STUDY OF FUNDAMENTAL LAMB MODES BEHAVIOUR WITH A SHARP CHANGE OF SECTION IN A PLATE

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Abstract

In order to improve transport safety in aircrafts by a fast and efficient way, a possible solution is to embed a system of piezoelectric transducers that generates ultrasonic Lamb waves in the structure. Indeed, the interaction of Lamb waves with damage enables its detection. However, due to the properties of Lamb waves, the understanding of these interaction phenomena is complex. Therefore, the problem of Lamb modes interaction with a sharp change of section in a plate is analysed in this paper with the help of a new hybrid method based on the finite element method and the normal mode analysis.

1 Introduction

Lamb waves [1] are more and more utilized in non-destructive applications because of their attractive characteristics. Since they are guided ultrasonic waves, they have the ability to propagate over long distances in plate-like structures and to be sensitive to different kinds of damage [2]. Consequently, their use allows a fast and an efficient inspection of large structures, unlike conventional NDE techniques based on ultrasonic bulk or surface waves. However, measurement analyzes remain relatively difficult since Lamb waves have a multimode and dispersive nature.

The main aim of this research effort is to treat a simple damage case, i.e. a sharp change of section. The study of such a simplified damage has for objective a deeper understanding of the interaction phenomena in the case of the propagation of the first symmetrical and anti-symmetrical Lamb waves modes.

After a short description of the Lamb waves theory in section 2, section 3 presents the finite element numerical modelling developed to allow the computation of transmitted and reflected transient signals at different positions along the structures. Section 4 deals with the normal mode analysis developed to compute the reflection and transmission coefficients from the preceding computed signals. Moreover, these coefficients are quantified for different thicknesses of section and the obtained results for both fundamental Lamb modes are discussed in section 5.

2 Lamb waves theory

Lamb waves are guided elastic perturbations propagated in a plate with free boundaries. They can be usually separated in two families [1], [3]: the symmetric and the anti-symmetric waves. The characteristic equations of these symmetric and anti-symmetric modes can be expressed respectively as:

$$\begin{aligned}&\left(k_s^2+s_s^2\right)^2\cosh(q_s d)\sinh(s_s d)\\&-4k_s^2 q_s s_s\sinh(q_s d)\cosh(s_s d)=0\end{aligned}\tag{1}$$

$$\begin{aligned}&\left(k_a^2+s_a^2\right)^2\sinh(q_a d)\cosh(s_a d)\\&-4k_a^2 q_a s_a\cosh(q_a d)\sinh(s_a d)=0\end{aligned}\tag{2}$$

With : $q_{s,a}=\sqrt{k_{s,a}^2-k_L^2}$, $s_{s,a}=\sqrt{k_{s,a}^2-k_T^2}$. d is the half-thickness of the plate, k_T and k_L correspond to the wave number of the transversal and longitudinal waves respectively and k_s and k_a are wave numbers for symmetric and anti-symmetric modes respectively.

From these equations, the wave number dispersion curves of Lamb waves as function of the frequency-half-thickness product can be computed (see Fig. 1). The multimode and dispersion nature of Lamb modes can be noticed. In this paper, the used frequency range for the study is reduced around 0.6 MHz.mm in order to generate only the A_0 and S_0 modes. This allows also to work in a few dispersive region.

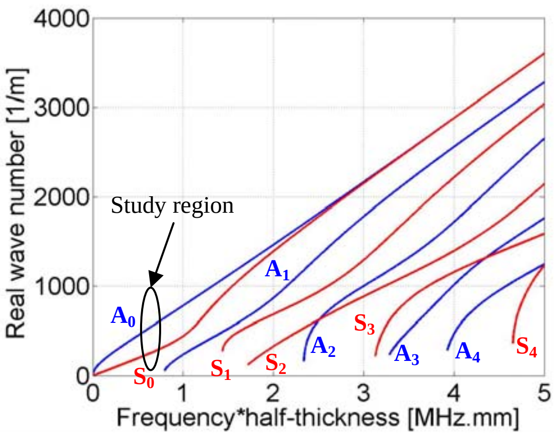

Figure 1 : Dispersion curves of Lamb waves in an aluminium plate.

3 Finite element method

In order to study the behaviour of Lamb waves when they interact with damage, displacements shape must be computed. These displacements are determined with a numerical method based on the Finite Element Method (FEM). In this study, a transient analysis is used and displacement values are derived from the following equation [4]:

$$[M]\underline{\ddot{U}}+\frac{1}{\omega_0}[K''_{uu}]\underline{\dot{U}}+[K'_{uu}]\underline{U}=\underline{F} \qquad (3)$$

Where $[K_{uu}]$ is the stiffness matrix and K'_{uu} and K''_{uu} are the real and imaginary parts respectively, $[M]$ is the consistent mass matrix, $\underline{U}$ and $\underline{F}$ are vectors of the nodal values of the components of the displacement field and applied forces respectively and ω_0 is the angular frequency at which materials losses are defined.

