can be represented as:

$$\begin{aligned} \mathbf{r}_r &\triangleq \left[x_r\ y_r\ z_r\right]^T \\ &= \mathbf{r}_T + \mathbf{C}(-\chi_T, 3)\mathbf{C}(-\gamma_T, 2)\Delta \end{aligned} \tag{24}$$

where χ_T and γ_T are the components of $\mathbf{V}_T$, and $\mathbf{C}(\cdot,\cdot)$ are the following Euler's rotation matrices.

$$\mathbf{C}(\gamma, 2) = \begin{bmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{bmatrix} \tag{25}$$

$$\mathbf{C}(\chi, 3) = \begin{bmatrix} \cos\chi & \sin\chi & 0 \\ -\sin\chi & \cos\chi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{26}$$

By tracking the reference trajectory, the UAV can track the aerial target without collision.

Note that the reference trajectory of the UAV is a time-varying vector. The error vector using $\mathbf{r}_r$ and χ_r is defined as:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \triangleq \mathbf{C}(\chi_r, 3)\begin{bmatrix} x_r - x \\ y_r - y \\ z_r - z \end{bmatrix} \tag{27}$$

where χ_r is a component of $\mathbf{V}_r$ obtained by differentiating $\mathbf{r}_r$. The configuration of Eq. (27) is shown in Fig. 2.

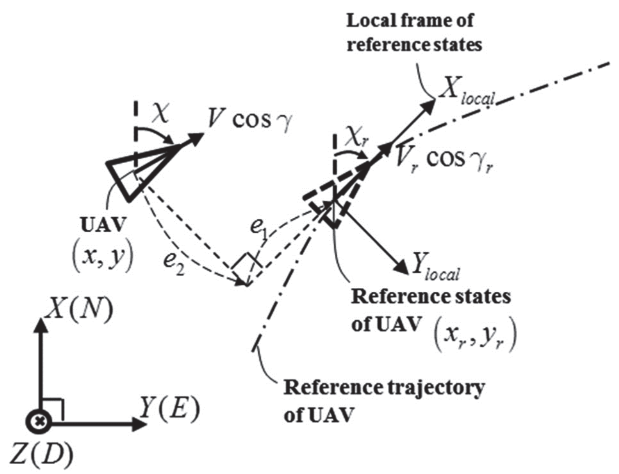

Fig. 2. Geometry of reference states tracking

By differentiating Eq. (27), the following equation can be obtained.

$$\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\end{bmatrix}=\begin{bmatrix}0 & \dot{\chi}_r & 0\\ -\dot{\chi}_r & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix}+\begin{bmatrix}V_r\cos\gamma_r\\ 0\\ -V_r\sin\gamma_r\end{bmatrix}-\begin{bmatrix}V\cos\gamma\cos(\chi_r-\chi)\\ -V\cos\gamma\sin(\chi_r-\chi)\\ -V\sin\gamma\end{bmatrix} \tag{28}$$

Consider a situation that Eq. (28) satisfies the following desired dynamics.

$$\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\end{bmatrix}=\begin{bmatrix}-k_1 & \dot{\chi}_r & 0\\ -\dot{\chi}_r & -k_2 & 0\\ 0 & 0 & -k_3\end{bmatrix}\begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix} \tag{29}$$

where k_1, k_2, and k_3 are positive constants. Then, the errors converge to zeros due to the desired dynamics. Let us define a command set (V_c, χ_c, γ_c) satisfying the desired dynamics, and substitute (V, χ, γ) to (V_c, χ_c, γ_c). Then, the following equation is obtained.

$$\begin{bmatrix}V_r\cos\gamma_r\\ 0\\ -V_r\sin\gamma_r\end{bmatrix}-\begin{bmatrix}V_c\cos\gamma_c\cos(\chi_r-\chi_c)\\ -V_c\cos\gamma_c\sin(\chi_r-\chi_c)\\ -V_c\sin\gamma_c\end{bmatrix}=-\begin{bmatrix}k_1e_1\\ k_2e_2\\ k_3e_3\end{bmatrix} \tag{30}$$

Each component of the command set can be calculated as:

$$V_c=\sqrt{\begin{aligned}&(V_r\cos\gamma_r+k_1e_1)^2+(k_2e_2)^2\\ &+(-V_r\sin\gamma_r+k_3e_3)^2\end{aligned}} \tag{31}$$

$$\chi_c=\chi+\tan^{-1}\left(\frac{k_2e_2}{V_r\cos\gamma_r+k_1e_1}\right) \tag{32}$$

$$\gamma_c=\sin^{-1}\left(\frac{V_r\sin\gamma_r-k_3e_3}{V_c}\right) \tag{33}$$

where V_c is the speed command, χ_c is the heading angle command, and γ_c is the flight path angle command of the command set. Note that these commands include forms of the position error terms.

