NON-LINEAR VIBRATION AND DAMPING CHARACTERIZATIONS OF DELAMINATED COMPOSITE LAMINATES BY USING MULTILAYERED FINITE ELEMENTS

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Abstract

In this study, non-linear iterative eigen-frequency analysis and modal-strain-energy-based damping characterization have been established for free vibration problems of composite laminated beams and plates with delaminations using multilayered isoparametric plate/shell finite elements. In the present finite element formulation, either of the un-delaminated displacement continuities or the delamination discontinuities at ply interfaces of laminates is modeled by invoking the penalty function method. Both of the material and state-change nonlinearities arising respectively from the frequency-dependent material properties and the over-lapping contact states at delaminated interfaces may be taken care of in the iterative manner. As the numerical examples, two types of delaminated carbon fiber reinforced plastic (CFRP) composite beams were taken from the existing literature. Each of the plies of the laminates with or without delaminations was modeled with an individual plate/shell element under the displacement assumption of first-order-shear-deformable (FOSD) model. Fairly good agreements between the present FE numerical results and the existing experimental measurements were basically confirmed for the natural resonant frequencies and modal shapes in the broad range of frequency. In addition, the modal loss factors predicted by the present FE indicated that damping performance tends to be improved just by the existence of delaminations. This is probably because of the transverse shear strain energy concentrations in the delaminated beams. This fact also implies that the delamination can be effectively used as ultimate damping material without any thickness and weight gains.

1 Introduction

Vibration analysis and damping characterization of composite laminated structures with damages and failures such as delaminations have been attracting much attention for more than two decades and is still one of the vital topics around the relevant research fields. The dynamic responses such as resonant frequencies, modal shapes and damping performance (modal loss factors) of composite laminates not only can tell us their structural integrity in terms of barely invisible damages such as embedded multiple delaminations but also may be one of the highest priorities when we think of letting composite laminates evolve smart materials and intelligent structures [1,2].

Up to the present, quite a few experimental, theoretical and numerical investigations on dynamics behaviors of composite laminates with delaminations have been carried out. Some of the earliest works for analytical modeling of free-vibrating composite laminated beams with a delamination were made by Ramkumar *et al.* [3], Wang *et al.* [4], and Mujumdar and Suryanarayan [5]. The so-called "free model", first proposed by Ramkumar *et al.* [3] and then modified later on by Wang *et al*. [4], is based on a straightforward combination of four Timoshenko beams connected to each other at the delamination fronts with flexural/ longitudinal kinematical couplings enforced. This "free model" was applied to natural frequencies and corresponding modal shapes at a few lower-order bending modes of some delaminated beams and then compared with experimental results, leaving a few technical difficulties in the delamination modeling. In order to avoid physically un-realistic overlapping situations at the delaminated interfaces and still keep a linear model, the so-called "constrained model" was next proposed by Mujumdar and Suryanarayan [5] and its accuracy improvement was confirmed through several case studies by several researchers [6,7]. Among them, Shen and Grady [8] investigated vibration phenomena in CFRP cross-ply laminated cantilever beams with a single through-width delamination introduced at mid-span with its length and its inter-ply location varied. They employed two different analytical vibration models, model A and model B, which were essentially based on the aforementioned "constrained" and "free" models, respectively, and then also carried out a corresponding experimental study to verify their predictions obtained by their analytical models. Another type of attempt for analytical modeling of delaminated beams was the one for post-buckled beams with delamination [9-11]. This kind of modeling methodology is important from a practical point of view. In addition, it happens to be free from the physically un-realistic over-lapping states.

Experimental measurements of dynamic characteristics in delaminated composites such as natural frequency, modal shape and damping coefficient, on the other hand, have also been carried out [12-17]. For instance, Kimpara *et al.* [17] employed the vibration pattern imaging technology for precisely measuring the resonant frequencies and the corresponding modal shapes over the wide range of frequency to assess the structural integrity of CFRP unidirectional and quasi-isotropic laminated cantilever beams with a delamination. It should be noted that all of these experimental investigations have revealed the non-linearity, more or less, in vibrations of delaminated composite beams/plates and consequently implied the limitations of the aforementioned "linear" analytical models (whichever "free" or "constrained" model).

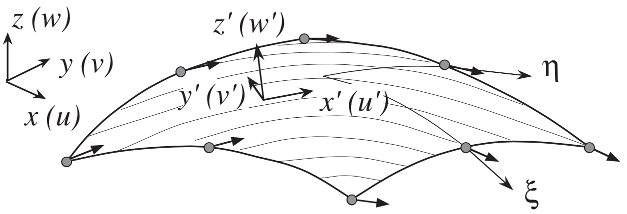
Recently, a few attempts for non-linear modeling of delaminated composite beams have been made. Luo and Hanagud [18,19] proposed a "piecewise linear spring model", which, they said, could simplify the problem while keeping the significant non-linear features of the vibrations of delaminated beam. This generalized model includes both of the "free" and the "constrained" models as its special cases. By using this "piecewise linear spring model", they analytically obtained vibration responses of delaminated composite beams in either frequency or time domain. Żak *et al.* [20] analyzed vibration of a laminated composite plate with closing delamination using the finite element method. In their finite element model, contact constrains between delaminated layers were introduced by using the penalty function method. They employed Fast Fourier Transfer (FFT) analysis to obtain dynamic responses in frequency domain of the delaminated plate analyzed from the ones in time domain. However, they did not perform the eigen-value analysis using their finite element models. In addition, in contrast with resonant frequencies and modal shapes, damping performance such as modal loss factors for those composite laminates with delamination was not evaluated in those investigations mentioned above, even though that dynamic quantity is generally said to be more sensitive to delamination.

