APPLICATION OF AN IMPROVED INTEGRAL TURBULENT BOUNDARY LAYER MODEL WITH MODERATE SEPARATION CORRECTION ON NACA SERIES AND LOW SPEED SUPERCritical AIRFOIL DERIVATIVES

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Abstract

The paper presents latest results in the development of a method for two-dimensional integral turbulent boundary layer analysis with moderate separation correction on general aviation airfoils. Initially it was based on a NASA's computer program and some empirical results, which gave general suggestions of how to solve such category of problems, but not with quite acceptable level of accuracy, specially in the case of lower speed wing section analyses. After introducing some minor, but relevant changes in the basic turbulent boundary layer model and some more remarkable changes in the moderate separation correction part of the analysis, an accurate algorithm for a large number of airfoils of 12-18% relative thickness was obtained. These results have covered range of lift coefficients that might be encountered on airfoils in cruising flight conditions and moderate maneuvers.

Results obtained by the modified numerical model have been proved by comparison with NACA/NASA experimental data for the analysed airfoils. So far, the modified model has been confined to lower Mach number verification with the data for existing airfoils, and the introduced algorithms are connected with particular categories and subcategories of airfoils. The aim is to spread its use to newly designed airfoils.

Symbols

- \( C_p \) pressure coefficient at separation point
- \( X_{sep} \) separation point position corresponding the \( \alpha \) angle of attack
- \( X_{sep}(\alpha) \) separation point position for angles of attack higher than \( \alpha_i \)
- \( C_{p\text{cc}}(X_{sep}, \alpha) \) base pressure coefficient correction factor
- \( \Delta C_{p\text{cc}}(X_{sep}, \alpha) \) base pressure coefficient correction factor gradient
- \( C_{p\text{cc}}(X_{sep}, \alpha_i) \) base pressure coefficient correction factor at \( \alpha_i \) angle of attack
- \( \alpha_i^* \) non-standard \( \alpha_i \), used in cases when transition from attached flow calculation to modified separation model can not be done through \( \alpha_i \) that belongs to introduced separation correction algorithm
- \( X_{sep}^* \) separation point position corresponding the \( \alpha_i^* \) angle of attack
- \( C_{p^*} \) base pressure correction factor at \( \alpha_i^* \)
- \( \delta \) boundary layer displacement thickness
- \( \theta \) boundary layer momentum thickness
- \( H \) boundary layer shape factor
- \( M_v \) local Mach number outside the t.b.l.
- \( u_e \) local velocity outside the t.b.l.
- \( \rho_e \) local density outside the t.b.l.

Introduction

This paper describes the results of an investigation aimed to modify and improve the turbulent boundary layer model such as the one applied in computer program Transes\(^{(1)}\) developed for NASA, widely and quite successfully used in transonic airfoil analysis and design. Unfortunately, when applied at lower subsonic speeds and on airfoils of higher relative thickness such as GA(W)-1, it misses experimental results for drag coefficient of the order of 7-10%, which is unacceptable even for preliminary engineering purposes. An attempt to spread the successful use of this program to lower Mach and
moderate Reynolds numbers by improving its boundary layer model was quite a challenge.

The key problem was that specific rectangular grid used in this program enforced the application of an integral model for boundary layer calculations, with adjacent approximations and averaging as possible sources of the uncertainty of the results. The flow over the displacement surface is calculated by full potential model. The boundary layer transition was fixed to 6% chord, corresponding to 'standard roughness' conditions, and momentum integral equation is solved for the momentum thickness using the equations of Nash and MacDonald for skin friction and shape factor. These equations contain correction factors for compressibility effects on boundary layer, but their influence at small Mach numbers becomes negligible and they can not be the cause of any mistakes. The boundary layer displacement thickness is smoothed everywhere and added to the airfoil ordinates to obtain the body boundary conditions for potential flow. For separated flow at higher angles of attack, it is suggested that from the point of separation on to the trailing edge an inverse approach is to be used. The initial value of displacement thickness should be determined by increasing the Cp distribution linearly from Cpsep at separation point up to the value Cps at the trailing edge (which for the aft-cambered airfoils corresponds to Cpm on at the rear part of the lower camber, according to the original model) and then the real displacement surface and Cp distribution in separated region is calculated during the iteration process.

