ANALYSIS OF APPROXIMATION OF AIRCRAFT STOCHASTIC MOTION BY MARKOV MODELS

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Abstract

A new method based on the multidimensional photogrammetry was developed for low cost investigation of the uncontrolled flights with use of free flying models. The method was tested in measurement of the poststall motion of small aircraft. The measurements of flight characteristics representing in time series can be approximated with Markov process for description of flight characteristics and flight situations.

The lecture deals with investigation on possible approximation of real stochastic flying processes by Markov process. In this paper the optimisation of the state space discretisation, complexity of the approximation processes, deterministicity of the measured processes and uncertainties in approximating models are analysed.

1. Introduction

The department of Aircraft and Ships at the Budapest University of technology and Economics has an interesting research called Unconventional Flight Analysis [1, 2]. One of the main topic of this research is investigation of aircraft motion at very high angles of attack, like poststall motion of fighters [3] or motion of large passenger aircraft after loosing the conventional control. Such motion of aircraft is very nonlinear and stochastic. A new method based on the multidimensional photogrammetry was developed for low cost investigation of such flights with use of free flying models [4, 5]. Principally all the critical flights can be measured by using this methods and photogrammetry can be applied as visual monitoring system for surveillance of air traffic in airport regions.

The newly developed method was tested in measurement of the postal motion of small aircraft [5]. The measurements of flight characteristics representing the time series can be approximated by Markov process for general description of flight characteristics and flight situations [5].

The lecture deals with investigation on possible approximation of real stochastic flying processes by Markov process. In this approximation the real flight is a stochastic process of continuous time and state space and the Markov chain is a process of continuous time and discrete state space.

Principally there is a lack of knowledge about the motion of aircraft at very high angles of attack. We have no accurate aerodynamic models, enough information about the appearing the stochastic disturbances and effects. So, there are four problems [5] associated with approximation at least:

- determination of the optimal state space discretisation – for better, finally for estimating the best approximation model,
- study of Markov process complexity – for finding the simplified approximation model,
- investigation on deterministicity – for definition of deterministic part of approximation process and losses of deterministic effects,
- investigation on uncertainties – for calculation the changes in uncertainties of approximation model, generated by stochastic disturbances.

This paper describes the solving of given problems and application of Markov model into stochastic, or better to say statistical flight dynamics.

So, the goal of this paper is the demonstration of possible use of newly developed methods to investigation of the aircraft stochastic motions and analysis of the possible approximation of the measured data by Markov processes.

The real data guided from flight measurements. The poststall motions of aircraft were recorded by two cameras. The images were used for determining the flight characteristics with application the photogrammetry. The flight data were applied in Markov model formations.

2. Measurement

The real motion of aircraft and free flying models can be investigated by using the methods of photogrammetry [4, 5]. In this case, the measurement is based on the movement grid of the multi dimensional photogrammetry. It means that, the 3D photogrammetric grid is defined to each measured time (Fig. 1.). The motion of aircraft is recorded by – at least – two cameras (or one stereo camera).

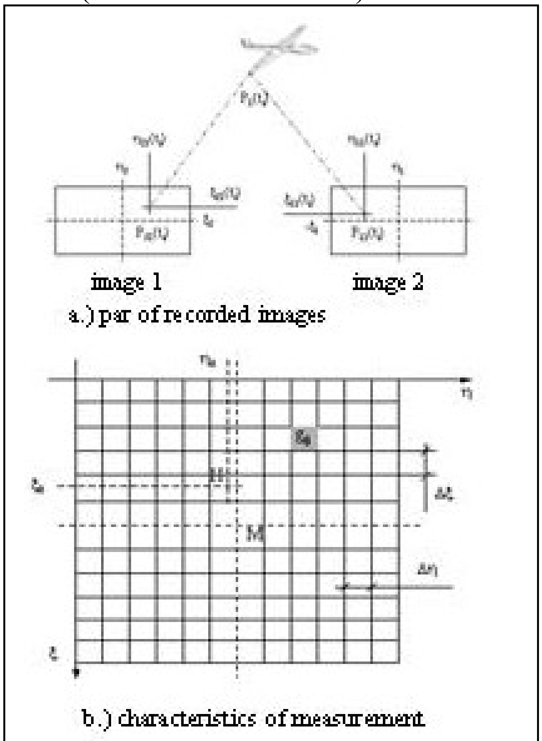
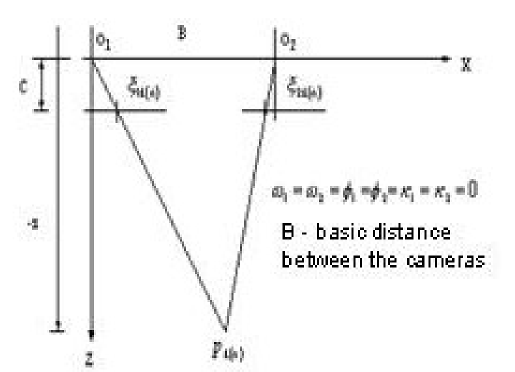