In this study, free vibration eigen-value analysis and modal-strain-energy-based (MSE-based) damping characterizations will be conducted for composite laminated beams with delaminations by using the multilayered isoparametric plate/shell finite elements of the present authors [21]. In the present finite element formulation, the displacement continuity constrains at the layer interfaces are enforced by invoking the penalty function method [20,21]. The natural resonant frequencies and the corresponding modal shapes will be obtained by solving generalized free-vibration eigen-value problems and the modal loss factors are estimated by modal strain energy (MSE) method [21]. Both of the material and the state-change non-linearity arising respectively from the frequency-dependent material properties and the over-lapping contact states at the delaminated interfaces may be taken care of in the iterative manner. As the numerical examples, the series of delaminated CFRP cross-ply and quasi-isotropic composite cantilever beams respectively investigated by Shen & Grady [8] and Kimpara *et al.* [17] will be considered. Each of the plies of the laminates will be modeled with an individual plate/shell element based on the displacement assumption of first-order-shear-deformable (FOSD) model.

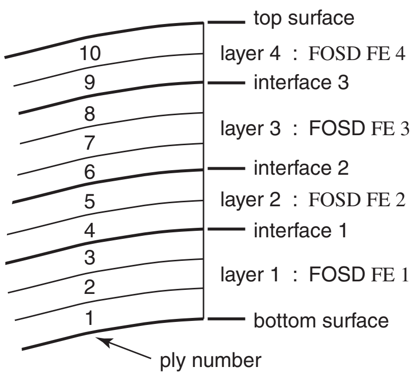
2 Finite Element Formulation

In Fig.1, the present eight-node multilayered 3-D degenerated plate/shell finite element is schematically shown.

The geometry of reference surface of k^{th} layer within an element can be expressed as,



(a)



(b)

Fig.1. The present multilayered plate/shell finite elements.

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^{(k)} = \sum_{i=1}^{8} \Phi_i \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}^{(k)} + \sum_{i=1}^{8} \Phi_i \frac{h_i^{(k)}}{2} \zeta \hat{\mathbf{V}}_{ni} \tag{1}$$

where (ξ, η, ζ): natural curvilinear coordinates, $\Phi_i(\xi,\eta); i = 1, 2, \cdots, 8$: shape functions, $\{x_i \ y_i \ z_i\}^{(k)T}$: sub-nodal coordinates for the k^{th} layer, $h_i^{(k)}$: thickness of the k^{th} layer and $\hat{\mathbf{V}}_{Ni}$ is the nodal unit vector normal to the mid-surface of the laminate. In addition, the nodal unit vectors tangential to the mid-surface of the laminate, $\hat{\mathbf{V}}_{Li}$ and $\hat{\mathbf{V}}_{Ti}$ are also introduced. These mutually perpendicular vectors $\hat{\mathbf{V}}_{Li}$, $\hat{\mathbf{V}}_{Ti}$ and $\hat{\mathbf{V}}_{ni}$ constitute a direction cosine matrix and then the local direction cosine matrix $\boldsymbol{\mu}_i$ is obtained by interpolating those nodal vectors as follows:

$$\boldsymbol{\mu}_i = \left[\hat{\mathbf{V}}_{Li} \ \hat{\mathbf{V}}_{Ti} \ \hat{\mathbf{V}}_{ni} \right] \tag{2}$$

$$\boldsymbol{\mu}' = \left[\hat{\mathbf{V}}'_L \ \hat{\mathbf{V}}'_T \ \hat{\mathbf{V}}'_n \right] = \sum_{i=1}^{8} \Phi_i \left[\hat{\mathbf{V}}_{Li} \ \hat{\mathbf{V}}_{Ti} \ \hat{\mathbf{V}}_{ni} \right] \tag{3}$$

The local displacements (u', v', w') in k^{th} layer are defined as the following first-order-shear-deformable (FOSD) assumptions.

$$\begin{Bmatrix} u' \\ v' \\ w' \end{Bmatrix}^{(k)} = \sum_{i=1}^{8} \Phi_i \left[\boldsymbol{\mu}'\right]^T \boldsymbol{\mu}_i \left[\begin{Bmatrix} U'_0 \\ V'_0 \\ W'_0 \end{Bmatrix} + Z \begin{Bmatrix} U'_1 \\ V'_1 \\ 0 \end{Bmatrix} \right]_i^{(k)} \tag{4}$$

in which $Z_i^{(k)} = \frac{h_i^{(k)}}{2} \zeta$

The rest of the finite element development procedure for composite laminated structures has already been established and hence it will not be repeated herein. One can refer to the works by Panda [22] and the present authors [21] for further details.

