A PROCEDURE TO DESIGN AIRCRAFT FLIGHT CONTROL LAWS TAKING INTO ACCOUNT ROBUSTNESS USING THE VARIABLE STRUCTURE CONTROL

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Abstract

In this paper, a procedure is presented to design aircraft flight control laws using the Variable Structure Control taking into account robustness to parametric uncertainties and chattering suppression, which is based in clear and straightforward numerical computations and relationships between design and performance parameters. Also, an example is presented to ilustrate the application and effectiveness of the procedure.

1 Introduction

In the aeronautical industry, robust control is usually adopted in the aircraft flight control law design in order to deal, for example, with the variation of aerodynamic coefficients along flight conditions. Some of the used techniques are linear, what place limitations, for example, on the envelope at which a given set of gains is valid.

In this document, a procedure is presented to design aircraft flight control laws, taking into account robustness to parametric uncertainties, using the non linear robust control technique called Variable Structure Control (VSC) or Sliding Mode Control (SMC). The procedure is such that control law adjustment and computation are performed via clear and straightforward numerical procedures and relationships between design and performance parameters.

In order to develop the procedure in question and, also, to ilustrate its application and effectiveness, a particular tracking and stabilization problem is considered for the longitudinal dynamics of an unstable hypersonic aircraft, represented by a nonlinear model.

The remainder of this paper is organized as follows. In section 2, a brief presentation of the main VSC concepts is performed. In section 3, the aircraft model and the control problem considered in the elaboration and presentation of the design procedure in question are presented. In section 4, the VSC controller considered in the design procedure and some of its properties are presented. In section 5, the design procedure and respective numerical implementation tasks are presented. In section 6, results of the application of the design procedure to the control problem considered are presented. Finally, section 7 contains some concluding remarks.

2 Variable Structure Control

Consider a state space and a differential equation that defines trajectories on it. Also, consider that the right-hand side of this differential equation is discontinuous in a manifold within the state space. According, for example, the references [8] and [9], under well defined conditions, trajectories defined by the differential equation that initiate outside the manifold progress in its direction. Once the manifold is reached, if other conditions are satisfied, the trajectories remain constrained to the manifold, in a kind of sliding motion along it. Such motion is called ‘sliding mode’, while the one outside the manifold that progresses in its direction is called ‘reaching mode’. Also, a condition that ensures the existence of the last motion is called a ‘reaching condition’.

As a sliding mode is a motion constrained in a particular manifold within a state space, in control applications, such manifold can be generated in order to ensure some desired behavior to the trajectories of a plant. In this case, the control input shall be such that the respective closed loop differential equation has a discontinuous right-hand side. This can be achieved by a discontinuous control, defined by total state feedback, that commutates instantly when the system closed loop trajectories reach an intended manifold, named ‘switching manifold’; which is composed by surfaces named ‘switching surfaces’, that are the sets of points at which scalar functions called ‘scalar switching functions’ are equal to zero. This discontinuous control is named ‘variable structure control’, which can be elaborated so that a reaching condition to the switching manifold and the conditions to the existence of sliding modes in its switching surfaces are satisfied. Also, the scalar switching functions can be designed in order to obtain sliding modes in the associated switching surfaces with desired properties.

As an example, in the figure 1, the trajectories of a variable structure control system of second order are presented. In such example, there is only one switching surface, which is a straight line with negative slope.

As a sliding mode is constrained to a surface that may be defined arbitrarily, it is reasonable to suppose that such movement is strongly dependent on the parameters that define these surfaces, regardless of the parameters that define the dynamic model of the plant. In fact, as discussed formally in the literature, for example, the references [2] and [8], under well defined conditions, a sliding mode is ‘invariant’ with respect to external disturbances and model uncertainties.

From the discussion above, in the implementation of a variable structure controller, it is necessary to generate a control action that can perform instantaneous commutations when a trajectory reaches a switching surface. However, in general control applications, this can not be achieved. In this case, commutations may be performed after a trajectory reaches a switching surface; what can cause an oscillation around it called ‘chattering’, which is, generally, an undesirable event. According to the reference [5], chattering reduction or suppression is an important research theme and a wide set of approaches has been proposed. Some of such approaches are feasible in general control applications and can eliminate the chattering. However, in these cases, the desirable property of invariance is lost and the sliding mode does not exist, but the control system can still possess acceptable robustness and trajectories with desired properties.