Fig. 1. Principle of measurement

The time series of pares of images are recorded. With use of the recorded images the positions, $\eta_{jk}(t_i)$, $\xi_{jk}(t_i)$ of preliminary identified control points can be defined. (Here the positions of the k-th identified control point measured by j-th camera at i-th time are defined). With using the methods of photogrammetry (Fig. 2.), special vectors describing the 3D position parallax of the each k-th identified points for series of t_i time [5]:

$$x_{k(ti)},\ y_{k(ti)},\ z_{k(ti)},\ p_{\xi(ti)} \qquad k=1,2,\cdots,n \quad i=1,2,\cdots,m. \tag{1}$$



$$-z_k(t_i)=\frac{c\,B}{\xi_{1k(ti)}-\xi_{2k(ti)}}=\frac{cB}{p_{\xi(ti)}},$$

$$y_k(t_i)=-z_{k(ti)}\frac{\eta_{1k(ti)}}{c}=-z_{k(ti)}\frac{\eta_{2k(ti)}}{c},$$

$$x_k(ti)=-z_{k(ti)}\frac{\xi_{1k(ti)}}{c}$$

Fig. 2. An x-z projection of the measurement and calculation of the positions of identified points

Finally, from the measurement the state vector defining the position of aircraft centre of gravity in 3D space and instantaneous turning position from initial one as angles between the initial and real positions of axis of applied reference system (as rotation around the centre of gravity) can be determined:

$$\left(x_{ti}^{0,j},\ y_{ti}^{0,j},\ z_{ti}^{0,j},\ \varphi_{ti}^{x,j},\ \varphi_{ti}^{y,j},\ \varphi_{ti}^{z,j}\right) \qquad i=1,2,\cdots,m \qquad j=1,2,\cdots,k \tag{2}$$

The developed method was tested in measurement (Fig. 3.) of poststall motion of small aircraft, Socata Tampico [5]. The recorded images are shown in Figure 4. The calculated changes in one of the coordinates (Fig. 5.), as an example shows real stochasticity of the measured motion of aircraft.