In the present finite element formulation, the displacement continuity constrains at the layer interfaces are enforced by invoking the penalty function method [20,21], in which the adhesions between the adjacent laminae can be achieved with the penalty parameter α set to be very large (say 10^8 in the case of double precision arithmetic), while the delaminated states at the lamina interfaces can easily be modeled with that parameter α set to be zero as illustrated in Fig.2.

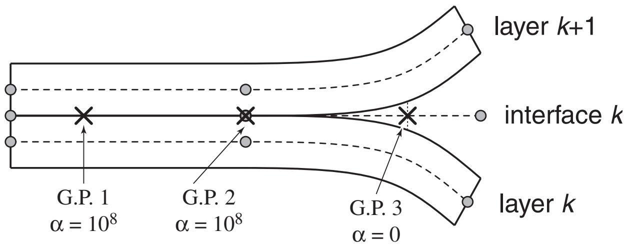


Fig.2. Adhesion and delamination modelings by the penalty function method.

The natural resonant frequencies and the corresponding modal shapes can be obtained by solving the generalized free-vibration eigen-value problems. From the finite element equations of motion governing the free vibration response of laminates, the following generalized eigen-problem can be finally derived:

$$\left(\mathbf{K}-\lambda\mathbf{M}\right)\cdot\mathbf{d}=\mathbf{0},\quad \lambda=\omega^2 \tag{5}$$

where $\mathbf{K}$ and $\mathbf{M}$ are, respectively, the global stiffness and mass matrices. After solving the above generalized eigen-problem by using an efficient solution algorithm such as the subspace iteration scheme [23], the lowest n eigen-values,

$$\lambda_m=\omega_m^2\ ;\ m=1,2,\cdots,n$$

and the corresponding eigenvectors $\boldsymbol{d}_m$ can be extracted.

Further, modal loss factor estimation by the modal strain energy method (MSE) [21], is also implemented into the present FE program. According to this method, The modal loss factor of a laminated plate, η_m^c, at the m^{th} mode is estimated by summing the products of the material loss factor, $\eta_m^{(k)}$, for each constituent lamina and the fraction of the modal strain energy stored in that lamina as the following:

$$\eta_m^c=\sum_{k=1}^{N_k}\eta_m^{(k)}W_m^{(k)}\bigg/\sum_{k=1}^{N_k}W_m^{(k)} \tag{6}$$

where $W_m^{(k)}$ is the strain energy stored in k^{th} layer under the m^{th} mode shape.

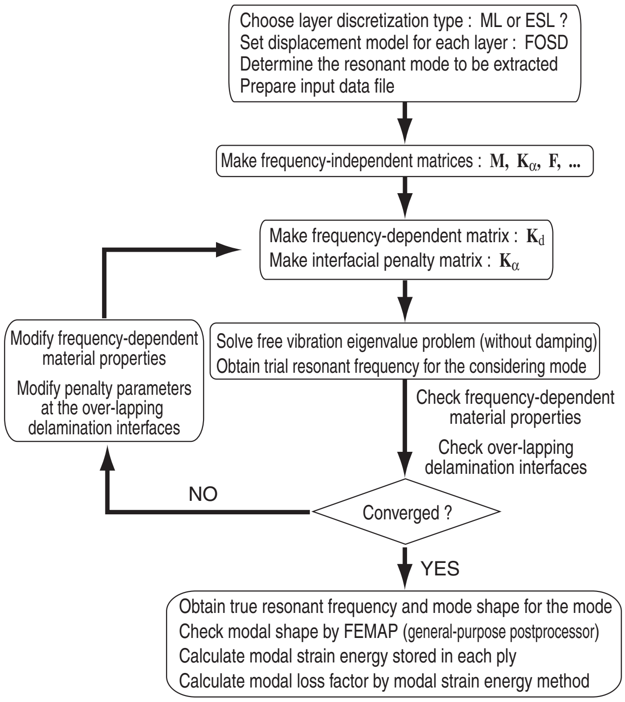


Fig.3. Program flow chart of the present non-linear iterative finite element free-vibration eigen-value analysis.

In Fig.3, the numerical analysis program flow of the present finite element calculation is schematically charted. In this calculation flow, first the number of layers for modeling the laminate should be specified. If it is preferable, the laminate can be numerically modeled as an equivalent-single-layer (ESL) plate/shell by smearing the layer-by-layer-different material properties. However, the composite laminated structures with delaminations should always be modeled as multilayered plate/shells since such laminated structures will inherently exhibit multilayered mechanical behaviors. Both of the material and the state-change non-linearity arising from the frequency-dependent material properties and the over-lapping contact states at the delaminated interfaces respectively may be taken care of in the iterative manner as shown in the flow chart. How many iterations will be needed for convergence in this non-linear calculation is considerably problem-dependent, but in cases of commonly-used composite laminated coupons with a single through-width delamination, a few iterations seem to be sufficient to converge the solution.

