ON THE GENERATION OF SOUND IN A SHEAR FLOW

L.M.B.C. Campos , M. H. Kobayashi
***Instituto Superior Técnico, Secção de Mecânica Aeroespacial, ISR, 1049-001 Lisbon, Portugal,**
****University of Hawai'i at Manoa, Department of Mechanical Engineering, 2540 Dole Street H302, 96822 Honolulu, HI, USA**

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Abstract

The Lighthill (1952) acoustic analogy describing the generation of sound by turbulence and inhomogeneities in an unbounded medium at rest is extended to a plane unidirectional shear mean flow. Unlike previous attempts at this result (Lilley 1974), there are no ambiguous terms: (i) the linear, non-dissipative terms form the acoustic wave equation in a plane unidirectional shear flow (Haurwitz 1932, Campos & Serrão 1998), with the acoustic pressure as variable; (ii) all the remaining terms are non-linear or dissipative, and specify the sources of sound in a shear flow, generalizing the original Lighthill tensor. The sources of sound consist of three terms, modelling turbulence, inhomogeneities and dissipative effects, broadly similar to the original Lighthill tensor, with additional three terms proportional to the vorticity of the mean flow. The Green's function for the acoustic wave equation in a plane unidirectional shear flow is obtained exactly for all frequencies. The result applies to any shear velocity profile and is illustrated for the hyperbolic tangent shear layer. The acoustic pressure due to a point monopole source is plotted as a function of the coordinate transverse to the shear layer, for several source positions in the shear flow, and several values of the shear layer thickness, free stream Mach number and angle of incidence.

1 Introduction

The original acoustic analogy (Proudman 1952, Lighthill 1952) concerned the generation of sound by turbulence (Lighthill 1954, 1961) in a region of an unbounded medium otherwise at rest. It was soon extended to include the effect of solid boundaries at rest (Curle 1955) or in motion (Ffowcs-Williams & Hawkings 1968). Other extensions include the generation of sound by two-phase flow (Crighton & Ffowcs-Williams 1969) and by fluid inhomogeneities (Howe 1975). The latter uses the stagnation enthalpy as wave variable (Campos 1978) and involves the high-speed wave equation, valid for sound in a steady potential mean flow of arbitrary Mach number (Campos 1986). Another extension concerns sound generation in a shear flow (Lilley 1974, Mani 1976); the latter has a formal inconsistency, in that a linear term involving the shear mean flow appears among the non-linear source terms. A complete separation of linear, propagation terms from non-linear, source terms is required for the acoustic analogy to hold (Doak 1998). In the present paper the acoustic analogy is extended to a two-dimensional unidirectional shear mean flow, enforcing a strict separation between linear, propagation terms and non-linear, source terms, so that no ambiguities arise.

The approach to ensure this result is fairly simple: the wave equation in a shear flow (Haurwitz 1932, Pridmore-Brown 1958, Möhring, Müller & Obermeier 1983, Campos & Serrão 1998, Campos & Kobayashi 2000) is rederived (§2), eliminating among the linear, non-dissipative terms exactly in the same way (§2.1), but retaining the non-linear and dissipative terms, to identify the sound sources (§2.2), in an inhomogeneous shear flow (§2.3). This method provides, in a single approach; (i) the wave operator describing linear, non-dissipative sound waves; (ii) the non-linear terms modelling the sources which generate sound; (iii) the dissipative terms specifying the mechanisms responsible the decay of acoustic energy. Whereas the Lilley equation has been solved for high-frequency sound (Goldstein 1976), the present acoustic wave equation with source in a shear flow can be solved exactly all frequencies; the sound fields are illustrated by the Green's function for the hyperbolic tangent (Campos & Kobayashi 2000). The modulus and phase of the acoustic pressure are plotted versus distance across the shear layer for several monopole source position (Figure 1), shear layer thickness (Figure 2), free stream Mach numbers (Figure 3) and angles of incidence (Figure 4).

2 Acoustic wave equation in a shear flow

In order to obtain the complete acoustic wave equation elimination is performed among the linear, non-dissipative terms (§2.1), leading to the wave operator (§2.2) describing propagation, while retaining all other terms (§2.3); the non-linear terms model the sources responsible for wave generation, and the dissipative terms specify the decay mechanisms.

