CONTROL ALLOCATION AND AUTOPILOT DESIGN FOR AGILE MISSILE

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Abstract

This paper is concerned with the control allocation strategies with the two-time scale dynamic inversion which generate nominal control input trajectories. In addition, an robust flight control design method is proposed by using a time-varying control technique which is a time-varying version of the pole placement of the linear time-invariant system for an agile missile with aerodynamic fin, thrust vectoring control, and side-jet thruster. The control allocation algorithms proposed in this paper are capable of extracting the maximum performance by combining each control effector. The time-varying control technique for the autopilot design enhances the robustness of the tracking performance for the wide angle of attack range. The main results are validated through the nonlinear simulations with aerodynamic data.

1 Introduction

The modern control system of an agile missile has the many challenges due to the stringent required performance such as fast time response, high angle of attack, and high maneuverability. Usually, to achieve the required performance, the agile missiles combine the new control effectors (thrust vectoring, side thrusters) with the conventional control surface(aerodynamic fin) because thrust vectoring control and side-jet thrusters can provide additional moments and forces to achieve the reference command [1],[2]. However, managing each of a group of control devices with the independent control logic sometimes can result in reduced missile controllability and efficiency [3]. For example, at the launch phase of the missile, the aerodynamic surface has low control authority due to low speed. Thus the missile must be controlled by simultaneously using the aerodynamic fin and the additional control effectors. For another case, the pitching moment due to aerodynamic forces on a conventional aerodynamic fin and the pitching moment due to engine thrust vectoring control are completely independent. If applied in unison, the moments sum. If applied out of phase, the moments may cancel [4]. Therefore, for the super-maneuverability of the agile missile, control allocation algorithm for control effector family is needed.

On the other hand, an agile missile has nonlinear, time-varying and highly coupled dynamics. Furthermore, this has many uncertainties due to the difficulty to obtain exact aerodynamic data for vehicles operating under such conditions and may in fact be poorly approximated to the actual dynamics. These and other concerns have prompted researchers to look beyond the classical methods. Most control techniques are based on linearizing the equations of motion at each equilibrium point by the Jocobian linearization or cancelling the nonlinear terms by the nonlinear feedback as known variously as gain scheduling or dynamic inversion (feedback linearization). However, these techniques rely heavily on the knowledge of the plant dynamics. That is, if the mathematical model has uncertainties, the linearization and the cancellation of the nonlinear dynamics will not be exactly performed. This may have serious consequences of system performance. A trial for the weak robustness in these control techniques is to apply the robust linear time-invariant system theory to each linear time-invariant controller, hoping that the extended stability margin at each design point would improve the overall performance and stability margin. However, the improvement of the overall performance does not seem to be proportional to the local improvement, and overall stability still hinges on the intrinsic limitation of slow time-variance [5]. Therefore, stability and performance robustness within these control frameworks must be addressed by robust control technique based on stability criterion for linear time-varying systems.

This paper is concerned with the control allocation algorithms with the two-time scale dynamic inversion and robust flight control design using a time-varying control technique for an agile missile with the aerodynamic fin, thrust vectoring control, and side-jet thrusters. The control allocation algorithms generate the nominal control inputs of each control effector to achieved the required moment which can be obtained from two-time scale dynamic inversion. They are capable of extracting the maximum performance from each control effector by combining the action of them. Time-varying control technique for flight control design enhances the robustness of tracking performance for a reference command. The main results will be validated through the nonlinear simulations with aerodynamic data. The schematic diagram of this paper is shown in the Figure 1.

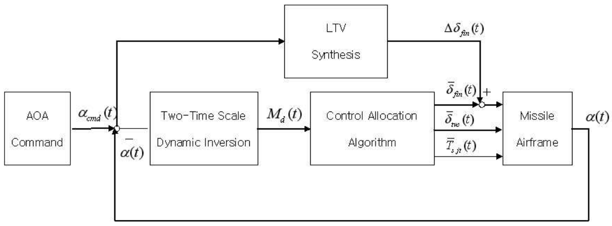

Fig. 1 Angle of attack tracking control system

2 Agile Missile Dynamics

The considered agile missile model with additional control effectors is a nonlinear pitch dynamics model. The equation of motion is given by