3 Numerical Examples

3.1 CFRP cross-ply laminates (1st bending)

As first of the numerical examples, a series of delaminated CFRP cross-ply composite cantilever beams, which were experimentally as well as analytically investigated by Shen and Grady [8], were picked up. The beam dimensions, its cross-ply laminate stacking sequence, the artificially induced delaminated regions and elastic/damping material properties are all together shown in Fig.4. A single through-width delamination is introduced at the mid-span of the laminate with its length (a = 0, 1, 2, 3, and 4inch) and through-thickness location (IF00, IF01, IF02, and IF03) varied. In the present finite element analysis, each of 8 plies of the laminates was modeled by an individual plate/shell element with the displacement assumption of first-order-shear-deformable (FOSD) model. As mentioned earlier, adhesions or delaminations at layer interfaces was introduced by setting the penalty parameter to be virtually infinite or zero.

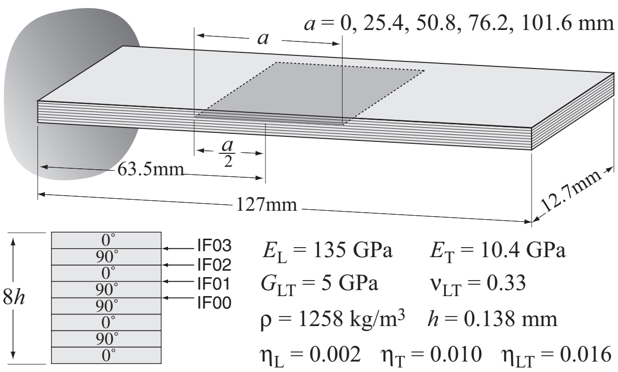
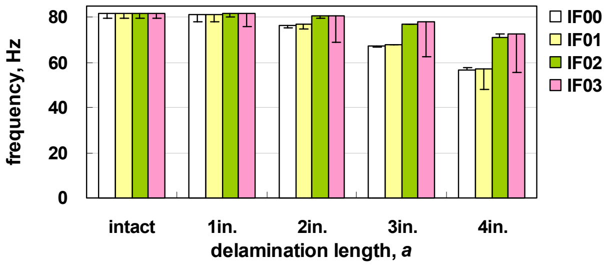
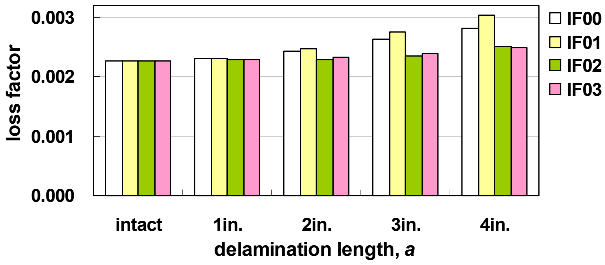


Fig.4. Delaminated CFRP cross-ply cantilever beams (Shen & Grady [Ref.8]).

In Fig.5 (a) and (b), resonant frequencies and modal loss factors at the first bending vibration mode obtained by the present FEM are shown. Note that the error bars shown in the graph for the resonant frequencies indicate the differences between the present FEM results and the existing experimental results [8]. As can be seen, fairly good agreements between the present FE results and the existing experimental



(a) resonant frequencies at 1st bending mode (the error bars in the graph show the differences between the present FEM results and the experiments)

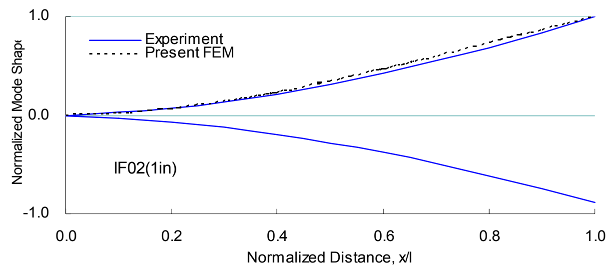


(b) modal loss factors at 1st bending mode

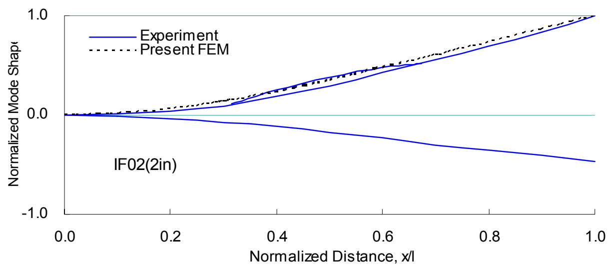
Fig.5. the present FEM results (resonant frequencies and modal loss factors at 1st bending mode).

measurements were basically confirmed for the natural resonant frequencies at this lowest order vibration mode of this type of laminates with or without delamination, and general tendency of decrease in resonant frequency and increase in modal loss factor can be clearly seen when a delamination is introduced within the laminate. In addition, the modal loss factors predicted by the present FE indicated that damping performance was apt to be improved (in the largest cast of IF01(4in), by 33.6%) just by the existence of delaminations. Those damping performance improvements should be attributed to the shear strain energy concentrations in the delaminated beams. Since the material loss factor under shear deformation, $\eta_{LT} = 0.016$, is assumed to be eight times larger than that under extension in the reinforcing fiber direction, $\eta_L = 0.002$, the more shear strain energy is stored in the laminates, the more damping performance will be realized.