4.2 Command Tracking Control

For the given speed, heading angle, and flight path angle commands, the control input in Eqs. (9)-(11) can be designed as:

$$\mathbf{u}=\begin{bmatrix}\sin\gamma+\dfrac{\dot{V}_c-k_V(V-V_c)}{g}\\ \dfrac{V\cos\gamma\left(\dot{\chi}_c-k_\chi(\chi-\chi_c)\right)}{g}\\ \cos\gamma+\dfrac{V\left(\dot{\gamma}_c-k_\gamma(\gamma-\gamma_c)\right)}{g}\end{bmatrix} \tag{34}$$

where k_V, k_χ, and k_γ are positive constants. The input in Eq. (34) is a PD (Proportional-Derivative) type control input, and therefore the stability of the closed-loop system can be proven by considering the following Lyapunov candidate function.

$$\begin{aligned}E_4=&\frac{1}{2}(V-V_c)^2+\frac{1}{2}(\chi-\chi_c)^2\\ &+\frac{1}{2}(\gamma-\gamma_c)^2\end{aligned} \tag{35}$$

By differentiating Eq. (35) with respect to time, and using the UAV model in Eqs. (9)-(11) and the control input in Eq. (34), the following equation can be obtained.

$$\dot{E}_4 = -k_V(V-V_c)^2 - k_\chi(\chi-\chi_c)^2 - k_\gamma(\gamma-\gamma_c)^2 \tag{36}$$

Then, $E_4 > 0$ and $\dot{E}_4 < 0$ except for the case of $V = V_c$, $\chi = \chi_c$, and $\gamma = \gamma_c$. Thus, (V, χ, γ) converge to (V_c, χ_c, γ_c) asymptotically. Consequently, the position errors in Eq. (27) also converge to zeros due to the dynamics in Eq. (29).

4.3 Excitation Signals for Estimation

The UAV can track the target by using the proposed guidance law when the state information of the target is given. For this, the estimation process in Section 3 has to satisfy the persistent excitation condition, which is related to the observability of the unknown parameter. According to Assumption 4 in Section 3.1, the persistent excitation condition is satisfied if $\boldsymbol{\psi}(\mathbf{x}, \mathbf{a})$ is persistently exciting. Then, the following condition is obtained.

$$\mathbf{a} \neq \mathbf{0}_{3\times1} \tag{37}$$

The meaning of the above equation is that the UAV is required to accelerate to estimate the unknown parameter. Therefore, in this study, the following sinusoidal signals are added to the guidance commands to satisfy the persistent excitation condition.

$$V_{c,E} = V_c + A_V \sin\left(\frac{2\pi t}{T_V}\right) \tag{38}$$

$$\chi_{c,E} = \chi_c + A_\chi \sin\left(\frac{2\pi t}{T_\chi}\right) \tag{39}$$

$$\gamma_{c,E} = \gamma_c + A_\gamma \sin\left(\frac{2\pi t}{T_\gamma}\right) \tag{40}$$

where $A_{(\bullet)}$ is the amplitude, and $T_{(\bullet)}$ is the period of the excitation signals, respectively.

5 Numerical Simulations

To demonstrate the effectiveness of the proposed nonlinear adaptive observer and guidance scheme of the aerial target tracking, numerical simulations are performed. It is assumed that the target velocity is constant and its speed is 25m/sec. The target size is set as $R = 2\text{m}$ $(\boldsymbol{\theta} = 0.5)$. Initial speed of the UAV is set as 25m/sec. Table 1 summarizes the parameters related to the estimation and guidance of the UAV.

Table 1. Parameters used in simulations

Observer (Sec. 3.1)	λ_1=3, λ_2=0.2
Guidance (Sec. 4.1)	$k_1 = k_2 = k_3$ =0.1
Guidance (Sec. 4.2)	$k_V = k_\chi = k_\gamma$ =10
Excitation signals (Sec. 4.3)	A_V =0, $A_\chi = A_\gamma = \pi/24$ $T_\chi = T_\gamma$ =2sec
Offset (Sec. 4.1)	$\triangle_1$ =-20m, $\triangle_2$ =10m, $\triangle_3$ =0m

Figure 3 shows the result of the unknown parameter estimation. Initially, the UAV assumes that the target size is $R = 1\text{m}$ $(\boldsymbol{\theta} = 1)$. As shown in Fig. 3, the estimated size of the target converges to the true value due to the nonlinear adaptive observer.