$$\dot{\alpha} = \frac{\frac{1}{2}\rho V^2 S}{mV}[C_{Z_0}(\alpha,M)+C_{Z_\delta}(\alpha,M,\delta_{fin})]+q$$

$$+\frac{T}{mV}\delta_{tvc}+\frac{1}{mV}T_{sjt}, \tag{1}$$

$$\dot{q} = \frac{\frac{1}{2}\rho V^2 SC}{I_{yy}}[C_{m_0}(\alpha,M)+C_{m_\delta}(\alpha,M,\delta_{fin})$$

$$+\frac{C}{2V}C_{mq}(M)]+\frac{T l_{tvc}}{I_{yy}}\delta_{tvc}-\frac{l_{sjt}}{I_{yy}}T_{sjt}, \tag{2}$$

$$\dot{V} = \frac{1}{m}[\tfrac{1}{2}\rho V^2 S\{C_{X_0}(\alpha,M)+C_{X_\delta}(\alpha,M,\delta_{fin})\}$$

$$+T\cos(\delta_{tvc})]\cos(\alpha)$$

$$-\frac{1}{m}[\tfrac{1}{2}\rho V^2 S\{C_{Z_0}(\alpha,M)+C_{Z_\delta}(\alpha,M,\delta_{fin})\}$$

$$+T\delta_{tvc}+T_{sjt}]\sin(\alpha) \tag{3}$$

where α, q, V, δ_{fin}, δ_{tvc}, T_{sjt}, M are angle of attack, pitch rate, missile velocity, aerodynamic fin deflection angle, thrust vectoring control deflection angle, side-jet thrust and Mach number, and m, ρ, S, C, T, l_{tvc}, l_{sjt} are mass, air density, reference area, reference length, thrust, moment arm of thrust vectoring control, and moment arm of side-jet thrust, respectively. C_{X_0}, C_{Z_0}, C_{m_0} are aerodynamic coefficients at $\delta_{fin}=0$, and C_{X_δ}, C_{Z_δ}, C_{m_δ} are variations of aerodynamic coefficients due to δ_{fin} deflection. Aerodynamic fin and TVC actuators have the limits within $\pm 30^\circ$ and $\pm 5.5^\circ$, and second-order dynamics with $\zeta = 0.7$, $\omega_n = 150$ and $\zeta = 0.7$, $\omega_n = 50$, respectively. A side-jet thruster has constant thrust during 30ms burning time like a pulse signal and maximum 10 side-jet thrusters can be simultaneously ignited at once.

Aerodynamic coefficients in Eqs. (1)-(2) are represented as the function of angle of attack at fixed Mach number:

$$\begin{aligned}\tilde{C}_{Z_0}(\alpha) &= a_1\alpha^4 + b_1\alpha^3 + c_1\alpha^2 + d_1\alpha \\ \tilde{C}_{Z_\delta}(\alpha) &= (a_2\alpha^3 + b_2\alpha^2 + c_2\alpha + d_2)\delta_{fin} \quad (4)\\ \tilde{C}_{m_0}(\alpha) &= a_3\alpha^4 + b_3\alpha^3 + c_3\alpha^2 + d_3\alpha \\ \tilde{C}_{m_\delta}(\alpha) &= (a_4\alpha^3 + b_4\alpha^2 + c_4\alpha + d_4)\delta_{fin} \quad (5)\end{aligned}$$

where the coefficients a_i, b_i, c_i, d_i in Eqs.(4)-(5) are constants obtained from curve-fitting of aerodynamic data.

3 Control Allocation Algorithms

Control allocation algorithm is to determine the amounts of deflection of the aerodynamic fin and thrust vectoring control, and the number of the ignited side-jet thruster to achieve a applied angle of attack command. In this paper, the two-time scale dynamic inversion is used to obtain the required moment for command tracking. Fast dynamic inversion, q inversion, calculates the required moment needed for the the actual pitch rate, q, to follow the commanded pitch rate q_{cmd} given by slow dynamic inversion, α inversion [6],[7].