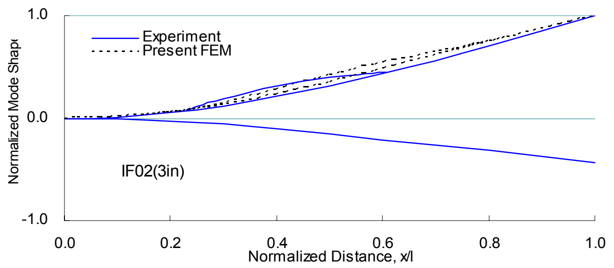
Finally, in Fig.6(a) through (d), normalized modal shapes at the first bending mode for the composite beams with a delamination at IF02 with its length varied are shown. Fairly good agreements between the modal shapes obtained by the present FE and those taken from the experimental report can be seen again. As can be seen, an opening mode in the delaminated region could be obtained by the present FEM, which was also experimentally observed.



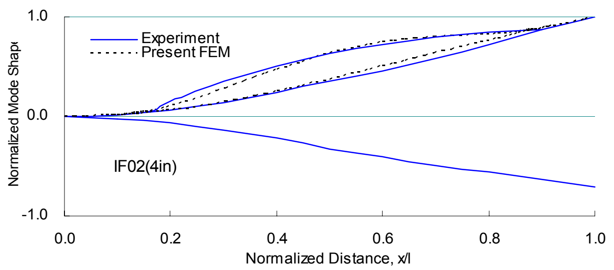
(a) $a = 1$in.



(b) $a = 2$in.



(c) $a = 3$in.



(d) $a = 4$in.

Fig.6. Normalized modal shapes (at 1st bending mode) for delaminated at IF02.

3.2 CFRP quasi-isotropic laminates (higher-order modes)

As the other numerical example, a series of delaminated CFRP quasi-isotropic composite laminated cantilever beams, which were experimentally as well as numerically with the conventional FEM investigated by Kimpara *et al.* [17], was next considered. The beam dimensions, its laminate stacking sequence, the artificially induced delaminated regions and elastic/damping material properties are all together shown in Fig.7. A single through-width delamination in the central plane, IF00, is introduced at the mid-span of the $[45/-45/0/90]_S$ quasi-isotropic laminates with its length (a = 0, 50, and 100mm) varied. In the present finite element analysis, each of 8 plies of the laminates was modeled by an individual plate/shell element with the FOSD displacement assumption.

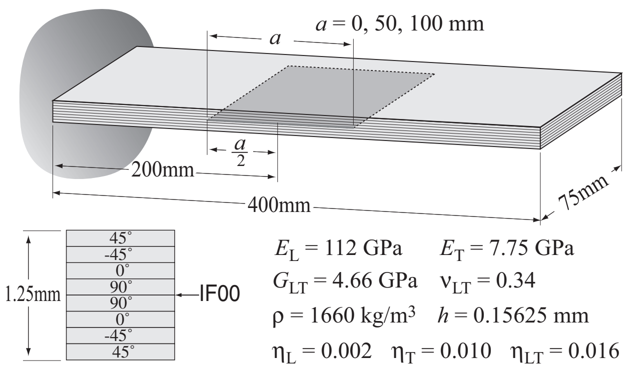
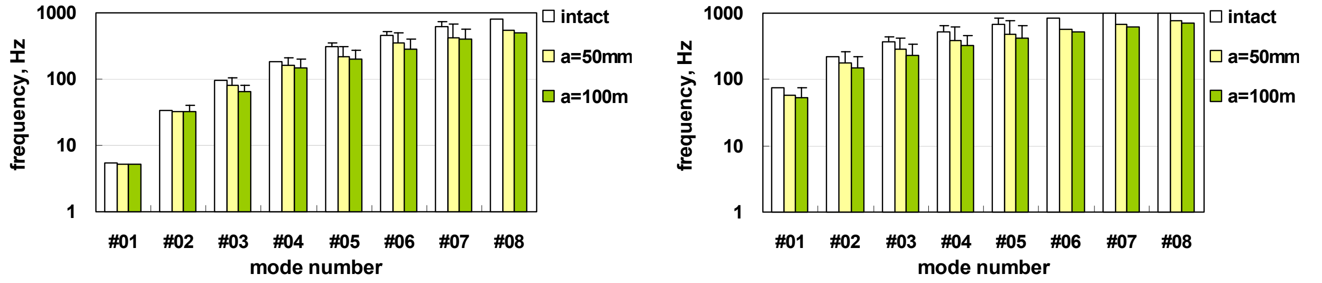
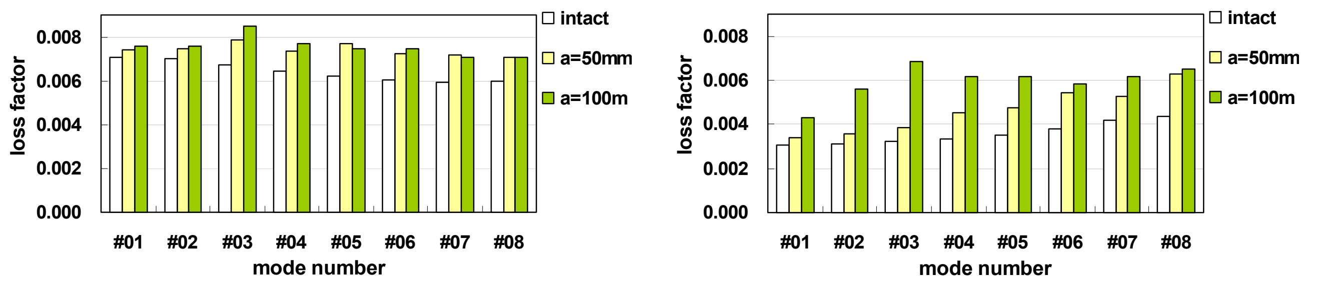


Fig.7. Delaminated CFRP quasi-isotropic beams (Kimpara *et al.* Ref.17).