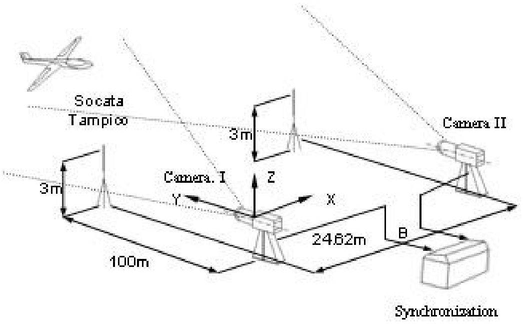


Fig. 3. Practical measurement

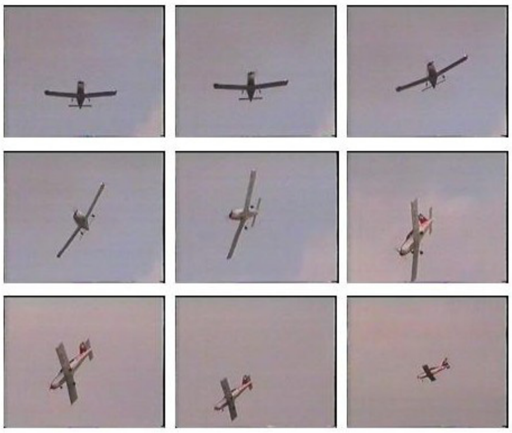


Fig. 4. Images recorded during investigation of the aircraft poststall motion

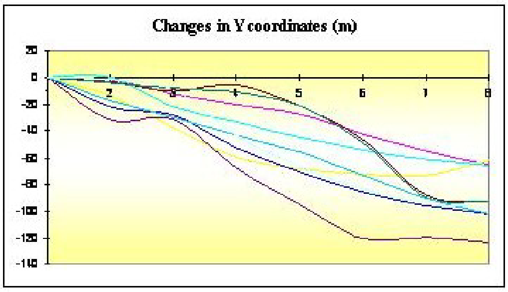


Fig. 5. Demonstration of real stochasticity of the measured data

3. Statistical flight dynamics

When examining the motion or technical condition of aircraft, it seems to be describable easily for an engineer [6] if the variation of its state vector **x** chosen appropriately is expressed as follows

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{u}, t) \tag{3}$$

In fact, the variation of state vector **x** is influenced by the variation in the instantaneous values of a number of factors (service conditions, methods of maintenance and repair applied, the realized management, the characteristics of the flight, the atmospheric conditions, etc.). These influences can be given in terms of stochastic processes, random variables or random space (turbulence of atmosphere). Moreover, state vector **x** can not generally be measured directly. Instead, some output signal vector **y** can be measured. Consequently, the controlled motion of the aircraft or their technical conditions, their dynamics can be described only by a much more complicated model than in (1), namely by the following general set of stochastic differential equations [6]:

$$\begin{aligned}
d\mathbf{x} &= f_x\,[\mathbf{x}(t),\mathbf{x}(t-\tau_x),\mathbf{p}(\mathbf{x},\mathbf{z},\omega,\mu,t),\mathbf{z}(\mu,t),\mathbf{u}(t),\omega,\mu,t]dt + \\
&\quad + \boldsymbol{\sigma}_x(\mathbf{x},\mathbf{p},\mathbf{z},\omega,\mu,t)\, d\mathbf{W}, \\
\mathbf{y} &= f_y\,[\mathbf{x}(t),\mathbf{x}(t-\tau_y),\mathbf{p}(\mathbf{x},\mathbf{z},\omega,\mu,t),\mathbf{z}(\mu,t),\mathbf{u}(t),\omega,\mu,t] + \\
&\quad + \boldsymbol{\sigma}_y(\mathbf{x},\mathbf{p},\mathbf{z},\omega,\mu,t)\,\boldsymbol{\xi}, \\
\mathbf{u}(t) &= f_u\,[\mathbf{x}(t),\mathbf{x}(t-\tau_u),\mathbf{p}(\mathbf{x},\mathbf{y},\omega,\mu,t),\mathbf{z}(\mu,t),\mathbf{u}(t),\omega,\mu,t], \\
\mathbf{x}(t=t_o) &= \mathbf{x}_o(t=t_o,\omega_o,\mu_o), \\
\mathbf{y}(t=t_o) &= \mathbf{y}_o(t=t_o,\omega_o,\mu_o);
\end{aligned} \tag{4}$$

where $\mathbf{x} \in R^n$ is the state vector, $\mathbf{p} \in R^k$ is the parameter vector characterizing the state of the aircraft, $\mathbf{z} \in R^l$ is the vector of environmental characteristics (vector of service conditions), $\mathbf{u} \in R^m$ is the input (control) vector, $\mathbf{y} \in R^r$ is the output (measurable) signal vector $\mathbf{W} \in R^s$ and $\boldsymbol{\xi} \in R^q$ are the noise vector (in simplified case the Wiener and Gaussian noise vectors respectively), $\boldsymbol{\sigma}_x$, $\boldsymbol{\sigma}_y$ are the noise transfer matrices, ω and μ are the random variables assigning the position of vectors **p** and **z** within admissible space Ω_p Ω_z described by density functions $f_p(\bullet)$, $f_z(\bullet)$, t is the time, and τ_x, τ_y, τ_u, are the time-delay vectors.