First, the slow dynamic inversion which transforms the angle of attack command into the derived pitch rate command has the following form:

$$\begin{aligned} q_{cmd} &= \dot{\alpha}_d - \frac{\frac{1}{2}\rho V^2 S}{mV}[\tilde{C}_{Z_0}(\alpha) + \tilde{C}_{Z_\delta}(\alpha)\bar{\delta}_{fin}] \\ &\quad -\frac{T}{mV}\bar{\delta}_{tvc} - \frac{1}{mV}\bar{T}_{sjt} \end{aligned} \quad (6)$$

where $\bar{\delta}_{fin}$, $\bar{\delta}_{tvc}$, and $\bar{T}_{sjt}$ are the nominal fin deflection, thrust vectoring control deflection, and side-jet thrust, respectively. $\dot{\alpha}_d$ is the desired angle of attack dynamics and defined by

$$\dot{\alpha}_d = \omega_\alpha(\alpha_{cmd} - \alpha_{meas}) \quad (7)$$

where α_{cmd} is angle of attack command and α_{meas} is measured(or estimated) angle of attack. ω_α is a design parameter.

Second, the fast dynamic inversion is applied to the dynamics of pitch rate q and calculates the required moment to achieve the reference command. With Eq. (2), the fast dynamic inversion has the following form:

$$\begin{aligned} &\dot{q}_d - \frac{\frac{1}{2}\rho V^2 SC}{I_{yy}}[\tilde{C}_{m_0}(\alpha) + \frac{C}{2V}\tilde{C}_{mq}] = \\ &\frac{\frac{1}{2}\rho V^2 SC}{I_{yy}}\tilde{C}_{m_\delta}(\alpha)\bar{\delta}_{fin} + \frac{T\,l_{tvc}}{I_{yy}}\bar{\delta}_{tvc} - \frac{l_{sjt}}{I_{yy}}\bar{T}_{sjt} \end{aligned} \quad (8)$$

Let this equation be briefly represented as follows:

$$M_d = M_f\bar{\delta}_{fin} + M_t\bar{\delta}_{tvc} + M_s\bar{T}_{sjt} \quad (9)$$

where M_f, M_t and M_s mean the control distribution functions of aerodynamic fin, thrust vectoring control, and side-jet thrust, respectively. In Eq. (9), the left-hand term means the required moment which makes pitch rate have the desired dynamics and can be given by

$$M_d = \dot{q}_d - \frac{\frac{1}{2}\rho V^2 SC}{I_{yy}}[\tilde{C}_{m_0}(\alpha) + \frac{C}{2V}\tilde{C}_{mq}] \quad (10)$$

where $\dot{q}_d$ is the desired pitch rate dynamics and defined by

$$\dot{q}_d = \omega_q(q_{cmd} - q_{meas}) \quad (11)$$

where q_{cmd} is pitch rate command obtained from the slow dynamic inversion and q_{meas} is measured pitch rate. ω_q is a design parameter. The right-hand term is the achievable moment which can be generated by using the aerodynamic fin, thrust vectoring control and side-jet thrust.

The considered agile missile has the conventional control surface, aerodynamic fin, and the additional control effectors, thrust vectoring control and side-jet thrust. The family of the control effectors can be divided into two groups. One (Group A) is a group of the aerodynamic fin and the thrust vectoring control, and the other (Group B) is a group of the aerodynamic fin and the side-jet thrust. The former is used during thrust propulsion, while the latter is used after burning out. Therefore, two control allocation techniques - a pseudo inverse method for a group of aerodynamic fin and thrust vectoring control, and a daisy-chain method for a group of aerodynamic fin and side-jet thruster - are used for allocating the pitch control moment obtained from two-time scale dynamic inversion with the conventional surface and the additional effectors.

3.1 Pseudo Control Method

Pseudo control allocation technique for aerodynamic fin and thrust-vectoring control is representative of the ganged configurations [8]. This is parallel or ganged arrangement. The ganged effectors always cooperate, that is, their control effort is coordinated and control effectiveness of each control effector is adjusted by the time-varying weighting functions w_i.