Original model, in case of the analyses mentioned above, in the minimum drag region misses the experimental polar quite remarkably. When an attempt is made to calculate the region of moderate separation (as originally suggested), it is not possible to join the part of the polar without correction smoothly, and beside that, it corresponds to the experiment in a very small domain.

Many attempts to improve the results, in the beginning, only seemed to make them worse. The turbulent boundary layer model was very sensitive to any changes, producing 'oscillating' polars and lift curves. Finally, by simply altering the last constant in the classic equation of Nash that relates Clauser parameters G and Gp for nonequilibrium boundary layer to a certain higher value (the idea which in the beginning seemed like a blasphemy) everything came to the proper place. Besides on GA(W)-1 airfoil, this change gave good adjustment of polar curves also on NACA 4 and 5 digit series airfoils for standard roughness conditions.

Keeping this change, the domain of moderate separation was tested including original correction model. The new troubles appeared, until a certain relation between the position of separation Xsep and fractions of Cps used for different angles of attack was established for certain categories and subcategories of airfoils. These results gave unique solutions for Cl and Cd that properly and smoothly matched the experimental curves. The mentioned relation between Cps fraction and Xsep is a very simple function both in case of GA(W)-1 airfoil and NACA 4 and 5 digit series of airfoils. Keeping this logic, good polars were calculated for some GAW airfoil derivatives designed at the Belgrade Faculty of Mechanical Engineering.

Description of the method

The method that will be described is developed from the Nash-Macdonald turbulent boundary layer model with included moderate separation correction such as the one applied in Tranxdes computer program. This program applies the full potential airflow calculation around the airfoil both in inverse design and airfoil analysis cases on series of specifically designed rectangular grids which enable very quick and efficient convergence of solution. The viscous interaction effects can be included as an optional mode while in that case the airflow is calculated about the surface that is obtained by adding the calculated local displacement thickness to the airfoil coordinates. Although nowadays there are more profound methods for calculation of airflow, like those based on Navier-Stokes equations, this program category has still kept its firm positions in many aerospace companies and institutions, although in many cases in some upgraded forms, since it requires very short computer run-time and gives good results.

(Such a program, named Tranpro(2), was developed at Belgrade Faculty of Mechanical Engineering from Tranxdes, but with completely automated trailing edge closure and some additional airfoil design input options. Results that will be presented in the paper were obtained by Tranpro's recently upgraded turbulent boundary layer model calculations).

Original turbulent boundary layer model

In brief, the original boundary layer model used in Tranxdes and initially in Tranpro computer programs is based on calculation of the distribution of displacement thickness $\delta$ on upper and lower camber of the airfoil by solving the momentum integral equation:

$$\frac{d\theta}{dx} + \left( H + 2 - M_c^2 \right) \frac{\theta}{u_c} \frac{du_c}{dx} = \frac{x}{\rho_c u_c} = \frac{1}{\xi^2}$$

(1)
for the momentum thickness $\theta$. In equation (1) subscript 'e' denotes the local values of Mach number, density and velocity just outside the boundary layer, while $H = \delta^*/\theta$ is the shape factor. Parameter $\zeta$ from the same equation is calculated as:

$$\zeta = f_1(M_e) \left[ 2.4711 \ln[f_2(M_e, u_e, \theta) + 4.75] + \frac{1724}{G^2 + 200} \right] + 1.5G + \frac{16.87}{G^2 + 200} \quad (2)$$

where $f_1$ and $f_2$ are known functions. Clauser parameters $G$ and $\beta_p$ are defined by equations (3) and (4):

