FEM BASED BENDING ANALYSES OF STIFFENED COMPOSITE PANELS

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Keywords: *Composite panel, stiffened panel, FEM*

Abstract

Solid composite panels in real structures often contain stiffeners to avoid excessive out-of-plane deformations or buckling problems. The stiffened panels can be analyzed using commercial finite element software but it may be troublesome and time consuming. There is a clear need for a fast and efficient analysis tool used by a design engineer. Such a tool has been developed and implemented into the ESAComp software.

The selected approach for the analysis tool is based on the MITC4 shell and Timoshenko beam elements. The plate is modeled as a shell and the stiffeners are included as beams. Equivalent properties of the composite stiffener are calculated for the beam element and the beam is connected to the plate using rigid links.

The selected analysis approach is summarized in the paper. It is seen to be a feasible method for stiffened composite panel analysis and to provide a fast and efficient tool for conceptual design and analysis.

1 Introduction

Composite panels are usually thin due to their high load carrying capability in the plane of the panel. High excessive out-of-plane deformations or buckling may become a serious problem in such panels. To overcome this problem, either a sandwich structure or stiffeners are used to increase the bending stiffness of the panel.

Sandwich structures can be analyzed using the first order shear deformation theory in order to include the deflection caused by the shear deformation of the core material. Any commercial finite element software can solve this case.

Analyses of stiffened composite panels are more complicated. Such problems can still be solved with FE software but it is time consuming to build up a model even with a simple geometry. In addition, beam elements in commercial FE software normally allow only homogeneous and isotropic material properties. Several simplifications have then to be made when modeling the composite stiffeners.

A special tool for basic analyses of stiffened composite panels would eventually speed up the design process and allow detailed stress analyses of stiffeners. Such a tool was developed and implemented into the ESAComp software [1].

The approach used is based on the computation of the equivalent stiffness properties of the stiffeners and on the use of corresponding beam element in the finite element model. The first order shear deformation theory is used for the plate and the Timoshenko beam theory for the stiffeners.

The principal assumptions adopted in the stiffened panel analysis are:

- The out-of-plane normal stress is negligible, i.e. plane stress state is assumed
- The stiffeners are straight and parallel to the x-axis
- The stiffeners are placed on one side of the panel only
- The stiffeners are perfectly bonded onto the plate

- The common normal to the plate and stiffener system before deformation remains straight after deformation
- The middle plane of the plate is used as a reference plane in the analysis.

Panels with different cross sections of stiffeners can be analyzed. No limitation exists on the number of stiffeners and distances between them.

This paper presents the analysis approach and the main equations of the selected finite element formulation used in the analyses.

2 Plate Element

The MITC family of plate elements with shear deformation was developed by Bathe and Dvorkin [2,3] for isotropic plates. A key aspect of the 4-node element is the mixed interpolation of the various strain components: the bending and membrane strain components are calculated as usual from the displacement interpolations of an isoparametric element, while the transverse shear strain components are interpolated differently. The MITC elements are locking free, do not contain any spurious zero-energy modes and have a good predictive capacity for displacements, bending moments and membrane forces. Their convergence properties have been mathematically analyzed and proved to be satisfactory [4].

The extension of the MITC plate elements to the case of composite laminates is done in Ref. [4]. In-plane displacement components are introduced as additional kinematic parameters and the laminate constitutive equations replace the corresponding isotropic equations.

A single 4-noded MITC element (MITC4) is considered in this paper. The formulation of the stiffness matrix is reviewed and the key equations presented below.

2.1 Constitutive Equations

Fig. 1 shows a four node element in global and natural coordinate systems. Each node has five degrees of freedom, i.e. the in-plane displacements u_i and v_i, the rotations $\psi_x^{\,i}$ and $\psi_y^{\,i}$, and the deflection w_i.