In Fig.8 (a) through (d), resonant frequencies and modal loss factors for intact or delaminated (a = 50 or 100mm) beams in the broad range of frequency, i.e. at eight 'bending-only' from #01 to #08 modes and another eight 'bending-with-torsion' from #01 to #08 modes, are summarized. Note that the error bars shown in the first two graphs for the resonant frequencies indicate the differences between the present FEM numerical results and the existing experimental results obtained with the vibration pattern imaging (VIP) sensor technology [17]. As can be seen, fairly good agreements between



(a) resonant frequencies for '**bending-only**' modes (b) resonant frequencies for '**bending-with-torsion**' modes



(c) modal loss factors for '**bending-only**' modes (d) modal loss factors for '**bending-with-torsion**' modes

Fig.8. Comparisons of the present numerical results (the error bars shown in the first two graphs for resonant frequencies are the differences between the present FE results and the experimental measurements).

the present FE results and the existing experimental measurements were basically confirmed for the natural resonant frequencies in these broad range of vibration modes of this type of laminates with or without delamination, and general tendency of decrease in resonant frequency and increase in modal loss factor can be nearly consistently observed when a delamination is introduced into those laminates. In addition, the modal loss factors predicted by the present FE indicated that damping performance was more significantly improved under the bending-with-torsion modes than under the bending-only modes, although the absolute values of the modal loss factors under the latter modes are always large compared to those under the former ones. This is probably because the bending-with-torsion modes are more sensitive to delamination than the bending-only modes, and hence may be more useful for delamination detection by examining the damping performance change.

Finally, in Fig.9 and 10, the two typical cases of modal shapes at the 'bending-only' 8th mode and the 'bending-with-torsion' 5th mode are respectively shown and compared to the corresponding experimental measurements. The detailed procedure for obtaining the actual vibration mode patterns by using VPI sensor in the experiment can be referred to the literature [17]. Agreements of the present FE numerical results with those of the experiment are remarkably good. The experiment tends to give higher resonant frequencies for the same vibration mode when compared to those predicted by the present FEM. This is probably because the frequency-dependent material properties are no longer negligible at those higher-order frequency modes and therefore more stiff elastic moduli should be used as an input data for the numerical calculations when one wants to bring together the numerical and experimental results in terms of the resonant frequency as well as the modal shapes.