$$G = \frac{H - 1}{H} \zeta \quad (3); \quad \beta_p = -H \zeta^2 \frac{\theta}{u_e} \frac{du_e}{dx} \quad (4)$$

and related by the empirical equation of Nash:

$$G = 6.1 (\beta_p + 1.81)^{1/2} - 1.7 \quad (5)$$

For calculated values of $G$ and $\zeta$, shape factor $H$ is obtained from equation (3), and corrected for compressibility effects (correction is relevant only for higher subsonic Mach numbers). Then from the value of $(d\theta / dx)$ calculated by equation (1), $\theta$ and $\delta^*$ are obtained. This computation is done on the same grid spacing as for the inviscid part of calculation. The local values of velocity over the displacement surface are calculated in the relaxation cycles converging to final values simultaneously with the boundary layer solution.

The calculated displacement surface is then smoothed over the upper and lower airfoil camber and extrapolated at the trailing edge. After that values of $\theta$ for upper and lower camber trailing edge are recalculated. Final solution of airflow is obtained using the smoothed displacement surface coordinates. Airfoil profile drag is calculated by the Squire-Young formula using final values of $\theta$ at the trailing edge.

Original separated flow correction

Previously described boundary layer calculation can be used in case of attached flow over the whole surface of the airfoil, i.e. in a narrow range of angles of attack corresponding to the minimum drag coefficient $C_{d_{\text{min}}}$. If the angle of attack is increased beyond that range, separation occurs and in the separated region boundary layer calculation cannot be applied. Still, in case of not to strong separation it is possible to prescribe some additional, in most cases purely empirical equations, which could lead to satisfactory solutions when combined with partly restricted boundary layer equations in the domain of separated flow. Strictly theoretically speaking, such an approach might not be exactly correct, but for engineering purposes it could give quite satisfactory results.

Such a moderate separation model was suggested in Trandes program. At the moment the program was issued, this model was suggested for use with caution, since not too much experience with its use had been gained.

The suggested model was based on the following:

(a) Since the turbulent boundary layer model can not be applied in the separated flow region, an additional relation must be introduced to 'close' the calculation cycles, by prescribing the initial values of displacement thickness in this case. Some experience showed that by setting a new, linear $C_p$ distribution from $C_{p_{\text{sep}}}$ at Xsep to the value $C_{p_{\text{base}}}$ at the trailing edge on the upper airfoil camber can 'artificially thicken' the initial $\delta^*$ values, which after the iteration process is completed may quite well converge to the real distribution (if Xsep position has been correctly chosen). Such $C_p$ distribution is only used to initialize the $\delta^*$ values and it by no means is an attempt to give a 'smart guess' of what $C_p$ should be like in that region.

(b) Base pressure coefficient $C_{p_{\text{base}}}$. has a typical value of $C_{p_{\text{base}}} = 0.6$ (approximately) for classical airfoils while for aft-cambered airfoils it should be set to $C_{p_{\text{base}}} = C_{p_{\text{max}}}$ where this value represents the maximum $C_p$ value close to the trailing edge at the lower surface of the airfoil.

(c) Not only at separation point, but in the whole region $r_m$ is set to zero in order to retain numerical consistency of the model (since in separated region it is negative). In that case, $\zeta$ and also $\beta_p$ become infinite. Instead of that, $\beta_p$ is set to high enough value of 10000, and other parameters calculated based on this value.

(d) The equations of the Nash-MacDonald model with mentioned restrictions as for the boundary layer are applied in the separated region.

(e) So calculated displacement thickness distribution is used only to determine the slope of the displacement surface at Xsep. This value is then used for extrapolation of new $\delta^*$ distribution which is linear from Xsep to the trailing edge.