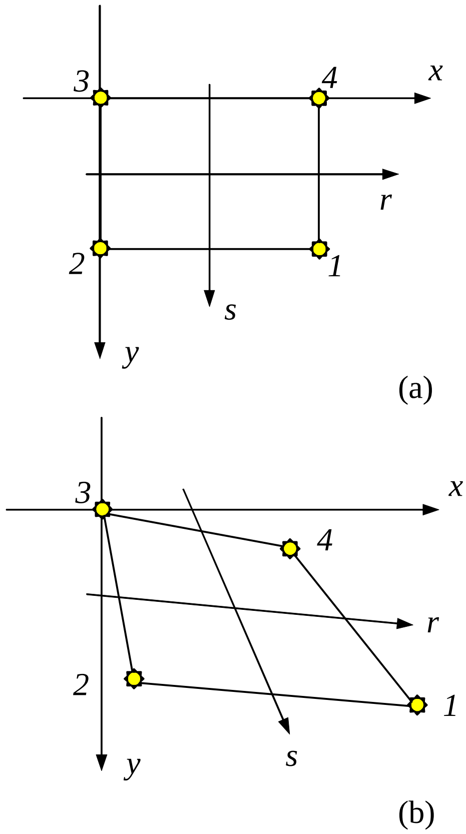

Fig 1. Node numbering sequence of (a) a rectangular, (b) a general MITC4 element.

The constitutive equations for a general laminate can be written as

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{Q\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & 0 \\ [B] & [D] & 0 \\ 0 & 0 & [kA]_{44/55} \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{\kappa\} \\ \{\gamma_z\} \end{Bmatrix}$$

where

$$\{N\} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \quad ; \quad \{M\} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \quad ; \quad \{Q\} = \begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix}$$

$$\{\varepsilon^0\} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \quad ; \quad \{\kappa\} = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad ; \quad \{\gamma_z\} = \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

$$[A]=\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}$$

$$[B]=\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}$$

$$[D]=\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}$$

$$[kA]_{44/55}=\begin{bmatrix} k_{44}A_{44} & k_{45}A_{45} \\ k_{45}A_{45} & k_{55}A_{55} \end{bmatrix}$$

2.2 Interpolation for In-Plane Strains and Curvatures

The generalized nodal displacements corresponding to the twenty degrees of freedom of an element are

$$\{\delta\}=\begin{Bmatrix} \{u^0\} \\ \{v^0\} \\ \{\psi_x\} \\ \{\psi_y\} \\ \{w\} \end{Bmatrix}=\begin{Bmatrix} \{u_1^0 \quad u_2^0 \quad u_3^0 \quad u_4^0\}^T \\ \{v_1^0 \quad v_2^0 \quad v_3^0 \quad v_4\}^T \\ \{\psi_x^1 \quad \psi_x^2 \quad \psi_x^3 \quad \psi_x^4\}^T \\ \{\psi_y^1 \quad \psi_y^2 \quad \psi_y^3 \quad \psi_y^4\}^T \\ \{w_1 \quad w_2 \quad w_3 \quad w_4\}^T \end{Bmatrix}$$

The coordinates and displacement fields within an element are interpolated using the interpolation functions of a 4-noded isoparametric element:

$$x=\sum_{i=1}^{4} n_i x_i \quad ; \quad y=\sum_{i=1}^{4} n_i y_i$$

$$u^0(x,y)=\sum_{i=1}^{4} n_i u_i^0 \quad ; \quad v^0(x,y)=\sum_{i=1}^{4} n_i v_i^0$$

$$w(x,y)=\sum_{i=1}^{4} n_i w_i$$

$$\psi_x(x,y)=\sum_{i=1}^{4} n_i \psi_x^i \quad ; \quad \psi_y(x,y)=\sum_{i=1}^{4} n_i \psi_y^i$$

where the bilinear shape functions are

$$n_1=(1+r)(1+s)/4$$

$$n_2=(1-r)(1+s)/4$$

$$n_3=(1-r)(1-s)/4$$

$$n_4=(1+r)(1-s)/4$$

The in-plane strains and bending curvatures of the mid-plane can be obtained from

$$\{\varepsilon^0\}=[B]_\varepsilon\begin{Bmatrix} \{u^0\} \\ \{v^0\} \end{Bmatrix}$$