2.1 Separation of linear, non-dissipative and other terms

The starting point to study the generation of sound in a shear flow are the general equations of fluid mechanics, namely the equation of continuity:

$$\frac{D\rho}{dt} + \rho\nabla.\mathbf{V} = 0, \tag{1}$$

where

$$\frac{D}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{V}.\nabla, \tag{2}$$

is the material derivative, ρ is the mass density and $\mathbf{V}$ the velocity; the equation of momentum:

$$\frac{\partial V_i}{\partial t} + V_j\frac{\partial V_i}{\partial x_j} + \frac{1}{\rho}\frac{\partial \mathrm{P}}{\partial x_i} = \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j}, \tag{3}$$

where P is the pressure field and σ the viscous stress tensor; and the equation of state $\mathrm{P} = \mathrm{P}(\rho, s)$, expressing pressure as a function of density and entropy.

$$\frac{D\mathrm{P}}{dt} = c^2\frac{D\rho}{dt} + \beta\frac{Ds}{dt}, \tag{4}$$

where c is the adiabatic speed of sound

$$c^2 \equiv \left.\frac{\partial \mathrm{P}}{\partial \rho}\right|_s, \tag{5}$$

and β a thermodynamic parameter

$$\beta \equiv \left.\frac{\partial \mathrm{P}}{\partial s}\right|_\rho \tag{6}$$

The total state is assumed to consist of a mean steady unidirectional shear flow in two dimensions:

$$\mathbf{V} = U(y)\mathbf{e}_x + u(x,y,t)\mathbf{e}_x + v(x,y,t)\mathbf{e}_y, \tag{7}$$

plus unsteady and non-uniform perturbations:

$$\rho = \rho_0(y) + \rho'(x,y,t), \tag{8}$$
$$\mathrm{P} = p_0 + p(x,y,t); \tag{9}$$

note that the continuity equation for the mean state allows the mean density ρ_0 to depend on the transverse coordinate:

$$\nabla.\left[\rho_0(y)\,U(y)\,\mathbf{e}_x\right] = 0, \tag{10}$$

and the momentum equation for the non-dissipative mean state implies a constant mean pressure p_0:

$$\nabla p_0 = \rho_0(y)U(y)\frac{\partial U(y)}{\partial x} = 0. \tag{11}$$

When substituting (7–9) in (1,3,4) the linear terms, involving the linearized material derivative

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + U(y)\frac{\partial}{\partial x}, \tag{12}$$

and sound speed for the mean flow:

$$c_0^2 \equiv \left.\frac{\partial p}{\partial \rho}\right|_{s_0} \tag{13}$$

are collected on the left hand side (lhs) and the other non-linear and dissipative terms, are collected on the right hand side (rhs). The preceding procedure leads to:

$$\begin{aligned}\frac{du}{dt} + vU' + \frac{1}{\rho_0}\frac{\partial p}{\partial x} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + \\ \left(\frac{1}{\rho_0} - \frac{1}{\rho}\right)\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \sigma_{xj}}{\partial x_j} \equiv X,\end{aligned} \tag{14}$$

for the x-component of the momentum equation, where prime denotes derivative with regard to y, e.g. $U' \equiv dU/dy = -\Omega_0$ specifies the mean flow vorticity. The y-component of the momentum equation:

$$\begin{aligned}\frac{dv}{dt} + \frac{1}{\rho_0}\frac{\partial p}{\partial y} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + \\ \left(\frac{1}{\rho_0} - \frac{1}{\rho}\right)\frac{\partial p}{\partial y} + \frac{1}{\rho}\frac{\partial \sigma_{yj}}{\partial x_j} \equiv Y,\end{aligned} \tag{15}$$

completes the system of equations, together with the equation of continuity (1) and state (4) combined:

$$\frac{1}{c^2}\frac{Dp}{dt} - \frac{\beta}{c^2}\frac{Ds}{dt} + \rho\nabla.\mathbf{V} = 0, \tag{16}$$

in the form separating linear, non-dissipative terms on the rhs:

$$\begin{aligned}\frac{1}{c_0^2}\frac{dp}{dt} + \rho_0\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \left(\frac{1}{c_0^2}\frac{dp}{dt} - \frac{1}{c^2}\frac{Dp}{dt}\right) + \\ (\rho_0 - \rho)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\beta}{c^2}\frac{Ds}{dt} \equiv Z,\end{aligned} \tag{17}$$

from other terms on the rhs.

2.2 Acoustic wave equation with source terms

In order to obtain the acoustic wave operator in a shear flow, the elimination is performed only among the linear, non-dissipative terms on the lhs of (14,15,17), although all other terms on the rhs are retained, to allow identification of sound sources and dissipation mechanisms. From (12) follow the commutation relations:

$$\frac{\partial}{\partial x}\frac{d}{dt} = \frac{d}{dt}\frac{\partial}{\partial x}, \tag{18}$$

$$\frac{\partial}{\partial y}\frac{d}{dt} = \frac{d}{dt}\frac{\partial}{\partial y} + U'\frac{\partial}{\partial x}, \tag{19}$$

which are used when applying the material derivative to (17), viz.:

$$\frac{d}{dt}\left(\frac{1}{c_0^2}\frac{dp}{dt}\right) + \rho_0\left(\frac{\partial}{\partial x}\frac{du}{dt} + \frac{\partial}{\partial y}\frac{dv}{dt} - U'\frac{\partial v}{\partial x}\right) = \frac{dZ}{dt}, \tag{20}$$

where can be substituted (14,15):

$$\begin{aligned}\frac{1}{c_0^2}\frac{d^2p}{dt^2} + \rho_0\frac{\partial}{\partial x}\left(-\frac{1}{\rho_0}\frac{\partial p}{\partial x} - vU' + X\right) + \\ \rho_0\frac{\partial}{\partial y}\left(-\frac{1}{\rho_0}\frac{\partial p}{\partial y} + Y\right) - \rho_0 U'\frac{\partial v}{\partial x} = \frac{dZ}{dt},\end{aligned} \tag{21}$$

leading to:

$$\begin{aligned}\frac{1}{c_0^2}\frac{d^2p}{dt^2} - \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) + \frac{\rho_0'}{\rho_0}\frac{\partial p}{\partial y} - \\ 2\rho_0 U'\frac{\partial v}{\partial x} = \frac{dZ}{dt} - \rho_0\left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y}\right),\end{aligned} \tag{22}$$

Note that, in the absence of mean shear flow $U' = 0$, the lhs of (22) is the high-speed wave operator (Howe 1975, Campos 1978, 1986) applied to the acoustic pressure, and thus the rhs represents the sources of sound for a uniform mean flow. In the presence of mean shear flow, the acoustic velocity perturbation is eliminated from the last term on the lhs of (22), by applying the linearized material derivative (12) once more:

$$\begin{aligned}\frac{d}{dt}\left[\frac{1}{c_0^2}\frac{d^2p}{dt^2} - \nabla^2 p + \nabla(\log\rho_0).\nabla p\right] - \\ 2\rho_0 U'\frac{\partial}{\partial x}\frac{dv}{dt} = \frac{d^2 Z}{dt^2} - \rho_0\frac{d}{dt}\left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y}\right),\end{aligned} \tag{23}$$

and substituting (15):

$$\frac{d}{dt}\left[\frac{1}{c_0^2}\frac{d^2p}{dt^2}-\nabla^2 p+\nabla(\log\rho_0).\nabla p\right]+ 2U'\frac{\partial^2 p}{\partial x\partial y}=q, \tag{24}$$

where

$$q\equiv\frac{d^2Z}{dt^2}-\rho_0\frac{d}{dt}\left(\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}\right)+2\rho_0U'\frac{\partial Y}{\partial x}. \tag{25}$$

The lhs of (24) is the acoustic wave operator for sound in a unidirectional shear flow (Campos & Serrão 1998, Campos & Kobayashi 2004), which, in the case of uniform mass density, simplifies to:

$$\rho_0'=0:\quad \frac{d}{dt}\left[\frac{1}{c_0^2}\frac{d^2p}{dt^2}-\nabla^2 p\right]+2U'\frac{\partial^2 p}{\partial x\partial y}=q, \tag{26}$$

which is well-known for a long time (Haurwitz 1932, Pridmore-Brown 1958, Möhring et al. 1983)). Unlike with the Lilley equation (Lilley 1974, Mani 1980, Goldstein 1976) all terms on the rhs of (24) are non-linear or dissipative, and thus there is no ambiguity with propagation terms.

The source term q in (24) involves the rhs X of (14), and Y of (15) and Z of (17), which must be substituted to allow the identification of sound sources and dissipation mechanisms. The term:

$$\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}=-\nabla.[(\mathbf{v}.\nabla)\mathbf{v}]+ \frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial\sigma_{ij}}{\partial x_j}\right)+\nabla.\left[\left(\frac{1}{\rho_0}-\frac{1}{\rho}\right)\nabla p\right], \tag{27}$$

involves the velocity perturbations $\mathbf{v}\equiv u\,\mathbf{e}_x+v\,\mathbf{e}_y$:

$$\nabla.[(\mathbf{v}.\nabla)\mathbf{v}]=\frac{\partial}{\partial x_i}\left(w_j\frac{\partial w_i}{\partial x_j}\right) =\frac{\partial^2}{\partial x_i\partial x_j}(w_iw_j)-\frac{\partial}{\partial x_i}\left(w_i\frac{\partial w_j}{\partial x_j}\right), \tag{28}$$

where the first term on the rhs of (28) is the double divergence of the Reynolds stresses per unit mass, and the second term involves the dilatation: $\nabla.\mathbf{v}=\frac{\partial w_i}{\partial x_i}$; the double divergence of the Reynolds stresses appears in the original acoustic analogy (Lighthill 1952, 1961, 1978) as the turbulent source of sound. An alternative way to write the first term on the rhs of (27):

$$\nabla.[(\mathbf{v}.\nabla)\mathbf{v}]=\nabla.[\mathbf{v}\times(\nabla\times\mathbf{v})]+\frac{1}{2}\nabla^2\mathbf{v}^2, \tag{29}$$

involves Lamb's vector:

$$\mathbf{L}=\mathbf{v}\times(\nabla\times\mathbf{v}), \tag{30}$$

which models the generation of sound by vorticity (Powell 1968, Howe 1975, Campos 1977).