The first equation from set (29) can be given in shorter form [7, 8]:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, t) + \mathbf{b}(t) + \boldsymbol{\sigma}(\mathbf{x}, t)\boldsymbol{\eta}(t) \tag{5}$$

here $\mathbf{b}(t)$ is the control vector and $\boldsymbol{\eta}$ is a noise-vector. This equation can be linearised statistically with change the state vector as sum of its mean values and deviations, $\mathbf{x} = \bar{\mathbf{x}} + \Delta\mathbf{x}$:

$$\dot{\bar{\mathbf{x}}} + \Delta\mathbf{x} = \mathbf{F}(\bar{\mathbf{x}}, t) + \mathbf{S}(\bar{\mathbf{x}}, t)\Delta\mathbf{x} + \mathbf{b}(t) + \boldsymbol{\sigma}(\mathbf{x}, t)\boldsymbol{\eta}(t), \qquad (6)$$

where $\mathbf{S}(\bar{\mathbf{x}}, t)$ - is the sensitivity matrix. This is a basis of the statistical dynamics [7, 8].

From another hand the set of equations (5) is a general form of the following simplified system of equations:

$$\dot{\mathbf{x}} = \mathbf{LA} + \mathbf{LC} + \mathbf{NL} \qquad (7)$$

where **LA** represents a linear aerodynamic, LC is a linear control, and **NL** describes the nonlinear dynamics.

4. Markov model

The general model (4) can be assumed [2, 7, 8] in the form of a set of the following stochastic (random) differential equation

$$\dot{x} = f(x,t) + \sigma(x,t)\eta(t) \ , \qquad (8)$$

called as diffusion process. Of course this equation as the set of equations can be rewritten in the vector form. The first part at right hand of equation describes the direction of the changes of the stochastic process passing through the $x(t) = X$ at the moment t, while the second part shows the scattering the random process. In case of uncontrolled aircraft motion the disturbances are mostly generated by air turbulence.

The equation (8) is called as Markov process [9], because its realization in future depends, only, on the present realizations and does not depend on the past. Such type of process can be fully described by giving its transition probability density function

$$p(x_2, t_2 | X_1, t_1), \quad (t_2 \succ t_1), \qquad (9)$$

which characterises the distribution probability of the continuous random process, $x(t)$, at the moment t_2, if it is passing through the $x(t) = X$ at the time, t_1.

The transition probability density function can be described by application of the Fokker - Planck - Kolmogorov equations like:

$$\frac{\partial p(x_2, t_2 | X_1, t_1)}{\partial t_2} = -\frac{\partial}{\partial x_2}\left[f(x_2, t_2)p(x_2, t_2 | X_1, t_1)\right] + \\ + \frac{1}{2}\frac{\partial^2}{\partial x_2^2}\left[\sigma^2(x_2, t_2)p(x_2, t_2 | X_1, t_1)\right] , \qquad (10)$$

or

$$\frac{\partial(x, t)}{\partial t} = -\frac{\partial}{\partial x}\left[f(x, t)p(x, t)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[\sigma^2(x, t)p(x, t)\right]. \ (11)$$

The statistic flight mechanics [8] has worked out already several methods for application of this type of models. For example the statistical linearization defined by (6) through the proof of the sensitivity function matrix to the flight mechanic models and generating out the set of equation for the moments of the investigated stochastic process can used for study the scattering of the process depending on the changes in the initial condition. However the flight after loosing the control has a mach more complicated picture depending on the unknown aerodynamics characteristics not studied yet in this high angle of regions and disturbance generated by air turbulence.