$$\{\kappa\}=[B]_\kappa\begin{Bmatrix} \{\psi_x\} \\ \{\psi_y\} \end{Bmatrix}$$

The in-plane strain-displacement transformation matrix $[B]_\varepsilon$ and the curvature-displacement transformation matrix $[B]_\kappa$ are the same and can be obtained according to the usual formulation for isoparametric elements:

$$[B]_\varepsilon = [B]_\kappa = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{bmatrix} n_1 & 0 \\ n_2 & 0 \\ n_3 & 0 \\ n_4 & 0 \\ 0 & n_1 \\ 0 & n_2 \\ 0 & n_3 \\ 0 & n_4 \end{bmatrix}^T$$

2.3 Interpolation for Out-of-Plane Shear Strain

The out-of-plane shear strains of the element are obtained via the interpolation of covariant shear strain components:

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = [B]_\gamma \begin{Bmatrix} \{\psi_x\} \\ \{\psi_y\} \\ \{w\} \end{Bmatrix} = \begin{bmatrix} [B]_{\gamma 1} & [B]_{\gamma 2} \end{bmatrix} \begin{Bmatrix} \{\psi_x\} \\ \{\psi_y\} \\ \{w\} \end{Bmatrix}$$

The shear strain-displacement transformation matrices $[B]_{\gamma 1}$ and $[B]_{\gamma 2}$ are defined as:

$$[B]_{\gamma 1} = [Tc] \begin{bmatrix} x_{14}R^+ & x_{23}R^- & x_{23}R^- & x_{14}R^+ & y_{14}R^+ & y_{23}R^- & y_{23}R^- & y_{14}R^+ \\ x_{12}S^+ & x_{12}S^+ & x_{43}S^- & x_{43}S^- & y_{12}S^+ & y_{12}S^+ & y_{43}S^- & y_{43}S^- \end{bmatrix}$$

$$[B]_{\gamma 2} = [Tc] \begin{bmatrix} R^+ & R^- & -R^- & -R^+ \\ S^+ & -S^+ & -S^- & S^- \end{bmatrix}$$

where

$$R^+ = \frac{1+r}{2} \quad ; \quad R^- = \frac{1-r}{2}$$

$$S^+ = \frac{1+s}{2} \quad ; \quad S^- = \frac{1-s}{2}$$

$$x_{12} = \frac{x_1 - x_2}{2}; \qquad y_{12} = \frac{y_1 - y_2}{2};$$

$$x_{14} = \frac{x_1 - x_4}{2}; \qquad y_{14} = \frac{y_1 - y_4}{2};$$

$$x_{23} = \frac{x_2 - x_3}{2}; \qquad y_{23} = \frac{y_2 - y_3}{2};$$

$$x_{43} = \frac{x_4 - x_3}{2}; \qquad y_{43} = \frac{y_4 - y_3}{2};$$

$$x_{13} = \frac{x_1 - x_3}{2}; \qquad y_{13} = \frac{y_1 - y_3}{2}$$

$$[Tc] = \begin{bmatrix} \cos\alpha & -\cos\beta \\ -\sin\alpha & \sin\beta \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$$

$$C_1 = \frac{\sqrt{[C_{x2} + sC_{x1}]^2 + [C_{y2} + sC_{y1}]^2}}{8\det[J_p]}$$

$$C_2 = \frac{\sqrt{[C_{x3} + rC_{x1}]^2 + [C_{y3} + rC_{y1}]^2}}{8\det[J_p]}$$

$$C_{x1} = x_1 - x_2 + x_3 - x_4$$
$$C_{x2} = x_1 - x_2 - x_3 + x_4$$
$$C_{x3} = x_1 + x_2 - x_3 - x_4$$
$$C_{y1} = y_1 - y_2 + y_3 - y_4$$
$$C_{y2} = y_1 - y_2 - y_3 + y_4$$
$$C_{y3} = y_1 + y_2 - y_3 - y_4$$