2.3 Sound generation in inhomogeneous shear flows

The source term (25) involves, besides (27) also (17), where the mean flow vorticity does not intervene:

$$\begin{aligned}q_1\equiv{}&\frac{d}{dt}\left[\rho_0\frac{\partial^2}{\partial x_i\partial x_j}(w_iw_j)\right]-\frac{d}{dt}\left[\rho_0\frac{\partial}{\partial x_i}\left(w_i\frac{\partial w_j}{\partial x_j}\right)\right]\\&-\rho_0\frac{d}{dt}\left[\frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial\sigma_{ij}}{\partial x_j}\right)\right]\\&-\rho_0\frac{d}{dt}\left\{\nabla.\left[\left(\frac{1}{\rho}-\frac{1}{\rho_0}\right)\nabla p\right]\right\}\\&+\frac{d^2}{dt^2}\left(\frac{1}{c_0^2}\frac{dp}{dt}-\frac{1}{c^2}\frac{Dp}{dt}\right)\\&+\frac{d^2}{dt^2}\left[(\rho_0-\rho)\frac{\partial w_i}{\partial x_i}\right]+\frac{d^2}{dt^2}\left(\frac{\beta}{c^2}\frac{Ds}{dt}\right),\end{aligned} \tag{31}$$

plus a term proportional to the vorticity of the mean flow:

$$\Omega_0\equiv-U':\quad q(\mathbf{x},t)\equiv q_1(\mathbf{x},t)+2U'q_2(\mathbf{x},t) \tag{32}$$

given by (15):

$$\begin{aligned}q_2(\mathbf{x},t)={}&\rho_0\frac{\partial Y}{\partial x}\\={}&-\rho_0\frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)\\&+\frac{\partial}{\partial x}\left[\left(1-\frac{\rho_0}{\rho}\right)\frac{\partial p}{\partial y}\right]+\frac{\rho_0}{\rho}\frac{\partial\sigma_{yj}}{\partial x_j},\end{aligned} \tag{33}$$

which contrasts with (31) because: (i) in (31) the spatial coordinates $x_i \equiv (x, y)$ and velocity components $\mathbf{v} = (u, v)$ appear symmetrically; (ii) in (33) there is a lack of symmetry between the mean flow x and mean shear y directions.

Turning to the non-shear sound sources (31), they consist of three sets of terms. The first:

$$q_{11} \equiv \frac{d}{dt}\left[\rho_0 \frac{\partial^2}{\partial x_i \partial x_j}(w_i w_j)\right] - \frac{d}{dt}\left[\rho_0 \frac{\partial}{\partial x_i}\left(w_i \frac{\partial w_j}{\partial x_j}\right)\right], \tag{34}$$

represents the generation of sound by turbulence; in the case of an homogeneous mean flow $\rho_0 = \text{const}$:

$$q_{11} = \frac{d}{dt}\left[\frac{\partial^2}{\partial x_i \partial x_j}(\rho_0 w_i w_j) - \rho_0 \frac{\partial}{\partial x_i}\left(w_i \frac{\partial w_j}{\partial x_j}\right)\right], \tag{35}$$

the first term is minus the linearized material derivative (12) of the double divergence of the Reynolds stresses $\rho_0 w_i w_j$ calculated for the velocity perturbation w_i and mean flow mass density ρ_0; it corresponds to the first term in the Lighthill tensor:

$$T_{ij} = \rho w_i w_j + (p' - c_0^2 \rho')\delta_{ij} + \sigma_{ij}. \tag{36}$$