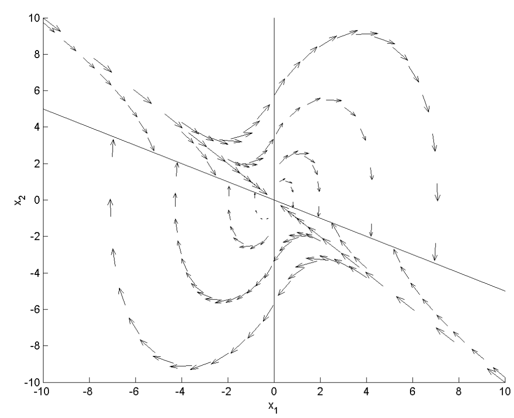

Fig. 1 Variable structure control system state space trajectories

3 Plant and Control Problem

In this section, the plant, a respective model and the control problem considered in the elaboration and ilustration of application of the procedure in question are presented. This plant and control problem are identical the ones treated in the reference [10] and are chosen in order to take advantage of the models, problem formulation, procedures and results presented in this reference.

Through the discussion, imperial units are considered. This metric system is adopted in order to allow better comparison with the reference [10].

3.1 Plant and Respective Model

The plant is a generic hypersonic flight vehicle presented at the reference [6]. It represents the longitudinal dynamics on cruise flight at the altitude $110000 ft$ and Mach 15. In this condition, the longitudinal dynamics is unstable and described by the set of equations 1, which is taken from the reference [10].

$$\begin{aligned}
\dot{V} &= \frac{T\cos\alpha - D}{m} - \frac{\mu \sin\gamma}{r^2} \\
\dot{\gamma} &= \frac{L + T\sin\alpha}{mV} - \frac{(\mu - V^2 r)\cos\gamma}{V r^2} \\
\dot{q} &= \frac{M_{yy}}{I_{yy}} \\
\dot{\alpha} &= q - \dot{\gamma} \\
\dot{h} &= V\sin\gamma
\end{aligned} \tag{1}$$

where V (ft/s), γ (rad), q (rad/s), α (rad) and h (ft) are the state variables speed, path angle, pitch rate, angle of attack and altitude, respectively; T (lbf), D (lbf) and L (lbf) are the thrust, drag and lift forces, respectively; M_{yy} $(lbf \cdot ft)$ is the pitching moment; m $(slugs)$ and I_{yy} $(slug \cdot ft^2)$ are the aircraft mass and the aircraft moment of inertia in the pitch axis, respectively; μ $(1.39 \times 10^{16} ft^3/s^2)$ is the gravitational constant; and $r = R_E + h$ is the distance from the aircraft center of mass until the center of the Earth, where R_E $(20,903,500 ft)$ is the Earth medium radius. D, L and M_{yy} are given by the aerodynamic model presented in the set of equations 2 and T is given by the engine dynamic model presented in the set of equations 3.

$$\begin{aligned}
D &= q_d S C_D, \;\; L = q_d S C_L \;\; M_{yy} = q_d S \bar{c} C_m \\
C_L(\alpha) &= 0.620\alpha \\
C_D(\alpha) &= 0.645\alpha^2 + 0.00434\alpha + 0.00377 \\
C_m &= C_m(\alpha) + C_m(q) + C_m(\delta_e) \\
C_m(\alpha) &= -0.035\alpha^2 + 0.0366\alpha + 5.33 \times 10^{-6} \\
C_m(q) &= \frac{1}{2}\bar{c}V(-6.80\alpha^2 + 0.302\alpha - 0.229)q \\
C_m(\delta_e) &= c_e(\delta_e - \alpha)
\end{aligned} \tag{2}$$

where δ_e (rad) is the control variable elevator angle of deflection, C_D, C_L and C_m are the drag, lift and pitching moment coefficients, respectively; S (ft^2) and $\bar{c}$ (ft) are the aircraft reference area and mean aerodynamic chord, respectively; $q_d = 1/2\rho V^2$ is the dynamic pressure, where ρ $(slugs/ft^3)$ is the air density; and c_e (rad^{-1}) is called 'elevator gain'.

$$\begin{aligned}
T &= q_d S C_T \\
C_T &= \begin{cases} 0.0258\beta & if \;\; \beta \leq 1 \\ 0.0224 + 3.36 \times 10^{-6}\beta & if \;\; \beta > 1 \end{cases} \\
\ddot{\beta} &= -2\zeta\omega_n\dot{\beta} - \omega_n^2\beta + \omega_n^2\beta_c
\end{aligned} \tag{3}$$

where β_c is the control variable throttle setting, C_T is called 'coefficient of thrust', β and $\dot{\beta}$ (s^{-1}) are engine state variables and ζ (0.5) and ω_n $(1 rad/s)$ are the damping ratio and the undamped natural frequency of the engine dynamic model, respectively.