Both in the calculation of separated flow and the turbulent boundary layer calculation, a certain sensitivity to grid spacing is reported. Since the computer program uses a specific series of rectangular grids with constant spacing in x - direction in the domain of the airfoil, some uncertainties may occur in the region of leading and trailing edge, where closer grid spacing in some cases might be necessary.
Modified turbulent boundary layer model

In order to satisfy some project requirements, a computer program *Tanpro* was used on PC486 category computers for the analyses of airfoils for general aviation use, 12 - 18% thick, at low Mach and moderate Reynolds numbers. (Initially, some minor adjustments of computer listing were done in order to match the results of *Trandes* test samples in the domain of transonic analysis and design, due to the different processor types of computers on which *Trandes* was originally run). *Tanpro* computer program in the beginning had the same viscous interaction model as *Trandes*, described in the two previous sections. In order to confirm the solutions for the mentioned airflow domain, results for some usual airfoils with transition fixed at 6% chord were compared with the experimental data. Surprisingly, quite remarkable discrepancy with the experiment was observed (figures 7 - 15.)

In all cases, program results gave drag coefficients 7 - 10% lower, and lift coefficients slightly higher than the experiment. Since drag coefficient was obtained through the boundary layer calculation, many steps were taken to detect the source of the problem there.

The integral boundary layer model does not give any insight in the internal boundary layer physicality, so the consequences of variation of some empirical relations used in the model could not have been examined directly. On the other hand, improper variation of some of them, as an example, caused ‘oscillating’ polars with the change of $\alpha$, which meant that physical consistency of the model was disturbed, and such modifications were abandoned.

At last, when analysis was focused to the equation of Nash (5), some promising results appeared. The equation was rewritten in the form:

$$ G = A_N (\beta_p + B_N)^{1/2} - C_N $$

(6)

and the introduced coefficients were varied separately for the fixed values of $\alpha$ approximately corresponding $C_{D_{min}}$ for various airfoils. Example of such an analysis is given in Table 1 and on figures 1 - 6, for GA(W)-1 airfoil (in case of the other tested airfoils conclusions are more or less the same).

From data presented in Table 1, the following is obvious:

- decreasing of only $A_N$ (figures 1. & 2.) causes uniform increase of drag coefficient, but non-uniform behavior of lift coefficient (first decreases, than gradually increases), which eliminated this parameter from further consideration;
- decreasing of only $B_N$ (figures 3. & 4.) causes an uniform increase of drag coefficient and uniform

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**Table 1.**

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<th>Variation of $A_N$</th>
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<th>$C_L$</th>
<th>$C_D$</th>
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<table>
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<td>0.01225</td>
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decrease of lift coefficient at the same time, which is a
desired tendency;

- increasing of only C_N (figures 5. & 6.) also causes an
uniform increase of drag coefficient and uniform
decrease of lift coefficient at the same time.

Both B_N and C_N variations can reach desired values of
drag coefficient. Beside that, C_N gives good match of
lift coefficient, very 'stable' polar curve and good
agreement with experiment in the range of α in which
turbulent boundary layer model without separation
correction may be applied, so C_N was the final choice
for further analyses. In case of a wide number of
tested airfoils including GA(W)-1, the value of C_N =
4.1 proved to give the best results (see C_{Dmin} region
of the polars of airfoils at figures 7. to 15.). Thus for
the modified equation of Nash the following form was
accepted:

\[ G = 6.1 (\beta_p +1.81)^{0.2} \cdot 4.1 \]  \hspace{1cm} (7)

In this paper no further efforts will be involved in
trying to explain the physical background of the
introduced change. The explanation that 'it simply
works well' in practice may be enough.

### Table 2.

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<tr>
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<th>Xsep_i</th>
<th>CCp*</th>
<th>(\alpha_i) deg.</th>
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<th>CCp(Xsep,α)</th>
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<td>0.96 = const.</td>
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**Modified separated flow correction**

In the next step, the original separated flow
correction was applied with the modified boundary
layer model in the analyses of airfoils at higher angles
of attack. Many tests were done, and the following
two main problems appeared: (1) the part of the polar
with the applied correction could not join the part of
the polar without correction smoothly (in many cases
it could not join it at all), and (2) the part of the polar
with the applied correction matched the experimental
polar curve in a very small domain, actually, it just
crossed it at a certain C_L value.

