A COMPLEX METHOD FOR NUMERICAL OPTIMIZATION ON AIRFOIL

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Abstract

An airfoil shape design method that couples viscous flow analysis and a complex optimum method is described in this paper. The method is applied to search an airfoil geometry with improved aerodynamic performance at single or multiple design-point while the specified design constraints being satisfied. A compressible viscous flow model based on the Reynolds-averaged Navier-Stokes equation is used in the viscous flow analysis in order to gain reliable design results. Based on present complex optimum method, the single-point and multipoint optimization algorithm run efficiently and robustly demonstrated by several cases. This method described in this paper can be found its application in aircraft industry when it is employed to improve the aerodynamic performance characteristics of an existing baseline airfoil to meet specified engineering requirement.

1 Introduction

Nowdays, methods for solving airfoil design problem can be distinguished into two classes [1,2]: (a) inverse method, (b) direct numerical optimization method. The distinction is based on how the design problem is formulated.

In this paper, the problem is solved by using the second method. The design problem is posed as a minimization problem of an aerodynamic objective function subjected to constraints on the geometric and aerodynamic performance. The aerodynamic objective function is described as some aerodynamic parameter such as: the lift C_l, the drag C_d, the ratio of lift to drag C_l/C_d or the pitching moment C_m, etc. Then the objective function is improved by the complex optimum method. A series of shape functions are defined as design variables. The new airfoil shape is obtained by the new shape functions plus the baseline of the initial airfoil. Multi-objective optimization problem of this paper is transformed into a single-objective problem by weighting the objective functions of different design points.

2 The Complex Optimum Method

Mathematically, the optimization problem can be defined as:

$$\begin{aligned} &\min \quad f(x) \\ &s.t \quad \begin{array}{ll} c_i(x)=0, & i=1,2,\cdots,m_e \\ c_i(x)\geq 0, & i=m_e+1,\cdots,m \end{array} \end{aligned} \tag{1}$$

where $f(x)$ denotes objective function, $c_i(x)=0$ and $c_i(x)\geq 0$ are the constraints. m is the number of the constraints. The objective function ($f(x)$) depends on the design variables: x, for the airfoil optimization problem, x is related to the airfoil geometry.

There are two classes of optimization methods, global methods and local methods. Global methods, such as the genetic algorithm [3,4,5] and the simulated annealing algorithm [5,6,7], are aimed at obtaining the global optimum, however they incur large computational effort. Local methods, mostly the gradient-based optimization algorithm, although limited to reduce only one of the minima, requiring much less computation cost compared to global methods, and are very widely and efficiently employed in engineering development.

The complex method described in this paper belongs to the local methods. The gradient is difficult to solve accurately for the gradient-based optimization, but the complex method doesn't need to calculate gradient and hence avoids the difficulties mentioned above. The method is outlined as follows: Assume the number of the design variables is n , then $2\times n$ complexes, at least $n+1$, must be found to form the initial complexes and the constraints must be satisfied for the $2\times n$ complexes. By using viscous flow analysis, we can judge whether the constraints are satisfied. In every iteration, the complex with the biggest value, that is the complex with the worst value, in the $2\times n$ complexes is eliminated and is substituted with the mapping of the eliminated complex. The mapping is computed as follows: the center of the complexes except for the eliminated complex is solved and is defined as the mapping of the eliminated complex. Hence the new complex that describes a new airfoil has been built. In the process of optimization, the optimized airfoil will be gained by constantly changing to the better solution.

There are four steps in the iteration of the complex optimum method as follows:

(1). A series of numbers can be used to create $k(n+1\le k<2\times n)$ vertexes (the initial complexes) and calculated randomly by computer as:

$$x^{(j)}=[\underline{x}_i+R(\bar{x}_i-\underline{x}_i)]^T \quad \begin{Bmatrix} i=1,2,\cdots,m \\ j=1,2,\cdots,k \end{Bmatrix} \qquad (2)$$

where R are a set of random numbers which be distributed in the space: [0,1], $\bar{x}_i$ is the Maximum value of the design variables and $\underline{x}_i$ is the minimal value respectively, the values of the $\underline{x}_i$ and $\bar{x}_i$ can be gained experiential in engineering, the first vertex must satisfy the constraints.