The models used to study the sharp changes of section are presented in Figure 2. Figure 2.a represents the meshed structure with a symmetric sharp change of section (SSS), whereas Figure 2.b shows the case of an anti-symmetric sharp change of section (ASS). The thickness of the plate in the two cases changes from *2d* to *2d p*, and *p* takes values from 0 to 1. The length of the structure is *2L*.

In both cases, the through thickness displacements shape of a transient selected Lamb mode is calculated and applied to the left edge nodes of the plate. In this paper, the study is developed by exciting either the A_0 mode or the S_0 mode.

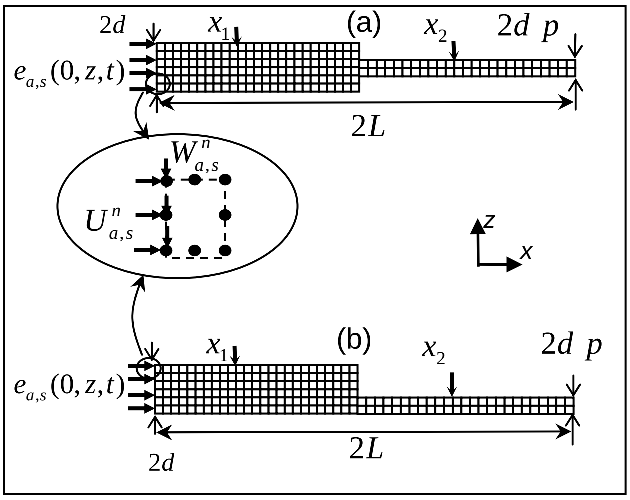

Fig. 2 : Mesh of an aluminium plate with a) a SSS and b) an ASS damage.

When the computation is performed, signals are acquired in two regions: at x_1 for the incident and reflected waves and at x_2 for the transmitted waves. These two points are both far from the edges of the plate and the sharp change of the section.

4 Reflection and transmission coefficients determination

The objective of this section is to compute reflection and transmission coefficients only from the surface displacements. For this purpose, a hybrid method based on the normal modes analysis and the FEM surfaces displacements is developed.

4.1 Hybrid method

From the Normal Mode Expansion Method (NMEM) [5], it can be shown that the acoustical power transported by the m^{th} mode at a distance x in harmonic regime can be obtained by:

$$P_m(x) = \left|a_m(x)\right|^2 P_{mm} \tag{4}$$

Where $a_m(x)$ is the modal amplitude, and P_{mm} is the average acoustic power flux per unit length (in 2-D) associated to the m^{th} mode, given by [6]:

$$P_{mm} = -\frac{1}{2}\mathrm{Re}\left\langle \int_{-d}^{+d} \left(V_x^* \sigma_{xx} + V_z^* \sigma_{xz}\right)_m dz \right\rangle \tag{5}$$

Where 'Re' denotes the real part, (*) is the complex conjugate, V_x and V_z represent the displacements velocities for tangential and normal displacements respectively and σ_{xx} and σ_{xz} are the stresses.

The advantage of this method is that the modal amplitudes of each mode can be derived directly from the surface displacements such as:

$$a_m(x) = \frac{M_m^{FEM}(x)\big|_{z=+d}}{M_m^{ANA}(x)\big|_{z=+d}} \tag{6}$$

Where *ANA* denotes the analytical method and M represents the normal or the tangential displacement maximum of the m^{th} mode.

Once the acoustical power is known, the classical reflection (R) and transmission (T) coefficients can be obtained easily. Two cases can be distinguished: first these coefficients can be computed for the excitation mode directly. Second, these coefficients can be also obtained for any mode issued from the excitation mode at the sharp change of section.

Hence, for the non converted modes, R and T are given at the central frequency by:

$$R_n\big|_{f_c}, T_n\big|_{f_c} = \frac{P_{nn}^{re,tr}\big|_{f_c}}{P_{nn}^{in}\big|_{f_c}} \tag{7}$$

Whereas for the converted modes, R and T can be expressed by:

$$R_m\big|_{f_c}, T_m\big|_{f_c} = \frac{P_{mm}^{re,tr}\big|_{f_c}}{P_{nn}^{in}\big|_{f_c}} \tag{8}$$

n corresponds to the excitation mode and m corresponds to the mode issued from the conversion. *in*, *re* and *tr* denote the terms incident, reflected and transmitted respectively and f_c is the central frequency.