According to the equations (10), (11) defining the Markov process the following definition [9] can be made:

$$p(X_2, t_2 | X_1, t_1) = \sum_{X(t)} p(X_2, t_2 | x, t)p(x, t | X_1, t_1), \\ (t_2 \geq t \geq t_1) \ , \qquad (12)$$

which equation is called as Chapman - Kolmogorov - Smoluchovski.

This equation gives possibility for approximation of the investigated non-linear stochastic process of continuous time and state space with the Markov chain of continuous time and discrete state space. In this case, the discretisation means that [5], the possible space of aircraft motion is divided into subspaces

$$\tilde{A}_{ti} = \{ \ A^i_{ti} \cap A^j_{ti} = 0 \quad \text{and} \quad \sum_{i \in \Lambda} A^i_{ti} = R^6 \ \} \ \forall t_i \qquad (13)$$

and the diffusion process of stochastic motion of aircraft is defined by transition matrix, $\mathbf{A}[k]$ as it shown in Figure 6. Here, in Figure 6, of course, the 3D space representation is demonstrated, only. In investigation the vector (2) was used as state vector **x**. The elements of matrix **A** depend on the dynamics of real motion of aircraft, e.g. aerodynamics, control and environmental characteristics (like temperature, pressure, air turbulence, wind, etc.)

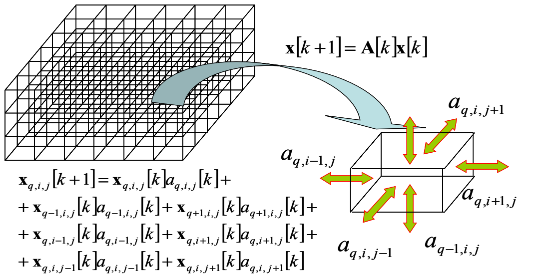


Fig. 6. Principle of approximation of the aircraft motion by Markov chain (discretisation of the space of motion)

Principally, the discretisation has to be connected with the measurement, i.e. sampling of measurement. Optimization of the discretisation means that the way of definition of the quantization levels, has to minimize the errors in approximation. In this case, the expected values and the variances for each elements of the state vector (2) can be calculated as:

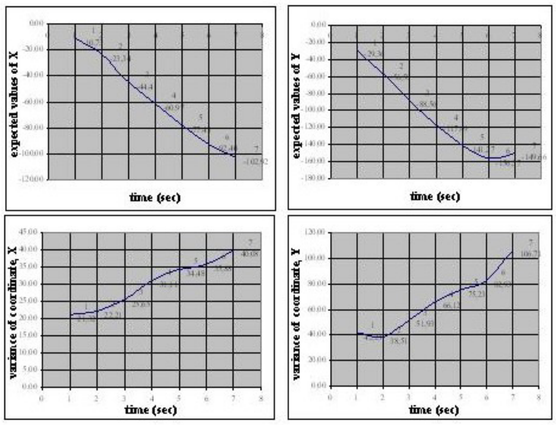
$$\overline{\mathbf{x}}(t) = \sum_{\mathrm{i}} \mathbf{P}_{\mathrm{i}}(t)\mathbf{x}_{\mathrm{i}}^{*}, \tag{14}$$

$$\boldsymbol{\sigma}(t) = \sqrt{\sum_{\mathrm{i}} \mathbf{P}_{\mathrm{i}}(t)\mathbf{x}_{\mathrm{i}}^{*2} = [\overline{\mathbf{x}}(t)]^{2}}\ , \tag{15}$$

where **P** matrix defines the probabilities of “staying” the aircraft in the given subspace of discretised state space, $\mathbf{x}^{*}$ is vector contains the quantization levels of the initial elements of state vector and **i** means $\mathbf{i}^{T} = (i, j, k, l, m)$ and the elements of this vector are changing as 0, 1, 2, … n. So, the optimal quantization can be found with minimization of the variance (15).

The statistical characterization of the measured changes in aircraft position (motion of centre of gravity) is demonstrated by Figure 7.

The discretisation was optimized. The quantization levels were defined as the middle values of the discrete subspaces that were calculated as prisms. Figure 8. demonstrates the optimised quantization determined for measured coordinates X.