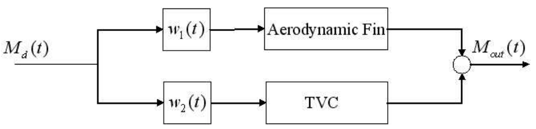

Fig. 2 Pseudo control method

For the case of Group A, to accomplish the desired command, the following equality must be satisfied with

$$\begin{aligned} M_d &= M_f\bar{\delta}_{fin} + M_t\bar{\delta}_{tvc} \\ &= \begin{bmatrix} M_f & M_t \end{bmatrix} \begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} \end{aligned} \tag{12}$$

From Eq. (12), the amount of the deflection of each control effector can be determined by matrix inversion as follows:

$$\begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} = \begin{bmatrix} M_f & M_t \end{bmatrix}^{-1} M_d \tag{13}$$

where the inverse of control distribution function matrix is not unique because of rank redundancy. Hence the control allocation function of each control effector can be obtained from using the pseudo-inverse property minimizing the following object function:

$$\min J = \begin{bmatrix} \bar{\delta}_{fin} & \bar{\delta}_{tvc} \end{bmatrix} \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix} \begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix}$$

$$\text{subject to } \begin{bmatrix} M_f & M_t \end{bmatrix} \begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} = v \tag{14}$$

where v is pseudo control. The pseudo control v is distributed in such a way that the weighted energy of the actual control input is minimized. The above optimization problem has an explicit solution which can be using several technique. But, by using the Lagrange multipliers, the optimal inputs are given by

$$\begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} = \left[\begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix}^{-1} \begin{bmatrix} M_f \\ M_t \end{bmatrix} \times \left\{ \begin{bmatrix} M_f & M_t \end{bmatrix} \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix}^{-1} \begin{bmatrix} M_f \\ M_t \end{bmatrix} \right\}^{-1} \right] v. \tag{15}$$

In Eq. (15), the effective control allocation algorithm can be designed by adjusting the weighting functions w_1, w_2 according to the flight conditions. Applying Eq. (15) to the given fast dynamic inversion Eq. (8) results in the following nominal control trajectories:

$$\begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} = \begin{bmatrix} \frac{M_f}{(M_f)^2+(\frac{w_1}{w_2})(M_t)^2} \\ \frac{(\frac{w_1}{w_2})M_t}{(M_f)^2+(\frac{w_1}{w_2})(M_t)^2} \end{bmatrix} M_d \tag{16}$$

where w_1, w_2 are the weighting values of each control effector, respectively.

3.2 Daisy-Chain Method

Daisy-chain allocation technique for aerodynamic fin and side-jet thrust allocates control effectors in prioritized manner [4]. That is, aerodynamic fin is not used until at least one side-jet thruster is ignited except for a case that the required moment is less than a side-jet thruster can generate.

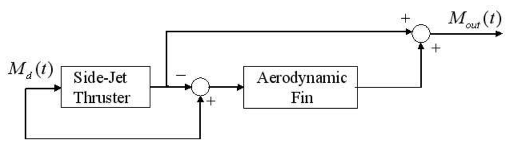

Fig. 3 Daisy-chain method

Daisy-chain control allocation for Group B is given by the following equation:

$$\begin{bmatrix} \bar{T}_{sjt} \\ \delta_{fin} \end{bmatrix} = \begin{bmatrix} F_s^{-1} M_d \\ F_f^{-1}\{M_d - F_s \bar{T}_{sjt}\} \end{bmatrix} \quad (17)$$

The side-jet thrust which has constant thrust during burning time is a pulse-like signal.

4 Time-Varying Control Technique

In this section, time-varying eigenvalue (SD-eigenvalue) is introduced into Extended-Mean Assignment(EMA) which is a time-varying version of pole placement for LTI systems. EMA is applied to stabilize a tracking error dynamics which is derived by linearizing a nonlinear dynamics through the nominal trajectories.

4.1 Extended-Mean Assignment

The EMA synthesis control technique is exemplified here with a generic second-order LTV system

$$\ddot{y} + p_2(t)\dot{y} + p_1(t)y = u \quad (18)$$

This LTV system can be written in an operator form $\mathcal{D}_p\{y\} = u$, where

$$\begin{aligned} \mathcal{D}_p &= D^2 + p_2(t)D + p_1(t) \\ &= (D - \lambda_2(t))(D - \lambda_1(t)) \\ &= D^2 - [\lambda_1(t) + \lambda_2(t)]D \\ &\quad + \lambda_1(t)\lambda_2(t) - \dot{\lambda}_1(t) \end{aligned} \quad (19)$$

is known as a polynomial differential operator and the factorization is known as Cauchy-Floquet factorization. The scalar functions $\lambda_1(t)$ and $\lambda_2(t)$ are called SD-eigenvalues for the LTV system (18) [9],[10].