However, even in an homogeneous shear flow, the sound generation by turbulence involves an additional term, the second on the rhs of (26), which relates to the dilatation. The second term in the Lighthill tensor (36), represents sound generation by fluid inhomogeneities, and is here replaced by:

$$q_{12} \equiv -\rho_0 \frac{d}{dt}\left\{\nabla.\left[\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)\nabla p\right]\right\} - \frac{d^2}{dt^2}[\rho'(\nabla.\mathbf{v})] + \frac{d^2}{dt^2}\left(\frac{1}{c_0^2}\frac{dp}{dt} - \frac{1}{c^2}\frac{Dp}{dt}\right), \tag{37}$$

where: (i) the first term involves the total mass density ρ and mean flow density ρ_0 and the pressure gradient due to waves (there is no pressure gradient in the mean flow); (ii) the second term involves the density perturbation $\rho' = \rho - \rho_0$ and the acoustic dilatation, (iii) the third term involves the exact, non-linear sound speed (5) and the linearized sound speed for the mean flow (13), and the exact, non-linear material derivative (2) and the linearized material derivative (12). The third set of terms unrelated to mean flow vorticity represents dissipation:

$$q_{13} = -\rho_0 \frac{d}{dt}\frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j}\right) + \frac{d^2}{dt^2}\left(\frac{\beta}{c^2}\frac{Ds}{dt}\right), \tag{38}$$

where $q_1 \equiv q_{11} + q_{12} + q_{13}$; the first term in q_{13} corresponds to the viscous stresses in the third term of the Lighthill tensor (36), except for density effects. If the mean flow is homogeneous $\rho_0 = \text{const}$:

$$q_{13} - \frac{d^2}{dt^2}\left(\frac{\beta}{c^2}\frac{Ds}{dt}\right) = -\frac{d}{dt}\frac{\partial}{\partial x_i}\left(\frac{\rho_0}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j}\right) \tag{39}$$

these density effects disappear $\rho_0/\rho \sim 1 - \rho'/\rho$ in the linear approximation to the dissipative terms. The second dissipative term involves the exact material derivative of the entropy and thermodynamic coefficient (6), and models the effect of heat release and mean flow entropy gradients.

Turning to the terms (33) proportional to the mean flow vorticity $\Omega_0 \equiv U'$—see (32), the dissipative component:

$$q_{23} \equiv -\frac{\partial \sigma_{yj}}{\partial x_j} = -\left(\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}\right), \tag{40}$$

involves only the viscous stresses in the shear direction, in contrast with the presence of all viscous stresses in the non-shear term (38). The shear term (33) also involves a term corresponding to sound generation by inhomogeneities:

$$q_{22} = \frac{\partial}{\partial x}\left[\left(1 - \frac{\rho_0}{\rho}\right)\frac{\partial p}{\partial y}\right] = \frac{\partial}{\partial x}\left(\frac{\rho'}{\rho}\frac{\partial p}{\partial y}\right), \tag{41}$$

again displaying an asymmetry relative to the corresponding term (37). The interaction of turbulence with mean flow shear is represented by the first term on the rhs of (33):

$$\begin{aligned} -\frac{q_{21}}{\rho_0} &= \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) \\ &= \frac{\partial^2}{\partial x^2}(uv) + \frac{\partial^2}{\partial x \partial y}(v^2) - \frac{\partial}{\partial x}\left[v\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right], \end{aligned} \tag{42}$$

which consists of: (i) some of the terms of the double divergence of the Reynolds stresses per unit mass: (ii) the x-term of the gradient of v times the acoustic dilatation. The source terms may be grouped:

$$q = \left(q_{11} + 2U' q_{21}\right) + \left(q_{12} + 2U' q_{22}\right) + \left(q_{13} + 2U' q_{23}\right) \qquad (43)$$

where: (i) the first set corresponds to sound generation by turbulence (34,42); (ii) the second set corresponds to sound generation by inhomogeneities (37,41); (iii) the third set corresponds to dissipation mechanisms (38,40), viz. viscous stresses and heating processes. Each of the three terms has a contribution independent of mean flow shear (34,37,38) which acts as a modified Lighthill tensor, and a contribution (42,41,40) proportional to mean flow vorticity, which forms a shear tensor.

3 Sound generation by a source in a shear flow

The preceding theory is illustrated by determining the exact Green's function for arbitrary frequency and wavenumber; it corresponds the sound field due to a point source in a shear flow, and is illustrated in the case of a hyperbolic tangent shear layer.

In (Campos & Kobayashi 2000) the authors determined the solution of the pressure sound spectrum for a flow without source terms. When the sound source is present, the Green's function $G(y;\xi)$ can be determined by a procedure explained in (Ince 1956). The Green's function is next plotted as a function of y/L, for selected values of the parameters

$$\mathsf{M} \equiv \frac{U_\infty}{c_0} \qquad (44)$$

$$\kappa \equiv kL \equiv (\omega L/c_0)\cos\theta \qquad (45)$$

$$\delta \equiv \lambda/L \equiv \frac{2\pi c_0}{\omega L} \qquad (46)$$

namely the Mach number of the free stream (44), the angle of the direction of propagation with the mean flow (45) and the ratio (46) of the length-scale of the shear flow L to the wavelength of sound in the free stream $\lambda \equiv \tau c_0$ with $\tau \equiv 2\pi/\omega$; and ξ/L, the position of the source.