As performed in the reference [10], m, I_{yy}, S, $\bar{c}$, c_e and ρ are considered uncertain parameters whose values can range in respective intervals defined by the set of equations and inequations 4.

$$\begin{aligned}
m &= m_n(1 + \Delta m), & |\Delta m| &\leq 0.03 \\
I_{yy} &= I_{yy_n}(1 + \Delta I_{yy}) & |\Delta I_{yy}| &\leq 0.02 \\
S &= S_n(1 + \Delta S), & |\Delta S| &\leq 0.03 \\
\bar{c} &= \bar{c}_n(1 + \Delta \bar{c}) & |\Delta \bar{c}| &\leq 0.02 \\
c_e &= c_{e_n}(1 + \Delta c_e), & |\Delta c_e| &\leq 0.02 \\
\rho &= \rho_n(1 + \Delta \rho), & |\Delta \rho| &\leq 0.03
\end{aligned} \tag{4}$$

where m_n $(9375 slugs)$, I_{yy_n} $(7 \times 10^6 slug \cdot ft^2)$, S_n $(3603 ft^2)$, $\bar{c}_n$ $(80 ft)$, $c_{e_n} = 2.92 \times 10^{-2} rad^{-1}$ and ρ_n $(2.432 \times 10^{-5} slugs/ft^3)$ are the nominal values of m, I_{yy}, S, $\bar{c}$, c_e and ρ, respectively, and Δm, ΔI_{yy}, ΔS, $\Delta \bar{c}$, Δc_e and $\Delta \rho$ are possible fractional increments in m, I_{yy}, S, $\bar{c}$, c_e and ρ, respectively, with respect to their nominal values.

Combining the equations given in 1, 2 and 3, a differential equation in vector form for the longitudinal dynamics is obtained, as presented in 5.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} \tag{5}$$

In the equation 5, $\mathbf{x}$ is the longitudinal dynamics state vector, defined in 6, $\mathbf{u}$ is the longitudinal dynamics control vector, defined in 7, $\mathbf{f}(\mathbf{x})$ is called 'state function' and defined in 8 and $\mathbf{B}(\mathbf{x})$ is called 'control gain matrix' and defined in 9.

$$\mathbf{x} = \begin{pmatrix} V & \gamma & q & \alpha & h & \beta & \dot{\beta} \end{pmatrix}^T \tag{6}$$

$$\mathbf{u} = (\beta_c \;\; \delta_e) \tag{7}$$

$$\mathbf{f} = \begin{pmatrix} \frac{(C_T(\beta)\cos\alpha - C_D(\alpha))q_d S}{m} - \frac{\mu \sin\gamma}{r^2(h)} \\ \frac{(C_L(\alpha)+C_T(\beta)\sin\alpha)q_d S}{mV} - \frac{(\mu - V^2 r(h))\cos\gamma}{V r^2(h)} \\ \frac{(C_m(\alpha)+C_m(q)-c_e\alpha)q_d S\bar{c}}{I_{yy}} \\ q - \frac{(C_L(\alpha)+C_T(\beta)\sin\alpha)q_d S}{mV} + \frac{(\mu - V^2 r(h))\cos\gamma}{V r^2(h)} \\ V\sin\gamma \\ \dot{\beta} \\ -2\zeta\omega_n\dot{\beta} - \omega_n^2\beta \end{pmatrix} \tag{8}$$

$$\mathbf{G} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{q_d S\bar{c}}{I_{yy}} c_e \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \omega_n^2 & 0 \end{pmatrix} \tag{9}$$

From the equations 5 to 9, the longitudinal dynamics is a system non-linear with respect to the state vector and linear with respect to the control input.

3.2 Control Problem

The control problem is to elaborate a VSC control law for the longitudinal dynamics described in the subsection 3.1, in order to satisfy, for any combination of uncertain parameters allowed by the equations and inequations given in 4, all the following requirements: the closed loop longitudinal dynamics shall be stable; the state variables V and h shall track desired patterns, defined by steps in their respective equilibrium values, with zero or negligible overshoots and steady state tracking errors; the chattering shall not exist and the control inputs shall not saturate.

4 Variable Structure Controller

The control law considered in the procedure in question is presented in the reference [7] and it is suitable for single or multiple outputs tracking problems. In this section, it is presented the scalar switching functions and the reaching conditions related to such control law and discussed their main properties; there are presented conditions that ensures that the closed loop control system is robust with respect to parametric uncertainties, as required in the subsection 3.2; and discussed an approach for chattering suppression.