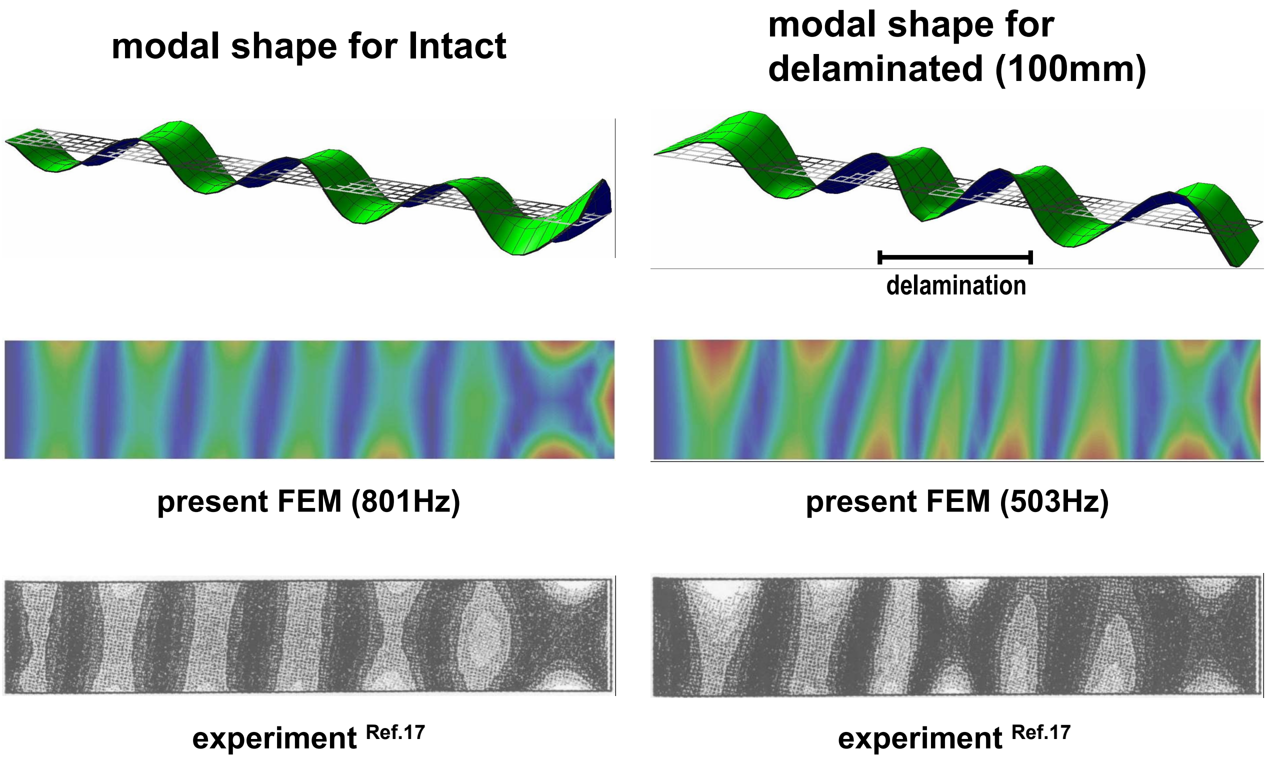


Fig.9. Modal shapes (‘**bending-only**’ 8th mode)

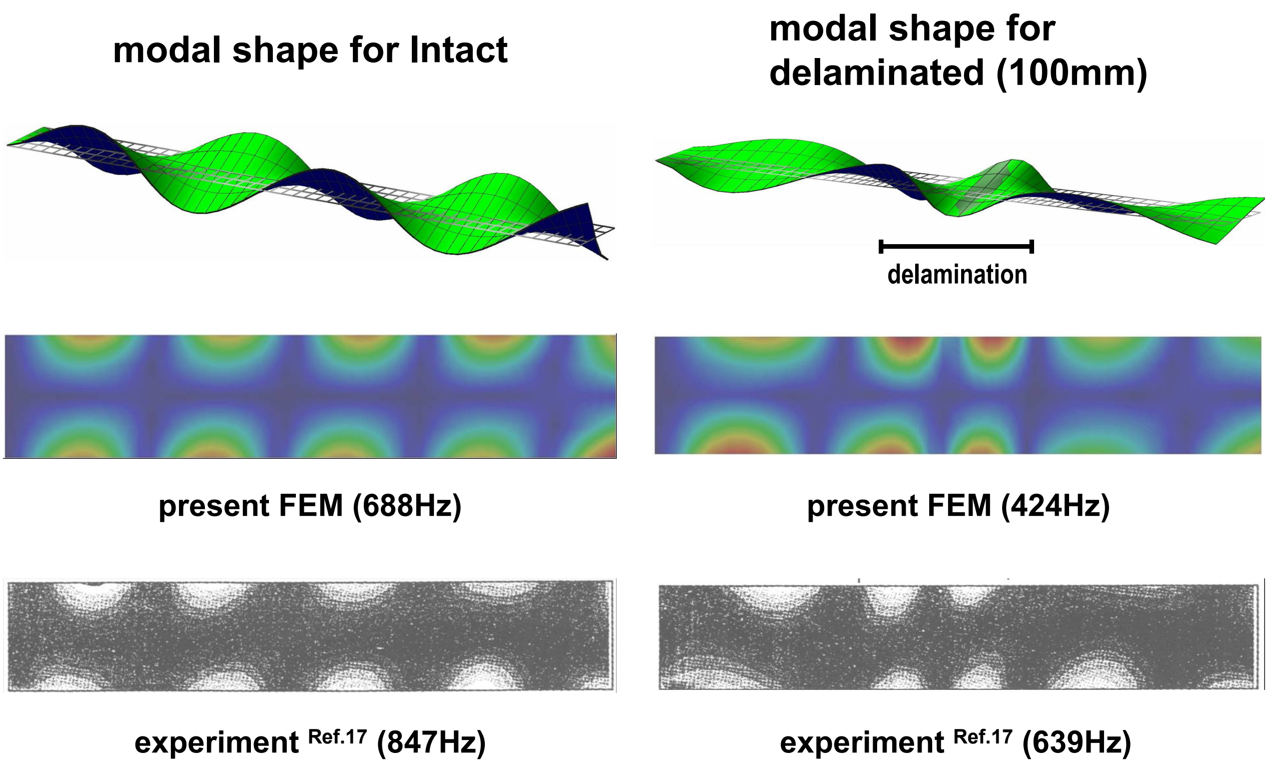


Fig.10. Modal shapes (‘**bending-with-torsion**’ 5th mode)

4 Conclusions

In this study, non-linear iterative free vibration eigen-value analysis and modal-strain-energy-based damping characterization have been conducted for composite laminated beams/ plates with delaminations by using multilayered isoparametric degenerated plate/shell finite elements of the authors. In the present finite element formulation, the displacement continuity constrains at the layer interfaces are enforced by invoking the penalty function method As typical numerical examples, two types of delaminated CFRP composite laminated cantilever beams were numerically investigated by the developed FEM program. Each of the plies of the laminates was modeled by an individual plate/shell element with the displacement assumption of first-order-shear-deformable (FOSD) model. Very good agreements between the present FE numerical results and the existing experiments were basically confirmed for the natural frequencies and modal shapes in the broad range of frequency. In addition, the modal loss factors predicted by the present FE indicated that damping performance tended to be improved by existence of delaminations. The reason for this damping improvements is probably due to the transverse shear strain energy concentrations in the delaminated beams. This fact apparently shows that the delamination can be effectively used as ultimate damping materials without any thickness and weight gains.