The baseline corresponds to: (i) a wavelength equal to the thickness of the boundary layer, since the present theory is unrestricted on frequency, and this is a case of strong interaction between the sound field and the boundary layer; (ii) transonic free stream Mach number; (iii) oblique angle of incidence of 60^{o}; (iv) sound source at the distance of one shear layer thickness:

$$\{\lambda, \mathsf{M}, \theta, \xi\} = \{L, 0.8, \pi/3, L\}. \qquad (47)$$

The modulus $|P|$ and phase $\arg(P)$ of the Green's function are plotted respectively at the top and bottom of Figures 1 to 4, as a function of distance y from the centreline of the hyperbolic tangent shear flow over a range of plus or minus five thicknesses:

$$P(Y;\lambda/L, \mathsf{M}, \theta, \xi/L) \equiv G(y;\xi), \;\; -5 \le Y \equiv y/L \le 5. \qquad (48)$$

Figure 1 shows the effect of changing source position in the shear layer:

$$\xi/L = 1/2, 1, 2, \qquad (49)$$

below or above one shear layer thickness L. The farther the source moves up the shear layer the smaller the modulus (top) of the acoustic pressure; the dip in the acoustic pressure indicates sound absorption by the shear flow. The lower acoustic pressure below the shear layer compared to above is due to sound reflection by the shear layer (there can be no sound absorption because the energy flux is conserved in the y-direction). It is seen from Figure 1 (bottom) that the phase of the acoustic pressure increases away from the shear layer, i.e. sound waves propagate upward above the shear layer and downward below; this is required by the radiation condition at $\pm\infty$, and implies that the shear layer reflects waves. Figure 2 shows the effect of wavelength smaller or larger than the shear layer thickness:

$$2, 1, 1/2 = \delta = \lambda/L = 2\pi c_0/\omega L = 2\pi/\Omega, \qquad (50)$$

which can be related to a dimensionless frequency. The modulus of the acoustic pressure

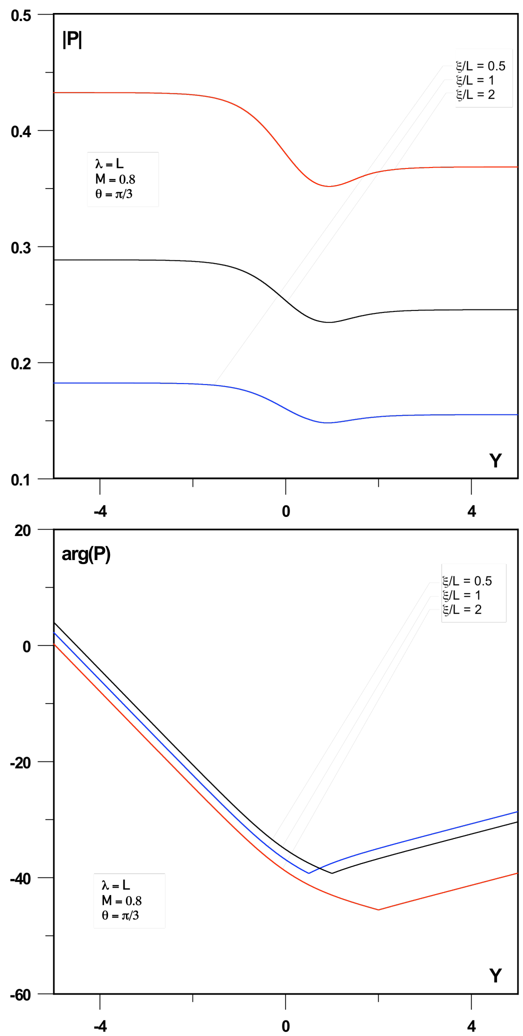

Fig. 1 Modulus (top) and phase (bottom) of the Green's function, for the acoustic pressure (48) due to an unit point source at position $y = \xi$, in an hyperbolic tangent shear flow, versus distance y from the centreline, made dimensionless by dividing (48) by the shear layer thickness. The baseline case (47) concerns incidence angle $\theta = \pi/3$ in a transonic flow $\mathrm{M} = 0.8$, and wavelength λ equal to the shear layer thickness L, and equal to distance ξ of sound source from centreline $\xi/L = 1$, which is halved $\xi/L = 1/2$ and doubled $\xi/L = 2$ for comparisons (49).