(2). k vertexes created in the first step are checked one by one in order to make the k vertexes satisfy the constraints.

Assume that there are $S(1\le S\le k)$ vertexes which satisfy the constraints, the center of the vertexes can be calculated by:

$$\bar{x}^{(S)}=\frac{1}{S}\sum_{j=1}^{S}x^{(j)} \qquad (3)$$

If the vertex $x^{(S+1)}$ doesn't satisfy the constraints, $x^{(S+1)}$ is reduced in the direction of $(x^{(S+1)}-\bar{x}^{(S)})$ by the formulation as:

$$x^{(S+1)}=\bar{x}^{(S)}+0.5(x^{(S+1)}-\bar{x}^{(S)}) \qquad (4)$$

then the new vertex $x^{(S+1)}$ is also checked by the constraints, if the constraints are satisfied, go to the next step, if not, $x^{(S+1)}$ is reduced according to the formulation (4) until the constraints are satisfied.

(3). After all the initial complexes satisfy the constraints, the objective function corresponded with every initial complex is calculated, we define the complex corresponding to the objective function with the biggest value as $x^{(h)}$, that with the smallest value as $x^{(e)}$, and the center of the vertexes except for the complex $x^{(h)}$ can be calculated by the following formulation:

$$\bar{x}=\frac{1}{k-1}\sum_{j=1}^{k}x^{(j)} \quad (j\ne h) \qquad (5)$$

then the complex $\bar{x}$ is checked by the constraints, if not, $\underline{x}_i$ is substituted with $x^{(e)}$ and $\bar{x}_i$ is substituted with $\bar{x}$, go to the first step; if the constraints are satisfied and the objective function $f(\bar{x})$ satisfy the condition:

$$\left\{\frac{1}{k}\sum_{j=1}^{k}[f(\bar{x})-f(x^{(j)})]^2\right\}^{1/2}\le\varepsilon_1 \qquad (6)$$

the optimum result has been obtained; where $\varepsilon_1\to 0$, if the condition (6) isn't satisfied, we can gain the reflection of the $x^{(h)}$ as follow:

$$x^{(\alpha)}=\bar{x}+\alpha(\bar{x}-x^{(h)}) \qquad (7)$$

the parameter α is the reflection coefficient, in general, $\alpha=1.3$; if the constraints are satisfied for the complex $x^{(\alpha)}$, go to the next step, but if the constraints are not satisfied, the value of α should be reduced to half of α and $x^{(\alpha)}$ can be substituted with the new $x^{(\alpha)}$ until the constraints are satisfied.

(4). Compare the value of $f(x^{(\alpha)})$ with the value of $f(x^{(h)})$, if $f(x^{(\alpha)})<f(x^{(h)})$, the

complex $x^{(h)}$ is substituted with the complex $x^{(\alpha)}$, go to the third step; if $f(x^{(\alpha)}) \geq f(x^{(h)})$, the progress employed in the third step will be repeated to compute $f(x^{(\alpha)})$ until $f(x^{(\alpha)}) < f(x^{(h)})$.

3 The Objective Function

The design of an optimal airfoil shape is a constrained optimization problem, the objective is to find an optimal airfoil that has a minimal value of the objective function subject to N-S equation and certain boundary constraints.

Generally, aerodynamic shape optimization of airfoils is concerned with obtaining the most aerodynamically favorable geometry that be provided with the better aerodynamic performance. In the approach, the aerodynamic objective function is described as an aerodynamic parameter for single-point design. For multi-point design, the aerodynamic objective function is gained by weighting the aerodynamic parameters of different design points. At the same time, except for the objective parameter, constraints will be imposed on some aerodynamic parameters and airfoil geometry according to sufficient strength and stiffness requirements.