4.1 Scalar Switching Functions

Consider a multiple output tracking problem for a given dynamic system. If e_i is the traking error for an output y_i relative to a respective desired pattern y_{d_i} and r_i is the relative degree[1] of the dynamic system with respect to y_i, it is defined the following scalar swicthing function.

$$s_i(t) = \left(\lambda_i + \frac{d}{dt}\right)^{r_i} \int_{t_0}^{t} e_i(\tau)d\tau \tag{10}$$

where λ_i is a real number that can be treated as a design parameter.

For $s_i(t) = 0$, it is obtained an homogeneous linear differential equation of degree r_i that describes the response of e_i during sliding mode, whose solution is given by the equation 11.

$$e_i(t) = k_{i_1}e^{-\lambda_i t} + k_{i_2}te^{-\lambda_i t} + \ldots + k_{i_{r_i}}t^{r_i-1}e^{-\lambda_i t} \tag{11}$$

where $k_{i_1}, k_{i_2}, \ldots$ are constants determined by initial conditions.

The equation 11 shows that the response of e_i during sliding mode is critically damped, with time constant related to the parameter λ_i and steady state error equal to zero. Also, the response of e_i during sliding mode is stable for $\lambda_i > 0$ and depends only on λ_i and initial conditions.

4.2 Reaching Conditions

One reaching condition for each scalar switching function (SSF) is defined. Each one has an identical form as presented in 12 for the SSF s_i.

$$s_i\dot{s}_i \leq -\eta_i|s_i| \tag{12}$$

[1] Intuitively, the relative degree of a dynamic system with respect to a given output y_i is the number of times that y_i has to be differentiated with respect to time in order to at least one control input to appear explicitly in the differentiation.

where η_i is a positive real number that can be treated as a design parameter.

If $s_i(t_0)$ is the value of s_i at the time t_0, it can be shown that the reaching time t_{r_i} of the switching surface $s_i = 0$, when the condition presented in 12 is satisfied, is finite and given by 13.

$$t_{r_i} \leq \frac{|s_i(t_0)|}{\eta_i} \tag{13}$$

The equation 13 shows that, if the order of magnitude of $s_i(t_0)$ is known, the design parameter η_i can be chosen in order to define an order of magnitude for maximum value of t_{r_i}.

4.3 Control Law

To present the control law, a tracking problem with two outputs and control inputs is considered as stated in the subsection 3.2.

The control law is defined in order to satisfy the proposed reaching conditions. As can be shown, the reaching condition for the SSF s_i depends only in $\dot{s}_i$, which, from the definitions of s_i and e_i ($e_i = y_i - y_{d_i}$) and the concepts of relative degree and Lie derivative [2], is given by 14.

$$\begin{aligned} \dot{s}_i \;=\; & L_{\mathbf{f}}^{r_i} y_i + (L_{\mathbf{g}_1} L_{\mathbf{f}}^{r_i-1} y_i) u_1 + (L_{\mathbf{g}_2} L_{\mathbf{f}}^{r_i-1} y_i) u_2 - \\ & y_{d_i}^{(r_i)} + r_i \lambda_i \left(L_{\mathbf{f}}^{r_i-1} y_i - y_{d_i}^{(r_i-1)} \right) + \\ & \frac{r_i(r_i-1)}{2!} \lambda_i^2 \left(L_{\mathbf{f}}^{r_i-2} y_i - y_{d_i}^{(r_i-2)} \right) + \\ & \ldots + \lambda_i^{r_i} (y_i - y_{d_i}) \end{aligned} \tag{14}$$

where $\mathbf{f}$ is the state function, $\mathbf{g}_i$, the i-th column of the control gain matrix, u_i, the i-th control input and $L_{\mathbf{f}}^{r} y$, the r-th Lie derivative of the scalar function y with respect to the vector function $\mathbf{f}$.

From $\dot{s}_i$ given by 14, it can be shown that a control vector that defines a closed loop system that satisfies the reaching conditions imposed to s_1 and s_2 is given by 15 and satisfies 16.

$$\begin{aligned} \mathbf{u}(\mathbf{x}) &= \mathbf{u}_{eq}(\mathbf{x}) + \mathbf{u}_{des}(\mathbf{x}) \\ \mathbf{u}_{eq}(\mathbf{x}) &= -\mathbf{B}(\mathbf{x})^{-1} \begin{pmatrix} v_1(\mathbf{x}) \\ v_2(\mathbf{x}) \end{pmatrix} \\ \mathbf{u}_{dis}(\mathbf{x}) &= -\mathbf{B}(\mathbf{x})^{-1} \begin{pmatrix} k_1 \mathrm{sgn}(s_1) \\ k_2 \mathrm{sgn}(s_2) \end{pmatrix} \end{aligned} \tag{15}$$

$$k_i \geq \eta_i \tag{16}$$

where $v_i(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ are given in 17.