Now define the Extended-Mean(EM) value of an integrable function $\sigma(t)$ by

$$\mathrm{EM}\{\sigma(t)\} = \limsup_{T\to\infty, t_0 \geq 0} \frac{1}{T}\int_{t_0}^{t_0+T} \sigma(\tau)d\tau. \quad (20)$$

Then the LTV sytem (18) with the bounded piecewise smooth coefficients $p_i(t)$ is exponentially stable for all $t_0 \geq 0$ if $\mathcal{D}_p$ has a bounded SD-eigenvalues $\{\lambda_1(t), \lambda_2(t)\}$ with EM values in the left half plane(LHP) [11]; i.e., for some $M > 0$,

$$|\lambda_i(t)| < M, \quad \mathrm{EM}\{\mathrm{Re}[\lambda_i(t)]\} < 0, i = 1,2 \quad (21)$$

If the LTV system (18) is unstable, a feedback control law

$$u = k_1(t)y + k_2(t)\dot{y} \quad (22)$$

can be synthesized so that SD-eigenvalues $\gamma_1(t)$ and $\gamma_2(t)$ of the closed-loop system $\mathcal{D}_h\{y\} = 0$, where

$$\begin{aligned} \mathcal{D}_h &= D^2 + h_2(t)D + h_1(t) \\ &= (D - \gamma_2(t))(D - \gamma_1(t)) \end{aligned} \quad (23)$$

has the desired EM values in the LHP.

Now implementing the control law (22) on the LTV plant (18) and comparing coefficients with the desired closed-loop system (23) yield

$$h_i(t) = p_i(t) - k_i(t). \quad (24)$$

Because $h_i(t)$ are related to $\gamma_i(t)$ by

$$\begin{aligned} h_1(t) &= \gamma_1(t)\gamma_2(t) - \dot{\gamma}_1(t) \\ h_2(t) &= -[\gamma_1(t) + \gamma_2(t)], \end{aligned} \quad (25)$$

the feedback control gains $k_i(t)$ can then be synthesized as

$$\begin{aligned} k_1(t) &= p_1(t) + \dot{\gamma}_1(t) - \gamma_1(t)\gamma_2(t) \\ k_2(t) &= p_2(t) + \gamma_1(t) + \gamma_2(t). \end{aligned} \quad (26)$$

4.2 Autopilot Design

For EMA autopilot design, let

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} \alpha \\ q \end{bmatrix} \quad (27)$$

be the state vector of the missile. Then, from Eqs. (1)-(2), the state equation is given by

$$\dot{\xi} \;=\; f(\xi, \delta_{fin}) = \begin{bmatrix} f_1(\xi_1, \xi_2, \delta_{fin}) \\ f_2(\xi_1, \xi_2, \delta_{fin}) \end{bmatrix}. \quad (28)$$

Now, for a given angle of attack command α_{cmd}, let $\bar{\delta}_{fin}$ be the nominal fin deflection and $\bar{\xi}$ be the nominal state trajectory such that

$$\dot{\bar{\xi}} = f[\bar{\xi}, \bar{\delta}_{fin}]. \quad (29)$$

Define the tracking errors by

$$x = \xi - \bar{\xi}, \tag{30}$$

and the tracking error control by

$$v = \delta - \bar{\delta}_{fin}. \tag{31}$$

Then the linearized tracking error dynamics is given by

$$\dot{x} = A(t)x + B(t)v \tag{32}$$

where

$$\begin{aligned} A(t) &= \left.\frac{\partial f}{\partial \xi}\right|_{\bar{\xi},\bar{\delta}_{fin}} = \begin{bmatrix} a_{11}(t) & 1 \\ a_{21}(t) & a_{22}(t) \end{bmatrix}, \\ B(t) &= \left.\frac{\partial f}{\partial \delta}\right|_{\bar{\xi},\bar{\delta}_{fin}} = \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix}. \end{aligned} \tag{33}$$