(Figure 2, top) is larger for longer wavelength, showing that shorter waves are more strongly affected by the shear flow; the phase variation (Figure 2, bottom) are smaller over the same distance for longer wavelengths. The phase is nearly a linear function of distance, both in Figures 1 and 2 (bottom), indicating a constant phase speed, except in the core part of the shear flow, where the curvature indicates varying phase speed. This occurs near to the 'kink' in the phase curve, indicating the change from downward to upward propagation; this 'point of reflection' is not affected by the wavelength (Figure 2, bottom) and moves down (Figure 1, bottom) as the source also moves down closer to the shear layer centreline. Figure 3 illustrates the effect of free stream Mach numbers:

$$\mathrm{M} = U_\infty/c_0 = 0.3, 0.8, 1.2, \tag{51}$$

by considering low and high subsonic and supersonic shear flows. The modulus of the acoustic pressure (Figure 3, top) varies little except in the case of supersonic shear flow, when there is a large variation across the shear flow, consisting of: (i) a moderate decay below the centreline; (ii) a small but sharp peak near the centreline; (iii) a fast decay above the centreline. The phase of the acoustic pressure (Figure 3, bottom) varies more rapidly for larger Mach number below the shear layer, and then increases more slowly above; in the case of the supersonic shear layer, the phase decreases rapidly below the 'kink', indicating upward propagation, and becomes constant above, confirming the wave absorption (seen in Figure 3, top). The condition of non-existence of a critical layer $\omega - kU >$ is satisfied

$$kU(y)/\omega < kU_\infty/\omega = (U_\infty/c_0)\cos\theta = \mathrm{M}\cos\theta < 1, \tag{52}$$

in all cases of subsonic flow $\mathrm{M} < 1$ and also in the supersonic flow $\mathrm{M} = 1.2$ in (51) because $\theta = \pi/3$ in (47). Figure 4 shows the effect of angle of incidence θ

$$\theta = \pi/4, \pi/3, 2\pi/3, 3\pi/4, \tag{53}$$

which appears in (45) the acoustic compactness, or horizontal wavenumber k made dimensionless

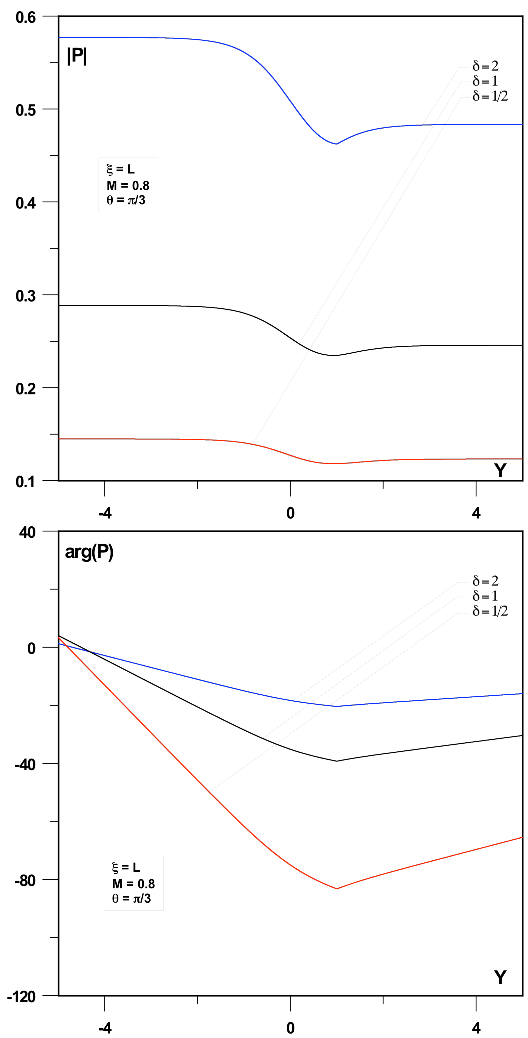

Fig. 2 As Figure 1, with baseline wavelength λ equal to shear layer thickness L, halved or doubled (50).