$$\begin{aligned} \mathbf{B}(\mathbf{x}) &= \begin{pmatrix} L_{\mathbf{g}_1} L_{\mathbf{f}}^{r_1-1} y_1 & L_{\mathbf{g}_2} L_{\mathbf{f}}^{r_1-1} y_1 \\ L_{\mathbf{g}_1} L_{\mathbf{f}}^{r_2-1} y_2 & L_{\mathbf{g}_2} L_{\mathbf{f}}^{r_2-1} y_2 \end{pmatrix} \\ v_i(\mathbf{x}) &= L_{\mathbf{f}}^{r_i} y_i - y_{d_i}^{(r_i)} + r_i \lambda_i \left(L_{\mathbf{f}}^{r_i-1} y_i - y_{d_i}^{(r_i-1)} \right) \\ & \quad + \frac{r_i(r_i-1)}{2!} \lambda_i^2 \left(L_{\mathbf{f}}^{r_i-2} y_i - y_{d_i}^{(r_i-2)} \right) + \\ & \quad \ldots + \lambda_i^{r_i} (y_i - y_{d_i}) \end{aligned} \tag{17}$$

In 15, $\mathbf{u}_{eq}$ is the ‘equivalent control’, that is such that $\dot{s}_i = 0$ when $s_i = 0,\ i = 1, 2$; $\mathbf{u}_{dis}$ is the ‘discontinuous control’, that is an ‘ideal relay control’; k_1 and k_2 are the control gains; and sgn is the ‘signal function’, which is such that $\mathrm{sgn}(s) = 1$ if $s > 0$, $\mathrm{sgn}(s) = -1$ if $s < 0$.

It can be shown that a closed loop system defined by the control law in question is stable in the sense of Lyapunov for $k_1 = k_2$, according the concept of stability in the sence of Lyapunov for VSC systems presented in the reference [9].

4.4 Reaching Conditions in Presence of Parametric Uncertainties

To obtain the condition given in 16, the control law is determined from a model that perfectly describes the plant. If any discrepancy between such model and the real plant behavior exists, the condition to be satisfied may differ from the one presented. In fact, in real applications, discrepancies exist, due to, e.g., uncertain parameters, external disturbancies or unmodeled dynamics. In the procedure in question, only discrepancies caused by uncertain parameters whose real values are limited as presented in the subsection 3.1 are considered.

[2] The definition of Lie derivative can be found in references such as the reference [7]. Briefly, Lie derivatives are defined by these recursive relations: $L^r_{\mathbf{f}(\mathbf{x})} y(\mathbf{x}) = L_{\mathbf{f}(\mathbf{x})} L^{r-1}_{\mathbf{f}(\mathbf{x})} y(\mathbf{x})$, $L^{(r-1)}_{\mathbf{f}(\mathbf{x})} y(\mathbf{x}) = L_{\mathbf{f}(\mathbf{x})} L^{r-2}_{\mathbf{f}(\mathbf{x})} y(\mathbf{x}), \ldots, L^1_{\mathbf{f}(\mathbf{x})} h(\mathbf{x}) = L_{\mathbf{f}(\mathbf{x})} h(\mathbf{x}) = \nabla_{\mathbf{x}} y(\mathbf{x})^T \mathbf{f}(\mathbf{x})$. Where $L^r_{\mathbf{f}(\mathbf{x})} h(\mathbf{x})$ is read as the Lie Derivative of order r of the scalar function h with respect to the vector field $\mathbf{f}$

As shown in the reference [1], the reaching conditions are satisfied, for any combination of uncertain parameters allowed by limitations as such presented in 4, if k_1 is given by 18 and k_2, by an analogous expression.

$$\begin{aligned} k_1 \quad = \quad & \max_{\Delta\mathbf{p}_u} \Bigg(\frac{\Delta B_{22}}{\Delta B_{11}\Delta B_{22} - |\Delta B_{12}|\,|\Delta B_{21}|} (\eta_1 + \\ & |1-\Delta B_{11}|\,|\hat{v}_1| + |\Delta B_{12}|\,|\hat{v}_2| + |\Delta v_1| + \\ & + \frac{|\Delta B_{12}|}{\Delta B_{22}} (\eta_2 + |\Delta B_{21}|\,|\hat{v}_1| + \\ & |1-\Delta B_{22}|\,|\hat{v}_2| + |\Delta v_2|)) \Bigg) \end{aligned} \tag{18}$$

where,

$$\Delta\mathbf{p}_u = \mathbf{p}_u - \hat{\mathbf{p}}_u \tag{19}$$

$$\Delta v_i = v_i(\mathbf{x}, \Delta\mathbf{p}_u) - \hat{v}_i(\mathbf{x}) \tag{20}$$

$$\begin{pmatrix} \Delta B_{11} & \Delta B_{12} \\ \Delta B_{21} & \Delta B_{22} \end{pmatrix} = \mathbf{B}(\mathbf{x}, \Delta\mathbf{p}_u)\hat{\mathbf{B}}^{-1}(\mathbf{x}) \tag{21}$$