In effect, multi-point optimization is more difficult than single-point optimization since that the parameters aren't minimized at the same time [5,8]. Mathematically, multi-point optimization problem can be described as the form:

$$
\begin{array}{ll}
\min & f_1(x) \\
\min & f_2(x) \\
\vdots & \vdots \\
\min & f_q(x) \\
s.t & g_i(x) \geq 0, \quad i = 1,2,\cdots,m
\end{array}
\tag{8}
$$

A straightforward approach to simplify the multi-point optimization problem is to generalize the objective in Eq.(8) to a combination of different parameters written as:

$$f(x) = f\{f_1(x), f_2(x), \cdots, f_q(x)\} \tag{9}$$

We can estimate the range of the value of every objective parameter according to practical engineering experience given as:

$$\alpha_i \leq f_i(x) \leq \beta_i, \quad i = 1,2,\cdots,q \tag{10}$$

where α_i, β_i are known numbers corresponding to $f_i(x)$. The objective functions in Eq.(8) are made to be standard as Eq.(11):

$$f_i^{'}(x) = \frac{f_i(x) - \alpha_i}{\beta_i - \alpha_i} \tag{11}$$

where the new objective function $f_i^{'}(x)$ is limited in the range:

$$0 \leq f_i^{'}(x) \leq 1$$

thus, the simplified objective function is written as:

$$f(x) = \sum_{i=1}^{q} \omega_i f_i^{'}(x) \tag{12}$$

where $\sum_{i=1}^{q} \omega_i = 1$, parameters $\omega_i (i = 1,2,\cdots,q)$ are the powers given by us according to their weightiness in the progress of the optimization. These parameters are very significant to the optimization result.

Multi-point optimization problem (8) can be transformed into a single-objective problem (12) as:

$$
\begin{array}{ll}
\min & f(x) \\
s.t. & g_i(x) \geq 0, \quad i = 1,2,\cdots,m
\end{array}
\tag{13}
$$

4 The Design Variables

The airfoil geometry is modified adding a linear combination of Hicks and Henne function [9] $f_i(\bar{x})$ as follows:

$$\Delta \overline{y_u}(\bar{x}) = \sum_{i=1}^{n_u} \delta_i f_i(\bar{x}) \tag{14}$$

$$\Delta \overline{y_l}(\bar{x}) = \sum_{i=n_u+1}^{n_u+n_l} \delta_i f_i(\bar{x}) \tag{15}$$

where the powers δ_i are the amplitudes of the shape functions $f_i(\bar{x})$ on the upper or lower surface of the airfoil and defined as design variables, $n_u + n_l$ the number of the design variables , u, l represent the upper and lower surface respectively. The design variables must be determined to obtain the optimal shape of the airfoil described as follows:

$$\overline{y_u}(\bar{x}) = \overline{y_{ub}}(\bar{x}) + \Delta \overline{y_u}(\bar{x}) \tag{16}$$

$$\overline{y_l}(\overline{x}) = \overline{y_{lb}}(\overline{x}) + \Delta\overline{y_l}(\overline{x}) \qquad (17)$$

where $\overline{x}$, $\overline{y}$ are non-dimension coordinate of airfoil surface, $\overline{y_{ub}}$ 、 $\overline{y_{lb}}$ non-dimension coordinate of base airfoil surface. Many computational results have shown that δ_i should be satisfied the following condition:

$$-\frac{5}{2}\cdot\frac{t_{\max}}{10^3} \le \delta_i \le \frac{5}{2}\cdot\frac{t_{\max}}{10^3}\ , \quad i = 1,2,\cdots,n \qquad (18)$$

where $t_{\max}$ is the Maximum thickness of the base airfoil.

10 Hicks and Henne functions and 16 Hicks and Henne functions are employed respectively in this paper on the basis of different requirements. 10 Hicks and Henne functions used for upper and lower surface perturbation are described as:

$$f_1(\overline{x}) = \overline{x}^{0.25}(1-\overline{x})e^{-20\overline{x}}$$

$$f_i(\overline{x}) = \sin^3\left[\pi\overline{x}^{e(i)}\right] \qquad i = 2,3,4,5$$

$$e(i) = \frac{\ln(0.5)}{\ln(\overline{x}_i)} \qquad i = 2,3,4,5$$

$$f_{i+5}(\overline{x}) = -f_i(\overline{x}) \qquad i = 1,2,3,4,5$$

$$\overline{x}_i = 0.2,0.4,0.6,0.8 \qquad i = 2,3,4,5$$

where the former five functions are distributed in upper surface and the other are distributed in lower surface. 16 Hicks and Henne functions are described as:

$$f_i(\overline{x}) = \sin[\pi(1-\overline{x})^{e(i)}] \qquad i = 1,2$$

$$f_i(\overline{x}) = \sin^3[\pi\overline{x}^{e(i)}] \qquad i = 3,\cdots,6$$

$$f_i(\overline{x}) = \sin[\pi\overline{x}^{e(i)}] \qquad i = 7,8$$

$$f_{i+8}(\overline{x}) = -f_i(\overline{x}) \qquad i = 1,2,\cdots,8$$

$$e(i) = \frac{\ln(0.5)}{\ln(1-\overline{x}_i)} \qquad i = 1,2$$

$$e(i) = \frac{\ln(0.5)}{\ln(\overline{x}_i)} \qquad i = 3,\cdots,8$$

$$\overline{x}_i = 0.06,0.13,0.20,0.40,0.60,0.80,0.87,0.94 \qquad i = 1,2,\cdots,8$$

where the former eight functions are distributed in upper surface and the other are distributed in lower surface.

5 Applications: Airfoil Optimization Design about RAE2822

The present method using the complex optimum method and N-S solver is applied to two cases. One is a single-point optimization and the other is a two-point optimization about RAE2822.

5.1 Single-Point Optimization about RAE2822

Case 1 can be described as follows:
Base airfoil: RAE2822
Design point:

$$Ma = 0.730, \mathrm{Re} = 6.5\times10^6, \alpha = 2.36°$$

Objective: minimize the drag C_d

Subject to: (1) $A/A_0 - 1 \ge 0$

(2) $C_l/C_{l0} - 1 \ge 0$

Governing equation: N-S equations
Number of design variables: 10

Table.1 The result on single-point optimization

Parameter	Base airfoil	Optimization	Δ	$\Delta\%$
C_l	0.8023	0.7952	-0.0071	-0.88%
C_d	1.5165×10^{-2}	1.1545×10^{-2}	-0.362×10^{-2}	-24%
C_m	-9.6508×10^{-2}	-9.4809×10^{-2}	0.169×10^{-2}	1.7%

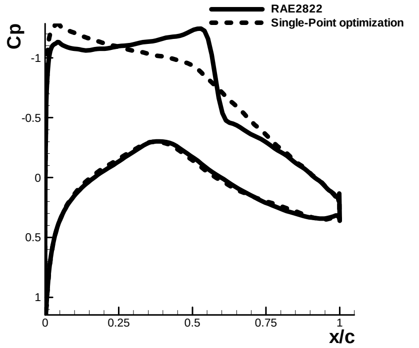

Fig.1 The comparison of pressure distributions

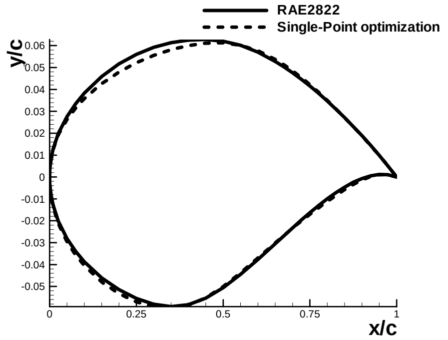

Fig.2 The comparison of geometries

The optimization results are described in Table.1. The minimization of the drag C_d is defined as objective. There are two constraints as follows: (1) the section area of base airfoil doesn't decrease, (2) the lift C_l doesn't decrease. The results are shown in Table.1. A better objective drag reduction is achieved by the present single-point optimization: the drag reduction at the design point is 24% whereas the lift reduction is only 0.88%. The comparisons of the pressure distribution and the airfoil geometries are plotted in Fig.1, Fig.2, respectively. In the overall optimum steps, this example costs 4 hours and 39 minutes on the Pentium IV personal computer.