The symbol α denotes the angle between the r- and x- axes and β between s- and x-axes, respectively:

$$\alpha = \tan^{-1}\left(\frac{y_{12} + y_{43}}{x_{12} + x_{43}}\right)$$

$$\beta = \tan^{-1}\left(\frac{y_{14} + y_{23}}{x_{14} + x_{23}}\right)$$

In the previous equations the $\det[J_p]$ is the determinant of the Jacobian and the subscript p denotes the plate element. The $\det[J_p]$ is obtained from

$$\det[J_p] = \det\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix}$$

$$= \left(x_{12}S^+ + x_{43}S^-\right)\left(y_{14}R^+ + y_{23}R^-\right) - \left(x_{14}R^+ + x_{23}R^-\right)\left(y_{12}S^+ + y_{43}S^-\right)$$

By combining the results from two different interpolations, the in-plane strains, curvatures and out-of plane shear strains are expressed as

$$\begin{Bmatrix} \{\varepsilon^0\} \\ \{\kappa\} \\ \{\gamma_z\} \end{Bmatrix} = \begin{bmatrix} [B]_\varepsilon & 0 & 0 \\ 0 & [B]_\kappa & 0 \\ 0 & [B]_{\gamma1} & [B]_{\gamma2} \end{bmatrix} \begin{Bmatrix} \{u^0\} \\ \{v^0\} \\ \{\psi_x\} \\ \{\psi_y\} \\ \{w\} \end{Bmatrix}$$

The stress resultants can then be obtained:

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{Q\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & 0 \\ [B] & [D] & 0 \\ 0 & 0 & [kA]_{44/55} \end{bmatrix} \begin{bmatrix} [B]_\varepsilon & 0 & 0 \\ 0 & [B]_\kappa & 0 \\ 0 & [B]_{\gamma1} & [B]_{\gamma2} \end{bmatrix} \begin{Bmatrix} \{u^0\} \\ \{v^0\} \\ \{\psi_x\} \\ \{\psi_y\} \\ \{w\} \end{Bmatrix}$$

The total strain energy of a plate element is the sum of the stretching, bending and shear energies:

$$U_p = \frac{1}{2}\int_S \begin{Bmatrix} \{\varepsilon^0\} \\ \{\kappa\} \\ \{\gamma_z\} \end{Bmatrix}^T \begin{Bmatrix} \{N\} \\ \{M\} \\ \{Q\} \end{Bmatrix} dS$$

Substituting strains and stress resultants from two previous equations yields

$$U_p = \frac{1}{2}\int_S \begin{Bmatrix} \{u^0\} \\ \{v^0\} \\ \{\psi_x\} \\ \{\psi_y\} \\ \{w\} \end{Bmatrix}^T \begin{bmatrix} [B]_\varepsilon^T & 0 & 0 \\ 0 & [B]_\kappa^T & [B]_{\gamma1}^T \\ 0 & 0 & [B]_{\gamma2}^T \end{bmatrix} \begin{bmatrix} [A] & [B] & 0 \\ [B] & [D] & 0 \\ 0 & 0 & [kA]_{44/55} \end{bmatrix} \begin{bmatrix} [B]_\varepsilon & 0 & 0 \\ 0 & [B]_\kappa & 0 \\ 0 & [B]_{\gamma1} & [B]_{\gamma2} \end{bmatrix} \begin{Bmatrix} \{u^0\} \\ \{v^0\} \\ \{\psi_x\} \\ \{\psi_y\} \\ \{w\} \end{Bmatrix} dS$$

The matrix product in the equation above is designated as the element stiffness matrix:

$$[EC_p] = \int_S \begin{bmatrix} [B]_\varepsilon^T & 0 & 0 \\ 0 & [B]_\kappa^T & [B]_{\gamma1}^T \\ 0 & 0 & [B]_{\gamma2}^T \end{bmatrix} \begin{bmatrix} [A] & [B] & 0 \\ [B] & [D] & 0 \\ 0 & 0 & [kA]_{44/55} \end{bmatrix} \begin{bmatrix} [B]_\varepsilon & 0 & 0 \\ 0 & [B]_\kappa & 0 \\ 0 & [B]_{\gamma1} & [B]_{\gamma2} \end{bmatrix} dS$$