In 19, $\mathbf{p}_u$ is a vector of real values of the uncertain parameters and $\hat{\mathbf{p}}_u$ is a vector of their respective nominal values. In 20, $\hat{v}_i$ is the nominal realization of the function presented in 17, determined by $\hat{\mathbf{p}}_u$, and v_i is its real realization, determined by $\hat{\mathbf{p}}_u$ and a set of increments $\Delta\mathbf{p}_u$, the same idea is valid for 21. Also, in 18, the maximum is constrained to the limits considered for the elements of $\Delta\mathbf{p}_u$.

4.5 Chattering Suppression

As discussed earlier, a control action that performs instantaneous commutations is, generally, not feasible, resulting in delays that may cause chattering. In the scenario in question, even in the case of an ideal control, the chattering is generated, because, in the presence of parametric uncertainties, $\dot{s}_i$ is not always zero when $s_i = 0$, since the equivalent control is such that $\dot{s}_i = 0$ when $s_i = 0$ for the nominal plant.

In order to suppress the chattering, the continuation approach, discussed, e.g., in the reference [5], is used. In this approach, a discontinuous control is substituted by a continuous approximation. The result is such that the conditions for the existence of sliding modes are not satisfied. However, if reaching conditions are satisfied, a region, of well defined width, surrounding both sides of each switching surface is reached, in a time smaller than the switching surfaces reaching time. This region is called 'boundary layer' and is such that, after it is reached, the system trajectories remain confined in its interior while the reaching conditions are satisfied. Also, the boundary layer width decreases as the continuous control tends to the discontinous one, and the trajectories inside the boundary layer tend to the sliding mode responses as the boundary layer width tends to zero.

To implement a continuous control, as proposed in the reference [7], the ideal relay control is changed by an 'ideal saturation control' given by 22.

$$\mathrm{sat}(s_i) = \begin{cases} \mathrm{sgn}(s_i), & if\ |s_i| > \phi_i \\ s_i/\phi_i, & otherwise \end{cases} \tag{22}$$

where $2\phi_i > 0$ is the boundary layer width around the switching surface $s_i = 0$.

As discussed in the reference [7], in the interior of the boundary layer, for $e_i(0) = \dot{e}_i(0) = \ldots = e_i^{(r_i)}(0) = 0$, e_i is limited as given in 23.

$$|e_i(t)| \leq \frac{\phi_i}{\lambda_i^{r_i - 1}},\ for\ t \geq 0 \tag{23}$$

5 Design Procedure and Numerical Implementation

In this section, a procedure to adjust and compute the control law presented in the section 4 is presented. The section will cover: the computation of relative degree and Lie derivatives; the adjustment of parameters λ_i, η_i and ϕ_i; the computation of closed loop responses and the maximization of gains k_i.

5.1 Relative Degree and Lie Derivatives Computation

From section 4, the system relative degrees with respect to the tracked variables are necessary to compute the scalar switching functions and the control law in question. These relative degrees can be determined via successive computation of tracked variables time derivatives, what can be done by algebraic computation of Lie derivatives, that can be performed using a symbolic computation software like Mathematica.

From equation 17, to compute the control law, it is necessary to evaluate the Lie derivatives; which can be performed via algebraic or numerical computation of partial derivatives. As discussed in reference [1], depending on the plant mathematical model, the algebraic expressions of the partial derivatives in question can be highly non linear and reasonably long, what may lead to high truncation errors. In other hand, numerical computation of partial derivatives may lead to high discretization errors.

From the results presented in the reference [1], it is verified that at least in the control problem in question, errors introduced by numerical computation of partial derivatives are smaller than errors introduced by truncation in the evaluation of Lie derivatives algebraic expressions. Also, in this reference, in order to reduce the error due to numerical computation of partial derivatives, there are derived equations for Lie derivatives using the notation of 'systems' [3] (which include as special cases the tensors). In such equations, a Lie derivative is determined by partial derivatives one degree lower than the partial derivatives required in the evaluation of the Lie derivative from Lie derivative definition. Also, rules for numerical computation of derivatives are compared and differentiation steps are chosen in order to reduce numerical differentiation errors.