By using equations 4, 6, 7 and 8, results lead for the non converted mode to:

$$R_n = \frac{M_n^{FEM,re}(x)\big|_{z=+d}}{M_n^{FEM,in}(x)\big|_{z=+d}} = \left|K_{R,n}^{MEF}\right|_{z=+d}^2 \tag{9}$$

$$T_n = \left|\frac{K_{T,n}^{MEF}}{K_{T,n}^{ANA}(f_c)}\right|_{z=+d}^2 \frac{P_{nn}^{ANA}(f_c)\big|_{2d.p}}{P_{nn}^{ANA}(f_c)\big|_{2d}} \tag{10}$$

For the converted modes, we obtain:

$$R_m = \left|\frac{K_{R,(m/n)}^{MEF}}{K_{R,(m/n)}^{ANA}(f_c)}\right|_{z=+d}^2 \frac{P_{mm}^{ANA}(f_c)\big|_{2d}}{P_{nn}^{ANA}(f_c)\big|_{2d}} \tag{11}$$

$$T_m = \left|\frac{K_{T,(m/n)}^{MEF}}{K_{T,(m/n)}^{ANA}(f_c)}\right|_{z=+d}^2 \frac{P_{mm}^{ANA}(f_c)\big|_{2d\ p}}{P_{nn}^{ANA}(f_c)\big|_{2d}} \tag{12}$$

$K^{MEF}_{R,(m/n)}$ and $K^{MEF}_{T,(m/n)}$ are the reflection and transmission ratios for the m^{th} mode with respect to the n^{th} mode computed numerically (FEM). $K^{ANA}_{R,(m/n)}$ and $K^{ANA}_{T,(m/n)}$ are the reflection and transmission ratios for the m^{th} mode with respect to the n^{th} mode computed analytically (ANA).

4.2 Signal processing

In order to quantify and separate each propagated mode, displacement signals must be treated with a power signal-processing tool. This tool is the Two-Dimensional Fourier Transform [7] (2D-FT) that allows a representation of temporal-spatial signals in the frequency-wave number domain. Then, the reflection and the transmission coefficients of any propagated Lamb mode can be computed by using the hybrid method presented previously The 2D-FT used for the analysis has the form:

$$S(k,f)=\int_x\int_t s(x,t)w(x)e^{-j(2\pi ft+kx)}dxdt \qquad (13)$$

Where x and t represents the space and the time respectively, k and f are the wave number and the frequency respectively. $s(x,t)$ is a spatial-temporal signal and $w(x)$ is the spatial Hanning function used to eliminate secondary lobes.

The application of the 2D-FT needs the acquisition of spatial-temporal signals. These signals are acquired on the plate surface at different positions along one direction with a fixed step. An example of an ASS damage is shown on figure 3.

For this example, the excitation signal corresponds to the A_0 mode and contains a tone burst of 10 sinusoidal cycles of frequency 0.6 MHz.mm windowed by Hanning function. Moreover, the step between each position is equal to 0.5 mm and the parameter p is equal to 1/2. On this figure, the incident mode is clearly identified but the reflected A_0 and S_0 modes are overlapped and their quantification is not possible at this stage.

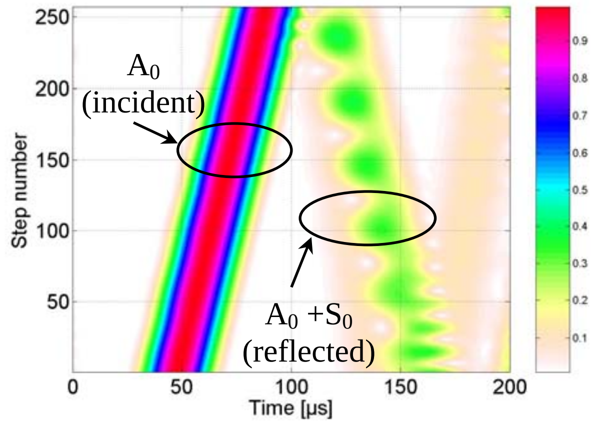

Figure 3 : spatial-temporal signals before an ASS damage when the A_0 mode is excited.

The result of the application of the 2D-FT on the spatial-temporal signal acquired before the damage (Fig. 3) is shown on figure 4. In this case, all the modes are easily identified. Therefore, reflection and transmission coefficients can be computed.