Changing the integration variable, the final form of the element stiffness matrix is

$$
\lfloor EC_p \rfloor = \det[J_p] \int_{-1}^{1}\int_{-1}^{1} \begin{bmatrix} [B]_\varepsilon^T & 0 & 0 \\ 0 & [B]_\kappa^T & [B]_{\gamma 1}^T \\ 0 & 0 & [B]_{\gamma 2}^T \end{bmatrix} \begin{bmatrix} [A] & [B] & 0 \\ [B] & [D] & 0 \\ 0 & 0 & [kA]_{44/55} \end{bmatrix} \begin{bmatrix} [B]_\varepsilon & 0 & 0 \\ 0 & [B]_\kappa & 0 \\ 0 & [B]_{\gamma 1} & [B]_{\gamma 2} \end{bmatrix} drds
$$

The Gauss quadrature of the second order is used for integration.

3 Stiffener Element

A 2-noded isoprametric beam (stiffener) element (Fig. 2) based on the Timoshenko beam theory is adopted for the analysis. Each node has four degrees of freedom, i.e. the displacement u_i, the rotations ψ_x^i and ψ_y^i and the deflection w_i.

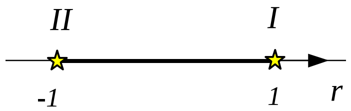

Fig 2. Node numbering sequence of the beam element.

The generalized nodal displacements corresponding to the eight degrees of freedom are

$$
\{\delta_b\} = \begin{Bmatrix} \{u_b^0\} \\ \{\psi_x^b\} \\ \{\psi_y^b\} \\ \{w_b\} \end{Bmatrix} = \begin{Bmatrix} \{u_{bI}^0 & u_{bII}^0\}^T \\ \{\psi_x^{bI} & \psi_x^{bII}\}^T \\ \{\psi_y^{bI} & \psi_y^{bII}\}^T \\ \{w_{bI} & w_{bII}\}^T \end{Bmatrix}
$$

The same interpolation functions are used for the generalized displacements as for the plate in order to satisfy the compatibility conditions between the plate and the stiffeners. The coordinates and displacement fields within a beam element are interpolated as

$$
x = \sum_{i=I}^{II} n_{bi} x_{bi}
$$

$$
u_b^0(x) = \sum_{i=I}^{II} n_{bi} u_{bi}^0 \quad ; \quad w_b(x) = \sum_{i=I}^{II} n_{bi} w_{bi}
$$

$$
\psi_x^b(x) == \sum_{i=I}^{II} n_{bi} \psi_x^{bi} \quad ; \quad \psi_y^b(x) = \sum_{i=I}^{II} n_{bi} \psi_y^{bi}
$$

where the interpolation shape functions are

$$
n_{bI} = (1+r)/2 \quad ; \quad n_{bII} = (1-r)/2
$$

3.1 Stiffness Matrix of the Stiffener

The deformation of a stiffener can be obtained from

$$
\{\varepsilon_x^{0b} \quad \kappa_x^b \quad \psi_{y,x}^b \quad \gamma_{zx}^b\}^T = \left[\frac{\partial}{\partial x}\right]_b [N_b(r)]\{\delta_b\} = [B]_b \{\delta_b\}
$$

where

$$
\left[\frac{\partial}{\partial x}\right]_b = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial x} & 0 \\ 0 & 1 & 0 & \frac{\partial}{\partial x} \end{bmatrix}
$$

$$
[B]_b = \left[\frac{\partial}{\partial x}\right]_b [N_b(r)]
$$

$$
= \frac{1}{x_I - x_{II}} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & (x_I - x_{II})n_{bI} & (x_I - x_{II})n_{bII} & 0 & 0 & 1 & -1 \end{bmatrix}
$$

$$[N_b(r)] = \begin{bmatrix} n_{bI} & n_{bII} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & n_{bI} & n_{bII} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & n_{bI} & n_{bII} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & n_{bI} & n_{bII} \end{bmatrix}$$