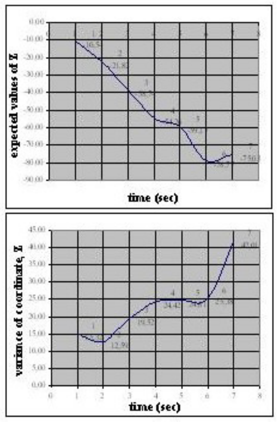


Fig. 7.: Statistical characterization of the measurement (motion of aircraft centre of gravity)
(Most interesting result is variance of coordinate Z, which is not changing between 4 and 5 seconds)

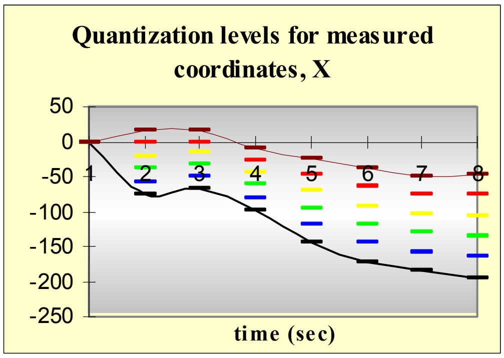


Fig. 8.: Result of the quantization optimization

5. Markov model features

Some features of the Markov models were studied with goal of analyzing the models applied for describing the aircraft poststall motion.

5.1. Complexity

Nowadays, the different types of Markov models (simple, semi-, maximum entropy, hidden, hierarchical, embedded, 2nd and more order, etc.) are applied. Principally, the different models try to take into account some specific features of the real stochastic processes, like unobservable states, superstates, or dependence on the previous states, etc. These models are developed for investigation of nonlinear dynamic systems, speech enhancement, face recognition, etc.

In our investigation the motion of aircraft was measured by using the video cameras. Speed of recording was high comparing to aircraft motion dynamics. So, all the possible states were observed. However the delay in changing in aerodynamics can generate some problems. In general case, the changes in the state vector elements can be defined as

$$P(X = X_{t1}X_{t2}\cdots X_{tn}) = P(X_{t1})P(X_{t2} \mid X_{t1})P(X_{t3} \mid X_{t1}X_{t2})\ldots \cdots P(X_{tn} \mid X_{t1}\cdots X_{tn-1})\,. \qquad (16)$$

In simple case, with use of Markov assumptions, the measured stochastic processes of changing in state vector can be approximated by discrete stochastic process that is characterized by

$$P\{X_{tn} \in A_{tn} \,|\, X_{t1}, X_{t2}, \cdots, X_{tn-1}\} = P\{X_{tn} \in A_{tn} \,|\, X_{tn-1}\} \qquad (17)$$

or

$$P\{X_{tn} = a_{tn} \,|\, X_{tn-1} = a_{tn-1}\}\,. \qquad (18)$$

This is a first order Markov chain. In more general case, instead of (17) the following models should apply:

$$P\{X_{tn} = a_{tn} \,|\, X_{tn-1} = a_{tn-1}, \cdots, X_{tn-S} = a_{tn-S}\} \quad S \geq 2,3,\cdots \qquad (19)$$

It means that, the embedded Markov models can be constructed with use of the model family:

$$M_{tn}^{S} \leftrightarrow P_{tn}^{S}\{X_{tn} = a_{tn} \,|\, X_{tn-1} = a_{tn-1}, X_{tn-2} = a_{tn-2} \cdots, X_{tn-S} = a_{tn-S}\},$$

$$\text{for} \quad S \geq 2,3,\cdots \quad t_{n-S} \leq t_{n-1}\,. \qquad (20)$$

In this way, the model series, $M_{tn}^1, M_{tn}^2, \cdots, M_{tn}^S$, is built up, in which the P_{tn}^i $i = 1,2,\cdots,S$ represents the transition probability function of the i times completed processes. For example, in case of M_{tn}^i $i = 1$, the associated transition probability function, P_{tn}^i $i = 1,$ $n = 1,2,\cdots,m$ results the mathematical model $\mathbf{P}_{tn}^1$ $n = 1,2,\cdots,m$ in form of series of the stochastic matrices:

$$\mathbf{P}_{tn}^1 \quad n = 1,2,\cdots,m \qquad p_{ij,tn}^1 \geq 0 \;\; \forall\, i, j, n$$
$$and \;\; \sum_{j=1}^{n} p_{ij,tn}^1 = 1 \;\; \forall\, i \qquad (21)$$