References

[1] Chattopadhyay A, Swann C, Kim, H S and Han Y. Characterization of Delamination in Using Damage Indices. *Proceedings of International Conference on Computational & Experimental Engineering and Sciences (ICCES'03)* , Corfu, 2003.

[2] Tan P and Tong L. Modelling for Delamination Detection in a Laminated Composite Beam Using Piezoelectric Layers. *Proceedings of 14th International Conference on Composite Materials (ICCM/14)*, San Diego, 2003.

[3] Ramkumar R L, Kulkarni S V and Pipes R B. Free Vibration Frequencies of a Delaminated Beam, 34th Annual Technical Conference. *1979 Reinforced Plastics/Composites Institute*, The Society of the Plastics Industry Inc., Section 22-E, pp 1-5, 1979.

[4] Wang J T S, Lin Y Y and Gibby J A. Vibrations of Split Beams. *Journal of Sound and Vibration*, Vol. 84, No. 4, pp 491-502, 1982.

[5] Mujumdar P M and Suryanarayan S. Flexural Vibrations of Beams with Delaminations. *Journal of Sound and Vibration*, Vol. 125, No. 3, pp 441-461, 1988.

[6] Tracy J J and Pardoen G C. Effect of Delamination on the Natural Frequencies of Composite Laminates. *Journal of Composite Materials*, Vol. 23, No. 12, pp 1200-1215, 1989.

[7] Hu J S and Hwu C. Free Vibration of Delaminated Composite Sandwich Beams. *AIAA Journal*, Vol. 33, No. 10, pp 1911-1918, 1995.

[8] Shen M H and Grady J E. Free Vibration of Delaminated Beams. *AIAA Journal*, Vol. 30, No. 5, pp 1361-1370, 1992.

[9] Yin W-L and Jane K C. Vibration of a Delaminated Beam-Plate Relative to Buckled States. *Journal of Sound and Vibration*, Vol. 156, No. 1, pp 125-140, 1992.

[10] Chen H-P. Free Vibration of Prebuckled and Postbuckled Plates with Delamination. *Composites Science and Technology*, Vol. 51, pp 451-462, 1994.

[11] Chang T-P and Liang J-Y. Vibration of Postbuckled Delaminated Beam-Plates. *International Journal of Solids and Structures*, Vol. 35, No. 12, pp 1199-1217, 1998.

[12] Lee B T, Sun C T and Liu D. An Assessment of Damping Measurement in the Evaluation of Integrity of Composite Beams. *Journal of Reinforced Plastics and Composites*, Vol. 6, No. 4, pp 114-125, 1987.

[13] Grady J E and Meyn E H. Vibration Testing of Impact-Damaged Composite Laminates. *Proceedings of 30th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, pp 2186-2193, 1989.

[14] Nagesh Babu G L and Hanagud S. Delamination in Smart Structures - A Parametric Study on Vibrations. *Proceedings of 31st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference*, pp 2417-2426, 1990.

[15] Hanagud S and Luo H. Modal Analysis of Delaminated Beam. *Proceedings of SEM Spring Conference on Experimental Mechanics*, Baltimore, pp 880-887, 1994.

[16] Luo H and Hanagud S. Delamination Detection Using Dynamic Characteristics of Composite Plates. *Proceedings of 36th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference*, pp 129-139, 1995.

[17] Kimpara I, Kageyama K, Suzuki T, Ohsawa I and Ide K. Vibration Mode Analysis of Delaminated Composite Laminates by Means of Vibration Pattern

Imaging and Finite Element Method. *Zairyo (Journal of the Society of Materials Science, Japan)*, Vol. 43, No. 487, pp 476-481, 1994, (in Japanese).

[18] Luo H and Hanagud S. Delaminated Beam Nonlinear Dynamic Response Calculation and Visualization. *Proceedings of 38th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference*, Vol. 1, 1997, pp 490-499, 1997.

[19] Luo H and Hanagud S. Dynamics of Delaminated Beams. *International Journal of Solids and Structures*, Vol. 37, pp 1501-1519, 2000.

[20] Zak A, Krawczuk M and Ostachowicz W. Vibration of a Laminated Composite Plate with Closing Delamination. *Key Engineering Materials*, Vols. 167-168, pp 17-26, 1999.

[21] Suzuki K, Kageyama K, Kimpara I, Hotta S, Ozawa T, Kabashima S and Ozaki T. Vibration and Damping Prediction of Laminates with Constrained Viscoelastic Layers - Numerical Analysis by a Multilayer Higher-Order-Deformable Finite Element and Experimental Observations. *Mechanics of Advanced Materials and Structures*, Vol. 10, No. 1, pp 45-73, 2003.

[22] Panda S. and Natarajan R. Analysis of Laminated Composite Shell Structures by Finite Element Method. *Computers and Structures*, Vol. 14, pp 225-230, 1981.

[23] Bathe K J. *Finite Element Procedures*. Prentice Hall, London, Chaps. 11, 1996.