The total strain energy of a beam element is the sum of the stretching, bending and shear energies:

$$U_b = \frac{1}{2}\int_l \begin{Bmatrix} \varepsilon_x^{0b} \\ \kappa_x^b \\ \psi_{y,x}^b \\ \gamma_{zx}^b \end{Bmatrix}^T \begin{Bmatrix} N_x^b \\ M_x^b \\ T_x^b \\ Q_x^b \end{Bmatrix} dx$$

By introducing the constitutive equations that are derived by following the approach presented in Ref. [6], the strain energy becomes

$$U_b = \frac{1}{2}\{\delta_b\}^T \int_{-1}^{1} \det[J_b][B]_b^T \begin{bmatrix} E_x A & 0 & 0 & 0 \\ 0 & EI_y & 0 & 0 \\ 0 & 0 & GJ_x & 0 \\ 0 & 0 & 0 & kG_z A \end{bmatrix} [B]_b \, dr \{\delta_b\}$$

The central part of the equation above is designated as the beam element stiffness matrix:

$$[EC_b] = \det[J_b]\int_{-1}^{1} [B]_b^T \begin{bmatrix} E_x A & 0 & 0 & 0 \\ 0 & EI_y & 0 & 0 \\ 0 & 0 & GJ_x & 0 \\ 0 & 0 & 0 & kG_z A \end{bmatrix} [B]_b \, dr$$

The det[J_b] is the determinant of the Jacobian for the beam element. For the beam element parallel to the x-axis the determinant of the Jacobian is

$$\det[J_b] = \det\left[\frac{\partial x}{\partial r}\right] = \frac{1}{2}(x_I - x_{II})$$

In order to avoid locking of the beam element, one point Gauss integration along the isoparametric beam axis is used [2].

3.2 Rigid Link between a Beam Element and a Plate Element

In Fig. 3 a beam element defined by the nodes I and II is connected to an edge (1-2) of a plate element by rigid links. The beam element has the same deflection and cross section rotations as those of the plate at the joint locations but the in-plane displacements are different. The rigid link is constructed between the stiffness weighted centroid C_{sw} of the beam and the plate reference plane, i.e. the mid plane.

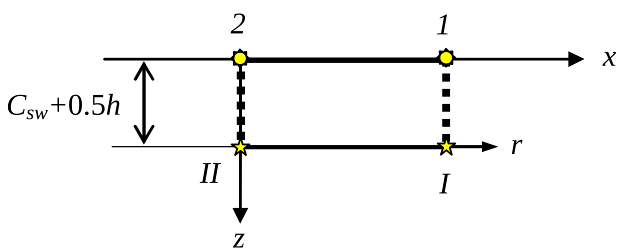

Fig 3. Beam element connected to the plate element using rigid links.

The relations between the generalized displacements of the beam and plate elements are expressed as

$$\begin{Bmatrix} u_{bI}^0 \\ \psi_x^{bI} \\ \psi_y^{bI} \\ w_{bI} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & \left(C_{sw} + \frac{1}{2}h\right) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1^0 \\ v_1^0 \\ \psi_x^1 \\ \psi_y^1 \\ w_1 \end{Bmatrix}$$

The deformation matrix is defined as

$$[T] = \begin{bmatrix} T_I & 0 \\ 0 & T_{II} \end{bmatrix}$$

where

$$[T_I]=[T_{II}]=\begin{bmatrix}1 & 0 & \left(C_{sw}+\frac{1}{2}h\right) & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

The transformation of the beam element stiffness matrix is therefore:

$$[EC_{bp}]=[T]^T[EC_b][T]$$

4 Cross Sectional Properties of Stiffeners

Five types of thin-walled stiffener cross sections are considered: I, Z, C, T, and Hat cross sections (Fig. 4). The major characteristics of the cross sections are:

- Laminate properties of all flanges and the web can be adjusted separately
- The web laminates of the I, Z, C and T cross sections are symmetric
- The widths of the flanges can be adjusted separately within a cross section
- The laminate in the flanges can be unsymmetric
- The hat section can be defined as an embedded section where the plate is thinner under the stiffener than elsewhere.