The complexity of the applied embedded Markov models was studied [5, 10] by using the method of statistical hypothesis investigation. We found that [5], the poststall motion of aircraft can not be approximated by steady Markov process, the non-steady models are the one times embedded (2nd order) models, the resulting models can be simplified with approximation of state vector elements separately.

5.2. Deterministicity

The poststall motion of aircraft or motion of aircraft after loosing its conventional control is a stochastic process. The state vector representing the dynamics of the motion has a deterministic part described by system of equations of motion developed by flight mechanics and a stochastic part appearing as deviations in characteristics.

There is an interesting and important question how far the deterministic part defines the real stochastic motion of aircraft, or measured processes of changing in state vector describing the investigated motion. This is the deterministicity (or the opposite phenomena, stochasticity) of the measured processes. We have introduced a merit of deterministicity [5] and we have used it for investigation of the aircraft poststall motion.

Let suppose that the results of measurement (2) result the set of variates

$$r_{ti} = \left(x_{ti}^{j,1}, x_{ti}^{j,2}, \cdots, x_{ti}^{j,6}\right) \in R^6 \quad i = 1, 2, \cdots, m$$
$$j = 1, 2, \cdots, k \qquad (22)$$

measured with use of principle independent methods of measurements $j = 1, 2, \cdots, k$.

Let the covariance matrix of the n dimensional variate vector

$$\mathbf{D}(t_i) \in A^{6\times 6}, \tag{23}$$

where $d_{ab}(t_i) = E\left\{\left(x_{ti}^a - m_{ti}^a\right)\left(x_{ti}^b - m_{ti}^b\right)\right\}$,
and $m_{ti}^a = E\left\{x_{ti}^a\right\}$.
The equation describes the concentration ellipsoid is

$$\sum_{a=1}^{n}\sum_{b=1}^{n}\frac{\Delta_{ab} y_a y_b}{\Delta} = n + 2, \tag{24}$$

where Δ_{ab} is the subdeterminant of matrix $\mathbf{D}$ associated with elements of d_{ab} and $\Delta = \left|\mathbf{D}_{ab}\right|$.

For this case, the concentration of the variate r_{ti} is

$$k(r_{ti}) = \frac{\Gamma\left(\frac{n}{2}+1\right)}{(n+2)^{\frac{n}{2}}\pi^{\frac{n}{2}}\sqrt{\Delta}}, \tag{25}$$

here $\sqrt{\Delta} = \sqrt{\det(\mathbf{D})}$.

The $k(r_{ti})$ defines the random deviation in the measured characteristics and it can be called as a merit of deterministicity, or in form of

$$K_{Stoc}(r_{ti}) = \frac{1}{k(r_{ti})} \tag{26}$$

as merit of stochasticity [5].

The $k(r_{ti})$ can be estimated by its statistical characteristics calculated for each t_i (Fig. 9.) and the stochasticity or deterministicity can be defined by operators

$$T_{\det}\left(x_{ti}^{j,z}\ i=1,\cdots,m\ \ j=1,\cdots,k\ \ z=1,\cdots,6\right) \to$$
$$\to \left(\frac{1}{k(r_{t1})},\frac{1}{k(r_{t2})}\cdots,\frac{1}{k(r_{tm})}\right) = K_{Stoc}, \tag{27}$$

and

$$T_{\det}\left(x_{ti}^{j,z}\ i=1,\cdots,m\ \ j=1,\cdots,k\ \ z=1,\cdots,6\right) \to$$
$$\to \left(k(r_{t1}),k(r_{t2})\cdots k(r_{tm})\right) = K_{\det}. \tag{28}$$

The stochasticity in measured characteristics is shown in Figure 10. [5].