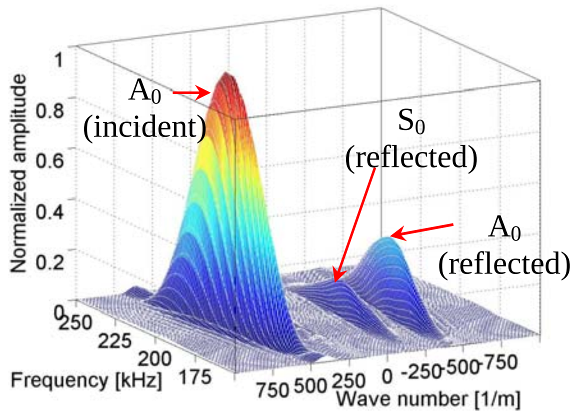

Figure 4 : 2D-FT of the spatial-temporal signals of the figure 3.

5 Results and discussion

Figure 5 represents a comparison between the reflection and transmission coefficients of the A_0 and the S_0 modes when they interact with a SSS damage. Results in the case of the excitation of each mode are similar when the ratio $p \geq 3/6$. On the contrary, when the ratio $p < 3/6$, the difference increases and it can be noticed that the A_0 mode is more sensitive than the S_0 mode. In fact, power distribution overall the thickness indicates that a great party of A_0 mode power is concentrated in vicinity of the plate surfaces which can explain the difference of sensitivity between the A_0 and S_0 modes.

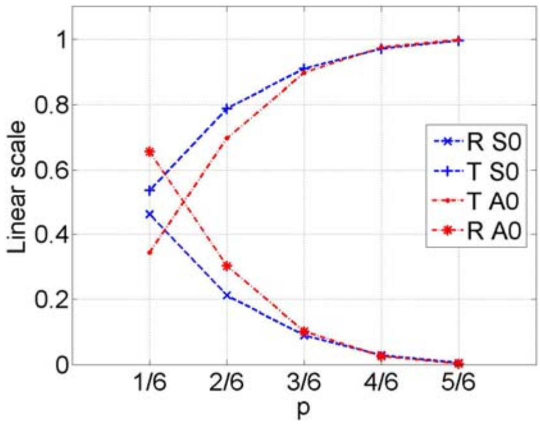

Figure 5 : **R**eflection and **T**ransmission coefficients of A_0 and S_0 when they interact with a SSS damage

Figure 6 and 7 illustrate the reflection and transmission coefficients when the A_0 and S_0 Lamb modes are respectively excited and interact with an ASS damage. In figure 6, it can be observed that the ASS damage leads to an important mode conversion from A_0 to S_0 mode especially for the transmitted signals. However, figure 7 show that the reflection of the excited mode is more important and a weak mode conversion from S_0 to A_0 mode. As before the power distribution can explain the high power migration from A_0 to S_0 mode.

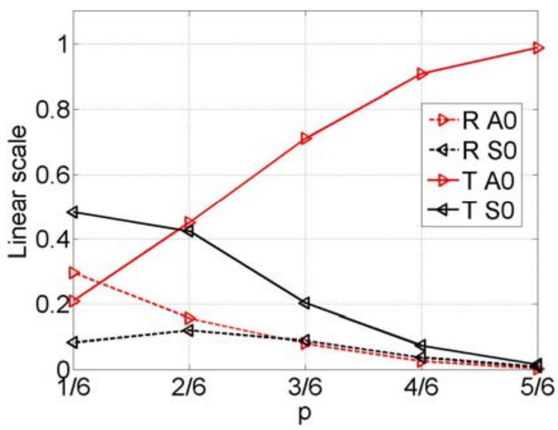

Figure 6 : **R**eflection and **T**ransmission coefficients of A_0 and S_0 when the A_0 mode is excited and interacts with an ASS damage.

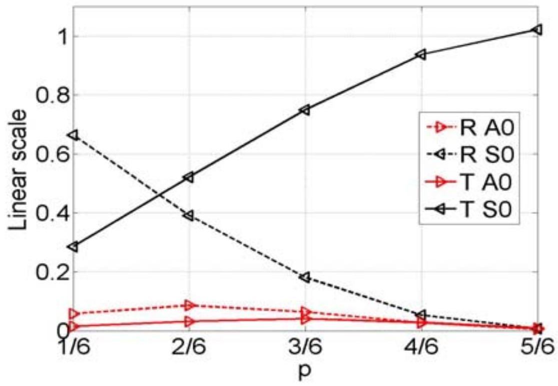

Figure 7 : **R**eflection and **T**ransmission coefficients of A_0 and S_0 when the S_0 mode is excited and interacts with an ASS damage.

6 Conclusions

A hybrid power method was developed analysed to study the A_0 and S_0 modes interaction with a sharp change of section. When this damage is symmetric, it can be noticed that there is no mode conversion observed for both excited modes. However, when the sharp change of section is asymmetric, we observe mode conversions from the A_0 mode to the S_0 mode when the A_0 mode is excited and from the S_0 mode to the A_0 mode when the S_0 mode is excited. Simplified damages studied in this paper could be combined to get complicated damages like notches.

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