5.2 Two-Point Optimization about RAE2822

Case 2 can be described as follows:

Base airfoil: RAE2822

Design points:

1) $Ma = 0.730, \mathrm{Re} = 6.5\times10^6, \alpha = 1.842°$

2) $Ma = 0.73, \mathrm{Re} = 6.5\times10^6, \alpha = 1.342°$

Objectives:

(1) minimize the drag C_d at design point 1)

(2) minimize the drag C_d at design point 2)

Subject to:

(1) $A/A_0 - 1 \geq 0$

(2) $C_l / C_{l0} - 1 \geq 0$ at design point 1)

(3) $|C_m| \leq |C_{m0}|$ at design point 1)

Governing equation: N-S equations

Number of design variables: 16

Table.2 The result on two-point optimization

Design point	Parameter	Base airfoil	Optimum airfoil	Δ%
$Ma=0.730$ $\mathrm{Re}=6.5\times10^6$ $\alpha=1.842°$	C_l	0.7023	0.6902	-1.7%
	C_d	1.174×10^{-2}	1.029×10^{-2}	-12.4%
	C_m	-9.651×10^{-2}	-9.574×10^{-2}	0.8%
$Ma=0.730$ $\mathrm{Re}=6.5\times10^6$ $\alpha=1.342°$	C_l	0.6025	0.5892	-2%
	C_d	1.005×10^{-2}	9.684×10^{-3}	-4%
	C_m	-9.412×10^{-2}	-9.597×10^{-2}	1.9%

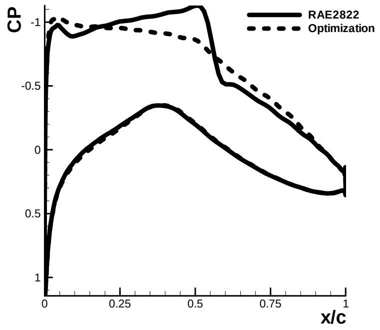

Fig.3 The comparison of pressure distributions on point 1)

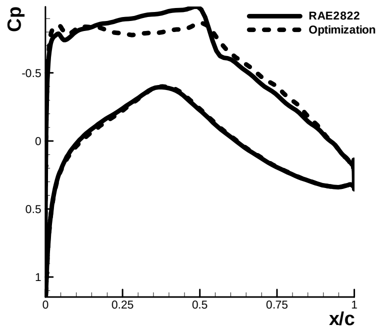

Fig.4 The comparison of pressure distributions on point 2)

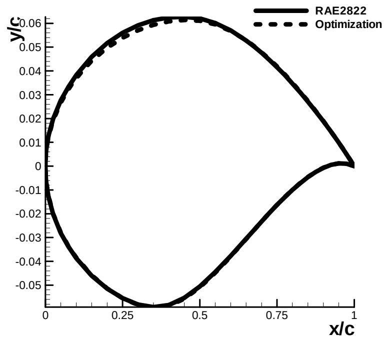

Fig.5 The comparison of geometries

The two design points described in the Table.2 have two weighting factors: 0.65 and 0.35. The minimizations of the drag C_d at the two design points are defined as objectives. Assume that the drag on point 1) is C_{d01} and the drag on point 2) is C_{d02}, respectively. Let:

$$\alpha_1 = 0.8C_{d01}, \alpha_2 = 0.8C_{d02}$$
$$\beta_1 = 1.2C_{d01}, \beta_2 = 1.2C_{d02}$$

the new aerodynamic objective function are obtained according to Eq.(11) and Eq.(12) as follows:

$$f(x)=0.65\frac{f_1(x)-0.8C_{d01}}{1.2C_{d01}-0.8C_{d01}}+0.35\frac{f_2(x)-0.8C_{d02}}{1.2C_{d02}-0.8C_{d02}}$$

There are three constraints as follows: (1) the section area of airfoil doesn't decrease, (2) the lift C_l of the first design point doesn't decrease, (3) the absolute value of the pitching moment C_m at the first-point doesn't increase. 16 design variables are employed in this example. The results are shown in Table.2. The results show that a better objective drag reduction is achieved by the present two-objective optimizations: the drag reduction at the design point 1) is 12.4% and at the design point 2) is 4%. The pressure distributions of two design points and the airfoil geometries are plotted in Fig.3, Fig.4 and Fig.5, respectively. In the overall optimum steps, this example costs 8 hours and 6 minutes on the Pentium IV computer.

6 Conclusions

A complex optimum method based on direct numerical optimization method has been presented in this paper, and two applications to airfoil optimization are described and analyzed. The results show that the robustness of an optimal solution can be achieved and the aerodynamic performances such as the lift, the drag, the pitching moment or the ratio of lift to drag are improved in the different design points.

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