The autopilot design task amounting to finding a control law such that the tracking error becomes zero exponentially for any admissible angle of attack command. This can be achieved using an EMA controller. However, to use prototype EMA controller, it is necessary to transform the linearized tracking error dynamics into the phase-variable canonical form. This can be done via Silverman's coordinate transformation, provided that $[A(t), B(t)]$ is uniformly completely controllable [12]. Whereas this approach will result in a minimal realization, the resulting system coefficients are very complicated. To simplify the matter, a nonminimal realization is adopted, which yields a phase-variable canonical form with very simple coefficients. To that end, apply the state coordinate transformation

$$x = L(t)z \tag{34}$$

where $L(t)$ is a time-varying coordinate transformation matrix given by

$$L(t) = \begin{bmatrix} 1 & 0 \\ -a_{11}(t) & 1 \end{bmatrix}. \tag{35}$$

Then the linearized system (32) in the z coordinates becomes

$$\dot{z} = A_c(t)z + B_c(t)v \tag{36}$$

where $A_c(t) = L^{-1}(t)[A(t)L(t) - \dot{L}(t)]$ is of the companion form

$$A_c(t) = \begin{bmatrix} 0 & 1 \\ -p_1(t) & -p_2(t) \end{bmatrix} \tag{37}$$

with $-p_1(t) = \dot{a}_{11}(t) + a_{21}(t) - a_{11}(t)a_{22}(t)$, and $-p_2(t) = a_{11}(t) + a_{22}(t)$, and

$$B_c(t) = \begin{bmatrix} b_1(t) \\ a_{11}(t)b_1(t) + b_2(t) \end{bmatrix}. \tag{38}$$

Note that $z_1 = x_1 = \alpha - \alpha_{cmd}$. By eliminating z_2 from Eq. (36), it is seen that this state equation is equivalent to a scalar equation

$$\begin{aligned} \ddot{z}_1 + p_2(t)\dot{z}_1 + p_1(t)z_1 = \\ b_1(t)\dot{v} + (\dot{b}_1(t) + b_2(t) - a_{22}(t)b_1(t))v \end{aligned} \tag{39}$$

To render this equation into the phase-variable form, the angle of attack zero dynamics is introduced as follows:

$$\dot{v} + \frac{\dot{b}_1(t) + b_2(t) - a_22(t)b_1(t)}{b_1(t)}v = \frac{1}{b_1(t)}u \tag{40}$$

Combining Eqs. (39) and (40) yields the desired form

$$\ddot{z}_1 + p_2(t)\dot{z}_1 + p_1(t)z_1 = u \tag{41}$$

Now an EMA control law u can be designed for the angle of attack tracking error dynamics (41) using the outlined in previous statements [9].

5 Simulation Results

Simulations with aerodynamic data are performed to validate the proposed schemes. In this study, there are two scenarios. One is subsonic flight condition($M = 0.6$) for Group A, and the other is hypersonic($M = 6.0$) for Group B.

Results for Scenario 1 and Scenario 2 are presented in Figures 4 - 5, and Figures 6 - 7, respectively. Figure 4 shows that an angle of attack command for Scenario 1 is well tracked within 5% steady-state error under various uncertainties such as poorly approximated aerodynamic data in curve-fitting, missile velocity variation, etc.

The distributed control efforts to follow the command are depicted in Figure 5. As approaching the steady state, the deflection of thrust vectoring control is growing down less and less while the deflection of aerodynamic fin is growing up more and more. It is because the authorities of control effectors are dependent on flight condition. Therefore, this fact reveals that pseudo control method for Group A is the efficient control allocation algorithm reflected on flight condition. Under similar circumstances, the angle of attack tracking performance for Scenario 2 is depicted in Figure 6. This shows that after burning out, the angle of attack command can be achieved by using side-jet thrust. The allocated control efforts by daisy-chain method for Group B are depicted in Figure 7. It can be inferred from this that side-jet thrust usage prior to aerodynamic fin increases the maneuverability of the missile in homing phase.