The numerical approach discussed above differs from an algebraic approach adopted in the reference [10], in which Lie derivatives are computed via reasonably complex algebraic expressions.

5.2 Parameters Adjustment and Evaluation

To implement the control law in question, besides to compute relative degrees with respect to tracked variables and evaluate Lie derivatives, it is necessary to adjust the parameters λ_i, η_i and ϕ_i and maximize the gains k_i.

Initial values to the parameters λ_i, η_i and ϕ_i can be determined from the relationships between them and performance parameters presented in the section 4. After, their values can be adjusted via computation and observation of closed loop responses; which, in flight control law design, should be done via methods for computation of numerical solution of stiff differential equations, since, in this case, scalar switching functions and plant dynamics can differ by several orders of magnitude.

Concerning the maximization of a gain k_i, this can be performed by trying to find the respective global maximum, using 16, in the region defined by the limits imposed to the increments in the nominal values of the uncertain parameters, via a combination of genetic and gradient algorithms. This approach differs from the one presented in the reference [10], in which a gain k_i is determined algebraically using approximations, such as linearization via Taylor Series.

5.3 Proposed Procedure

From the discussion presented in the last two subsections, it is proposed that the design procedure is defined by the following steps.

1: Computation of the plant model relative degrees with respect to the tracked variables;

2: Definition of initial values to the parameters η_i and λ_i from expected reaching times and transient responses during sliding mode, respectively, and adjustment of such parameters by computation and observation of closed loop responses for the nominal plant with discontinuous control;

3: Maximization of the gains k_i using successive tentatives to find respective global maximums via genetic algorithms and improvement of the results via maximization using gradient algorithm with initial guesses equal to the genetic algorithm results;

[3] Definitions of system and tensor can be found in references such as the reference [3].

4: Definition of initial values to the parameters ϕ_i from expected maximums tracking errors inside the boundary layer, and respective adjustment via closed loop responses computation and observation, taking into account a non nominal plant.

In the procedure above, to perform the evaluation of the control law, it is proposed the numerical computation of Lie derivatives via the equations presented in the reference [1] derived using the notation of systems. Also, to determine closed loop responses, the use of a numerical method for solution of stiff differential equations is suggested .

6 Results

This section covers the application of the procedure presented in the last section to the control problem defined in the section 3 and presented numerical and graphical results.

From algebraic computation of time derivatives, applying the concept of Lie derivative, performed using symbolic computation via the software Mathematica, it is found that the relative degrees r_1 and r_2 are equal to 3 and 4, respectively.

The parameters λ_1 and λ_2 are determined, from time constants equal to $3s$, where each time constant is the one obtained when the sliding mode response given by the equation 11 is the one of a first order system. So, it is taken $\lambda_1 = \lambda_2 = 1/3Hz$.

The parameters η_1 and η_2, that define the gains k_1 and k_2 when the nominal plant is considered, are computed from the equation 13, taking $s_1(0) = s_2(0) = 1 \times 10^{-3}$ and $t_{r_1} = t_{r_2} = 1s$. So, $\eta_1 = \eta_2 = 1 \times 10^{-3}$.

The values for λ_1, λ_2, η_1 and η_2 presented above define sliding mode responses for the nominal plant, taking $k_1 = k_2 = \eta_1 = \eta_2$, that safisfy the control problem. Therefore, they are adopted in the remaining design procedure steps.

To compute the responses refered above, and all other closed loop responses discussed in this section, it is used the MATLAB function for computation of numerical solutions of stiff differential equations called `ode15s`.

To maximize the gains k_1 and k_2, it is used a MATLAB genetic algorithm toolbox presented in the reference [4] and the MATLAB function `fmincon`, that implements a gradient method. To perform the evaluation, it is considered a particular point in the longitudinal dynamics state space, that is the equilibrium point for the flight condition Mach 15 and $110000ft$. In spite of this maximum is computed for a particular point, as the control gains are continuous with respect to the state variables, the computed maximums are, at least, approximately constant in a region around the point.

The set of increments that maximize a control gain is called 'critical parametric combination' (CPC). For both control gains, using the approach presented in the subsection 5.2, it is found the same CPC, which is defined by the set of fractional increments $(0.03 \quad 0.02 \quad -0.03 \quad -0.02 \quad 0.02 \quad -0.03)$, where the order is that presented at the equation 4. In the same context, in the reference [10], using the algebraic and approximated approach refered in the subsection 5.2, it is found the CPC $(-0.03 \quad -0.02 \quad 0.03 \quad 0.02 \quad 0.02 \quad 0.03)$. This discordance is investigated in the reference [1], where it is verified that the former CPC determines greater control gains with respect to the last one.