Based on the constitutive relations of wall laminates of a stiffener cross section the cross sectional properties of I, Z, C, T and Hat stiffeners are derived.

The coordinate systems for the wall laminates of the cross sections are shown in Fig. 4. The stiffness weighted centroid C_{sw} in the z-axis direction is measured from the bottom of the cross section, i.e. from the upper side of the plate, so as the shear center d_z in the same direction. The stiffness weighted centroid C_{swy} in the y-axis direction is measured from either the mid-plane of the only vertical web or from the central symmetry line, so as the shear center d_y in the same direction.

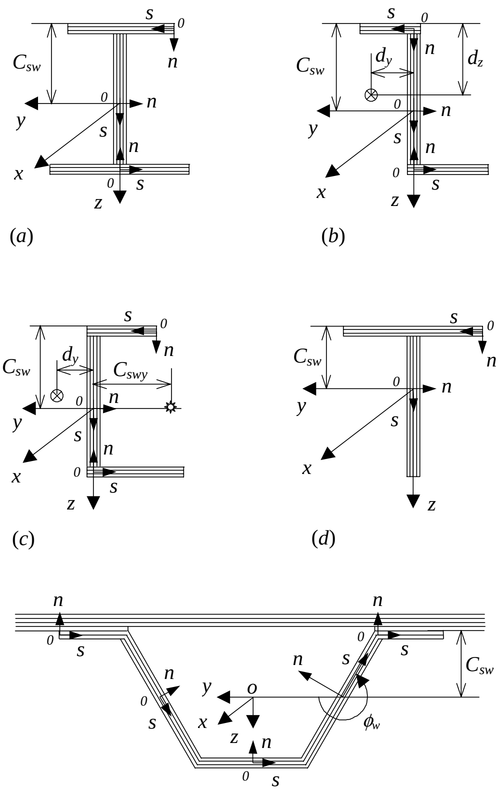

Fig 4. The main cross sectional properties of the stiffeners.

5 Numerical Examples

The reference results used for comparison are obtained with a shell model where both the plate and stiffeners are modeled using parabolic shell elements. The software used was I-DEAS Master Series [7]. The analyzed case is presented in Fig. 5 and the material properties and laminate lay-ups are presented in Tables 1 and 2. The analysis case consists of a symmetric plate with two stiffeners. The cross section of the stiffeners is an I-section without flanges. The loading is a uniform pressure 1 kPa on the plate.

The stiffeners are modeled using shell elements and connected to the plate using rigid links. This is done to ensure that the stiffener offset and cross sectional area are correct.

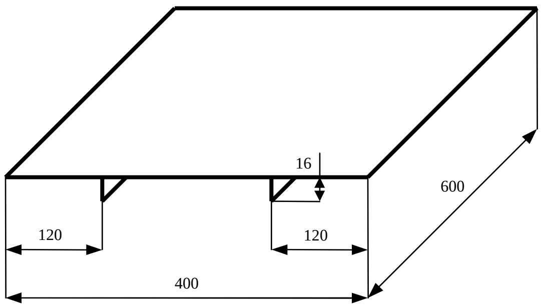

Fig 5. The analysis case used as a reference.

Table 1. Typical material properties for T800/epoxy ply used in analyses

Property	Value
E_1	155 GPa
E_2	8.5 GPa
G_{12}	5.5 GPa
v	0.3
Thickness	0.2 mm

Table 2. Laminate lay-ups used in the reference analysis case.

Laminate	Lay-up	Thickness
Plate	$[0/30/-30/45/-45/90]_s$	2.4 mm
Stiffener	$[0/45/-45/90]_s$	1.6 mm

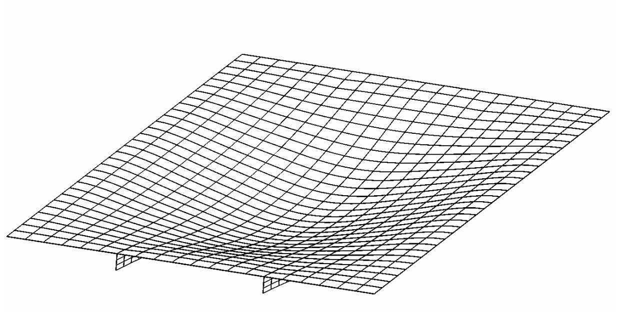

Fig 6. Typical deformation of the reference case, all edges are clamped. Stiffeners are modeled using shell elements.