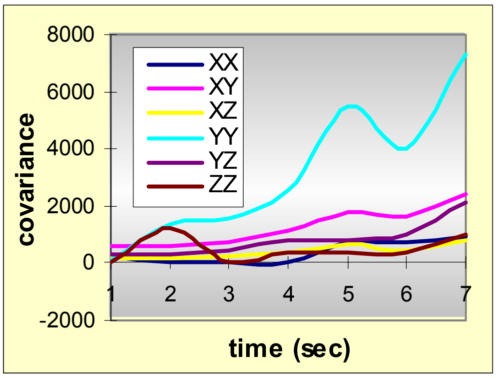


Fig. 9.: Statistical characteristics of measured changes in coordinates defining the position of aircraft centre of gravity

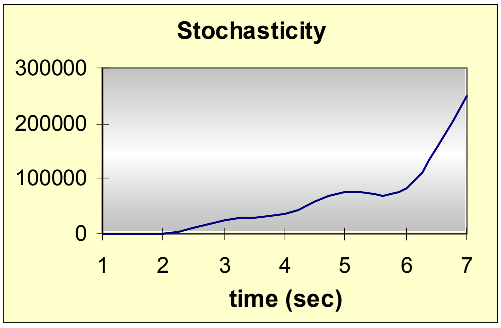


Fig. 10.: Calculated merit of stochastic-

5.3. Uncertainty

The uncertainties in measured stochastic processes as important information about the features of the investigation can be evaluated by determining the entropy that is defined in Mathematics [11] for X random variable with P(x) probability that X is in the state x, as:

$$H(X) = -\sum_x P(x)\log_2[P(x)]. \tag{29}$$

In our investigation [5, 10], the measured vector filed (2) as it given by d into disjunct subspaces A_{li} defined by (13). The series of distributions

$$P_{ti} = \left\{A_l \to \tilde{P}_{ti}(A_l)\quad \forall l \in \Delta\right\}\quad i = 1,2,\cdots m \tag{30}$$

was estimated from measurement for each t_i. Finally a special operator,

$$T_{unc}\left\{\left(\widetilde{P}_{t1}(A_l)\ \ l\in\Delta\right),\left(\widetilde{P}_{t2}(A_l)\ \ l\in\Delta\right),\cdots,\left(\widetilde{P}_{tm}(A_l)\ \ l\in\Delta\right)\right\}\rightarrow$$

$$\rightarrow (H_{t1},H_{t2},\cdots,H_{tm})=K_{unc}\in R^m\,, \qquad (31)$$

was determined for evaluating the uncertainties in measured processes.

One of the interesting results of investigation [5] is shown in Figure 11. On another hand, we have could not find any trend in changes of uncertainties.

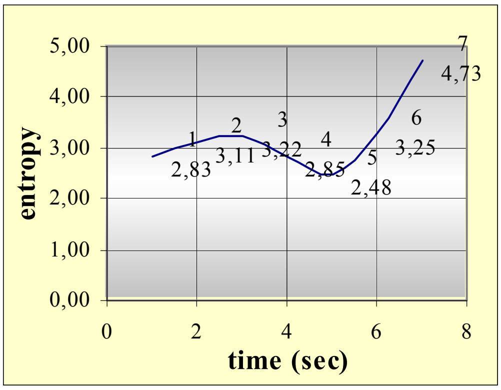


Fig. 11.: Entropy associated with mo-

6. Conclusions

A new method of measuring the aircraft stochastic motion, like its poststall motion was described. This method is based on the photogrammetry. The results of measurement were analysed. The measurement shows that, the flying characteristics, or elements of state vector describing the aircraft poststall motion are changing in form of non-steady stochastic processes, which can be described by Gaussian processes. The method of approximation of measured characteristics with discrete Markov processes was applied to description of the aircraft postal motion. The features of Markov approximation like optimization of the discretisation, stochasticity, deterministicity, uncertainty were analysed, too.

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