The parameters ϕ_1 and ϕ_2 are determined from the equation 23 taking maximum values for e_1 and e_2 equal to $1ft/s$ and $20ft$. So, $\phi_1 = 1/9$ and $\phi_2 = 20/27$.

To verify the control law defined by the parameters presented above, it is computed the closed loop response for the flight condition Mach 15 and $110000ft$ taking into account the non nominal plant defined by the CPC presented above. Also, it is considered references for V and h determined by steps of amplitude $100ft/s$ and $2000ft$ at the respective values on the flight condition.

Results obtained in the scenario in question are presented in the figures 2 and 3.

In the figure 2, `DV` and `Dh` are the increments in V and h with respect to the trimmed condition. Also, in the figure 3, s_1, s_2, `beta` and `delta`$_e$ are the responses of the scalar switching functions and control variables.

In the figure 3, $|s_1| < 1/9$ and $|s_2| < 20/27$ for every t. So, the closed loop responses are always inside the boundary layer. Also, there is no manifestation of chattering.

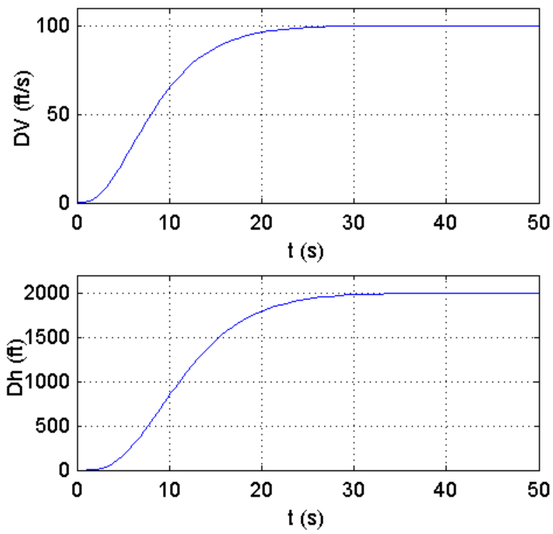

Fig. 2 Tracked variables

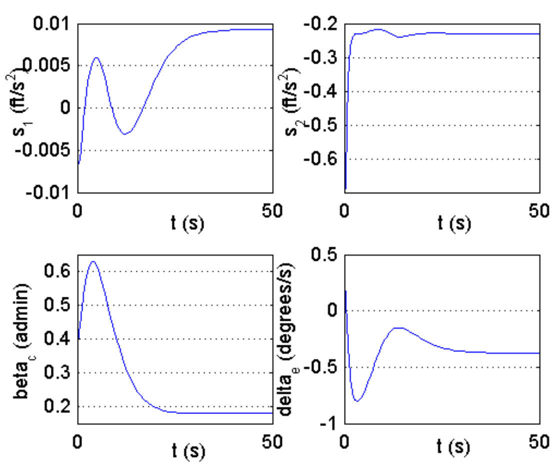

Fig. 3 Scalar switching functions and control variables

In the figure 2, the transient responses of `DV` and `Dh` are close to responses of first order systems and the steady state tracking errors have negligible amplitude, similar to the sliding mode responses. Also, the closed loop system is, at least, BIBO-stable. Additionally, from the figure 3, there is no saturation in the control variables. These results are very similar to the ones presented in the reference [10].

As the CPC has the following property (reference [7]): it is the combination of parametric uncertainties (CPU) that requires the greatest control gains in order to the reaching conditions be satisfied; the closed loop system trajectories remain inside the boundary layer of any other CPU. So, for any other CPU, the system responses are similar to that presented above and the control problem is satisfied.

7 Conclusion

A procedure to design aircraft flight control laws, using VSC, taking into account robustness to parametric uncertainties and chattering suppression is presented.

The procedure is implemented from straightforward numerical computations and clear relationships between performance and design parameters.

A control problem that illustrates the application of the procedure and defines results demonstrates the procedure effectiveness.

Also, new approaches, with respect to the ones presented in the reference [10], for robust control gains evaluation and for VSC control law computation are discussed.

Concerning future developments, the following is considered:

- In order to perform a better evaluation of the control solution and design procedure, it is proposed comparisons with other non linear and linear robust control techniques, such as backstepping and LQG/LTR;
- Due to the complexity of the proposed control law, that makes its analogical implementation difficult, study of the respective digital implementation is proposed;
- The application of the control solution and procedure in question to models of civil and militar aircrafts is also proposed.

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8 Copyright Statement