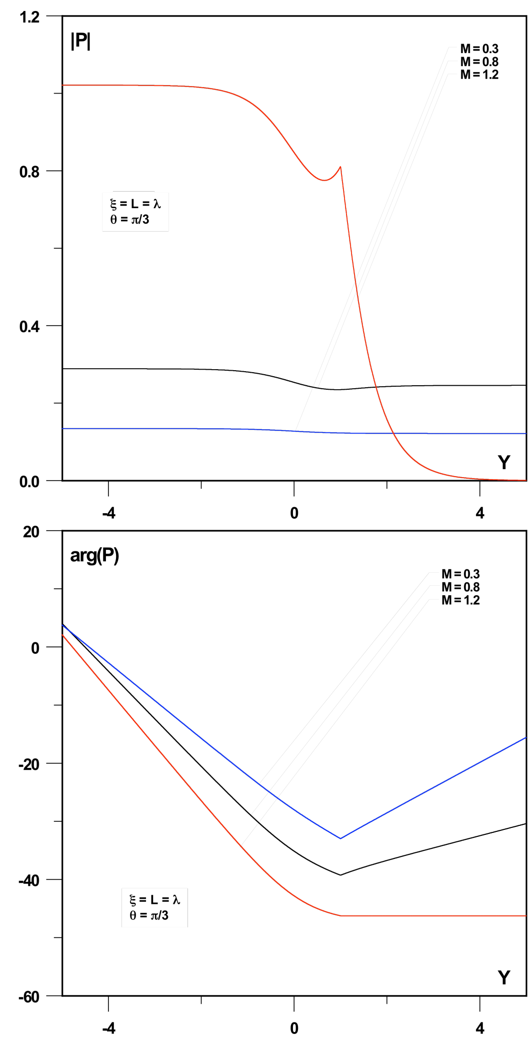

Fig. 3 As Figure 1, with in addition to baseline transonic shear flow $\mathrm{M} = 0.8$, also subsonic $\mathrm{M} = 0.3$ and supersonic $\mathrm{M} = 1.2$ cases (51).

multiplying by the shear layer thickness L, viz.:

$$\kappa \equiv kL \equiv (\omega L/c_0)\cos\theta = \Omega\cos\theta = (2\pi/\delta)\cos\theta, \quad (54)$$

which is smaller than the dimensionless frequency in (50). The modulus of the acoustic pressure (Figure 4, top): (i) for incidence in the forward arc not far from the vertical $\theta = \pi/3$, the acoustic pressure decreases from below to above the shear layer, with a dip in the shear layer; (ii) for incidence in forward arc far from the vertical $\theta = \pi/4$, the acoustic pressure is smaller than before (i) below the the shear layer, has a sharp peak in the shear layer, and then decays to zero above, indicating total reflection; (iii) for incidence in the rear arc not far from the vertical $\theta = 2\pi/3$, the acoustic pressure is small below the shear layer and increases by a small amount above the shear layer with an almost imperceptible dip in the shear layer; (iv) for incidence in the rear arc far from the vertical $\theta = 3\pi/4$, the acoustic pressure is negligible below the shear layer, and larger than case (iii) above the shear layer, with amplitude oscillations in the shear layer. The phase of the acoustic pressure (Figure 4, bottom) increases away from the shear layer as before, but displays two new features: (i) when the phase ceases to be a linear function of distance, in the shear layer, the curvature is opposite for propagation in the rear arc (negative for $\theta < \pi/2$), indicating a decrease in phase speed in the former case, and an increase in the latter case; (ii) the nodes of the acoustic pressure in the amplitude plot (Figure 4, top) in the case of incidence in the rear arc far from the vertical $\theta = 3\pi/4$, correspond to phase jumps in Figure 4, bottom.

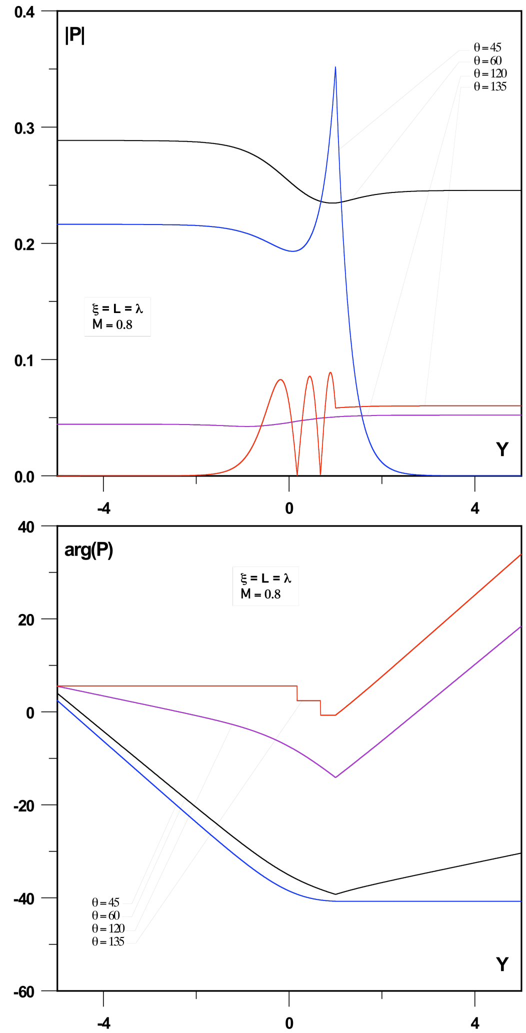

Fig. 4 As Figure 1, adding to the baseline case of incidence in the forward arc not far from the vertical $\theta = \pi/3$, a case far from the vertical $\theta = \pi/4$ and also the symmetric cases of incidence in the rear arc far $\theta = 3\pi/4$ and not far $\theta = 2\pi/3$ from the vertical (53).

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