Table 3 presents the deflection results of the reference model and the beam model used in the analysis tool. Table 3 also includes the first three eigenfrequencies. Most of the results are obtained using simply supported panel edges. The static load case is also presented with clamped support.

Table 3. Deflection and first three eigenfrequencies of reference structure compared with beam model.

Case	Reference model	Beam model
Static simply supported	1.38 mm	1.4 mm
Static clamped	0.409 mm	0.424 mm
Mode 1 simply supported	86.08 Hz	85.45 Hz
Mode 2 simply supported	145.17 Hz	146.54 Hz
Mode 3 simply supported	246.25 Hz	252.81 Hz

The correspondence of the deflection values can be considered good especially with simply supported edges. The correspondence is also good in first eigenfrequencies. The difference becomes larger with higher eigenmodes.

With clamped edges the difference of the deflection values is higher between the beam model and shell model. The difference is most likely caused by different amount of degrees of freedom in the stiffener models. By restraining all dofs in the shell stiffener, a stiffer structure is produced than in beam stiffener with four dofs.

5.1 The Effect of Plate Thickness

In the analysis tool the rigid link between the plate and the stiffeners include the half of the plate thickness. In shell models such as in Fig. 6 this is easily forgotten. However, it can have a significant effect on the plate deflection.

The Table 4 presents the deflection results when the plate half thickness is included or excluded. The results are obtained using the reference (shell) model.

Table 4. Deflection of the reference structure with different values used in the rigid link. All edges are simply supported.

Analysis case	Deflection
Half plate thickness included	1.38 mm
Half plate thickness excluded	2.03 mm
Difference	47 %

The analysis results reveal that the results may be seriously erroneous in case the effect of half plate thickness is neglected. Naturally the magnitude is dependent on the general dimensions of the plate and stiffener as well as on laminate structures.

5.2 Boundary Conditions

When stiffeners are modeled as beams and extended to the panel edge, special care must be taken in the use of boundary conditions, especially in case of clamped boundary conditions. Due to the modeling technique only one plane of nodes exists in the model. This means that the boundary conditions of the panel edge are applied also to the stiffener. Depending on the case to be analyzed this may or may not be a desirable feature.

Table 5 presents the deflection results when the boundary conditions are applied also to the stiffeners or only to the panel edge. The results are obtained using the shell model.

Table 5. Deflection of the reference structure with different boundary conditions. All edges are clamped.

Analysis case	Deflection
Boundary conditions on stiffener	0.409 mm
Boundary conditions only on edge	0.649 mm
Difference	59 %

Generally, if it is desired that the boundary conditions should not be applied to the stiffener the following tricks can be used in modeling:

- The stiffeners are not extended to the panel edge
- The simply supported boundary conditions are used at the edge. The plate is extended further and the clamped conditions are created using additional boundary conditions.
- Shell elements are used in stiffeners
- Offset is used in the plate element definition

Currently none of the tricks is planned to be included in the analysis module. Therefore, only ‘fully clamped’ boundary conditions are applicable.

6 Conclusions

Based on the work presented in this paper it can be concluded that:

- A feasible analysis method for stiffened composite panels is obtained by combining the MITC4 shell and Timoshenko beam elements.
- The half thickness of the plate must be included in the rigid links between the plate and the stiffeners.
- The restrictions in the boundary conditions may cause undesirable results. The user must be aware of the difference in actual and modeled boundary conditions especially at the clamped edge.
- The selected approach for modeling the stiffened composite panel provides a fast and easy to use tool for conceptual design and analysis.

Acknowledgement

The research described in this paper was carried out at Helsinki University of Technology, Laboratory of Lightweight Structures under a contract and co-operation with Componeering Inc. as part of development work for the ESAComp software.

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