6 Conclusions

Autopilot for the agile missile with conventional control surface - aerodynamic fin - and additional thrust - thrust vectoring control, side-jet thruster - was designed. Moment required to achieve the angle of attack command was obtained by the two-time scale dynamic inversion. A family of control effecors were divided into two groups according to the coverage of each effector in flight envelop, and two control allocation algorithms were used to effectively distribute the control effrots for each group to achieve the required moment. Linear time-varying control technique which is the time-varying version of pole placement for LTI systems was applied to the control of aerodynamic fin to enhance the robustness of two-time scale dynamic inversion. The features of the proposed schemes include (1) effective control allocation for each control effector (aerodynamic fin, thrust vectoring control, side-jet thruster) to achieve the angle of attack command, (2) good tracking performance for angle of attack command without scheduling of any constant design parameters throughout a wide range of angle of attack, and (3) time-varying control

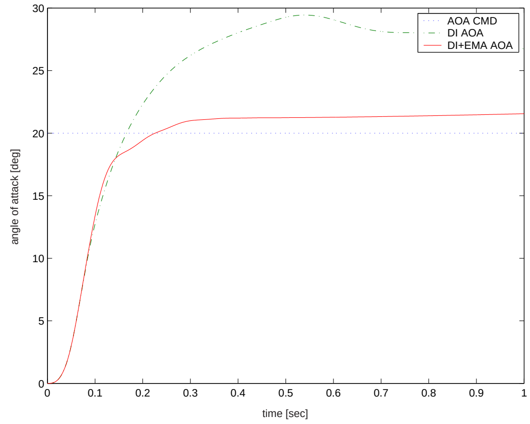

Fig. 4 Angle of attack output for Scenario 1

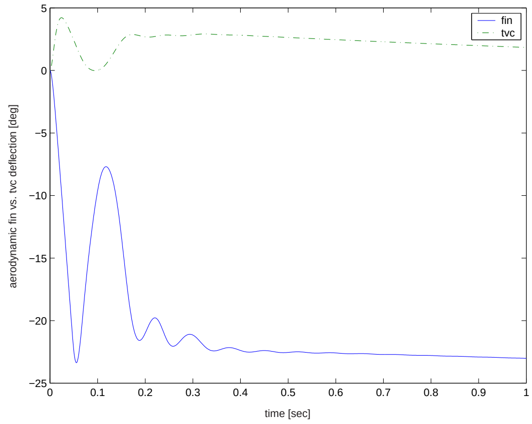

Fig. 5 Distributed aerodynamic fin vs. thrust vectoring control for Scenario 1

gains to improve the robustness for the unstructured uncertainties. The proposed schemes will be validated by nonlinear simulations with aerodynamic data.

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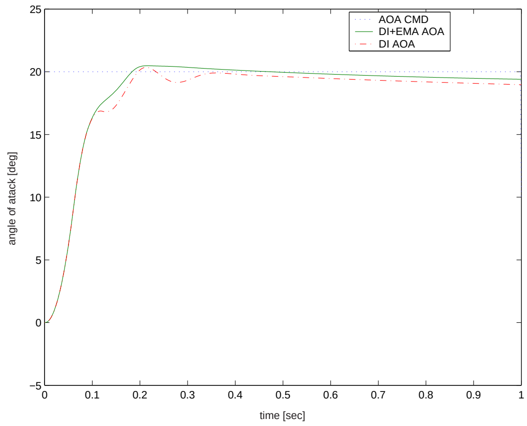

Fig. 6 Angle of attack output for Scenario 2

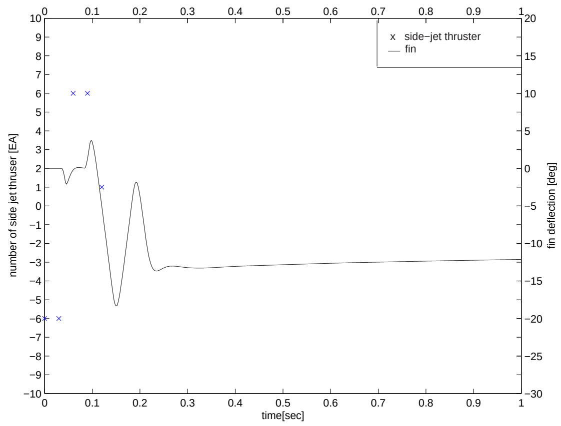

Fig. 7 Distributed aerodynamic fin vs. side-jet thrust control for Scenario 2

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