The observer estimates the unknown parameter of the target using measurement states, and therefore the information of the target position and velocity can be provided. Using the estimated information of the target, the UAV can be guided to track the target. Figure 4 shows the trajectories of the two vehicles. Figure 5 shows tracking errors between the UAV and the target. Because the UAV is supposed to track the target with a constant offset, the tracking errors converge to the offset values. The oscillatory motion of the UAV is due to the excitation signals, which is required to estimate the parameter of the target.

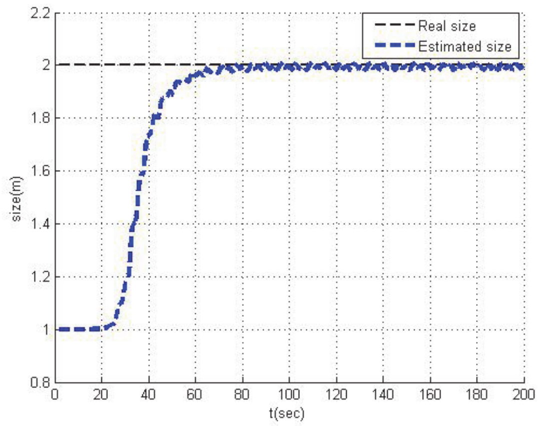

Fig. 3. Result of the parameter estimation

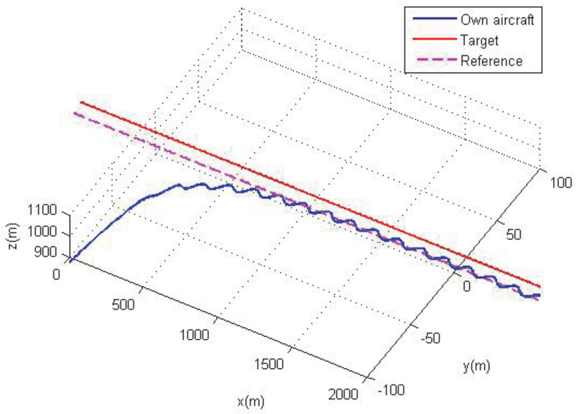

Fig. 4. Trajectories of the two vehicles

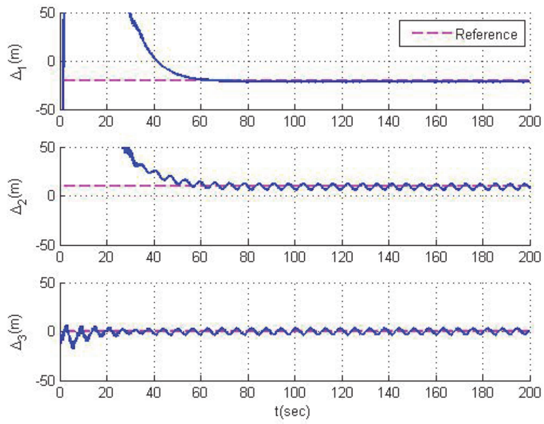

Fig. 5. Tracking errors between the UAV and the target

6 Conclusions

The aerial target tracking problem of a vision-based UAV was considered. A single vision sensor was used for the sensor of the UAV, which has problem of observability. By adopting a measurement model of the vision sensor based on a specific image processing technique, the unknown parameter estimation problem was constructed. Then, nonlinear adaptive observer was adopted to estimate the unknown parameter and states of the target. The nonlinear adaptive observer can be implemented to higher relative degree systems and its stability is guaranteed under the boundedness, Lipschitz, and persistent excitation conditions. Finally, UAV guidance scheme was proposed to track the target by using the result of the nonlinear adaptive observer. The performance of the proposed method was verified through numerical simulations. The proposed vision-based target tracking algorithm can be extended to autonomous and decentralized formation flight or collision avoidance problems of vision-based UAVs. The adaptive nonlinear observer can be implemented to various observation problems related to unknown parameters such as sensor bias, target's uncertain properties, or environmental disturbances. The proposed guidance scheme can be also extended to various vehicle systems including UGVs (Unmanned Ground Vehicles) and AUVs (Autonomous Underwater Vehicles).

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