After many introduced changes in order to make a
model that will work well in a range of angles of
attack where moderate separation occurs, an
algorithm which gave very good results has been
derived. This model is based on the following:
(A) Recommendations (a) and (c) of the original model should be accepted completely.

(B) Instead of the original equation of Nash, in the restricted boundary layer model equations set, the modified equation (7) should be used.

(C) For categories and/or subcategories of airfoils, the angle of attack \( \alpha \), from which moderate separation correction is to be introduced, should be defined (Table 2.)

(D) For categories and/or subcategories of airfoils, the relative position of initial separation point Xsep, corresponding the \( \alpha \), angle of attack should be defined (Table 2.)

(E) Equation which defines the separation point position Xsep for angles of attack \( \alpha \) beyond \( \alpha_s \), for categories and/or subcategories of airfoils should be defined. For all tested airfoils (except the NACA 230xx category) it is given by the relation:

\[
X_{sep}(\alpha) = X_{sep} - 0.03 \cdot (\alpha - \alpha_s)
\]  

(F) Base pressure coefficient \( C_{p_{base}} \) must be defined as a function of \( C_{p_{\max}} \):

\[
C_{p_{base}} = C_{p_{max}}(X_{sep}, \alpha) \cdot C_{p_{\max}}
\]  

where \( C_{p_{\max}} \) represents the maximum \( C_p \) value close to the trailing edge at the lower surface of the airfoil, and \( C_{p_{max}}(X_{sep}, \alpha) \) is the base pressure coefficient correction factor.

(G) Base pressure coefficient correction factor \( C_{p_{\max}}(X_{sep}, \alpha) \) must be determined for categories and/or subcategories of airfoils (Table 2.) as a function of the angle of attack or the separation point relative position:

\[
C_{p_{\max}}(X_{sep}, \alpha) = C_{p_{\max}}(X_{sep}, \alpha_s) + \Delta C_{p_{\max}}(X_{sep}) \cdot (X_{sep} - X_{sep}) \quad (10.6)
\]

or

\[
C_{p_{\max}}(X_{sep}, \alpha) = C_{p_{\max}}(X_{sep}, \alpha_s) + \Delta C_{p_{\max}}(\alpha) \cdot (\alpha - \alpha_s) \quad (10.6)
\]

where \( \Delta C_{p_{\max}}(X_{sep}) \) or \( \Delta C_{p_{\max}}(\alpha) \) represent the base pressure coefficient correction factor gradient, and \( C_{p_{\max}}(X_{sep}, \alpha_s) \) base pressure coefficient correction factor at \( \alpha_s \), angle of attack.

(H) In case of some airfoils, transition from attached flow calculation to modified separation model can not be done through \( \alpha_s \), that belongs to introduced separation correction algorithm. In that case, the non-standard values of \( \alpha_s \), Xsep, \( \alpha \) and \( C_{p_{\max}} \) must be determined (Table 2.).

(I) Values of \( \delta^* \) and \( \theta \) calculated by presented modified separated flow correction and smoothed should be accepted as final (i.e. recommendation (e)) of the original model must not be applied.

Described model when separated correction must be applied gives very good results compared with the experimental data (results for higher \( C_i \)-s, figures 7. to 15.). All parameters needed for its application which are different from the original model are given in Table 2.

Presentation of the results

Results obtained by the modified model and compared with the results of the original model and experiment are given in figures 7. - 15. At high \( C_i \)-s slight divergence of the modified model results from the experiment announces that moderate separation changes to massive flow separation, which can not be treated by the models based on momentum integral equation (1) and separated flow corrections, described in this paper.
Analysis of the modified model results

Presented results show the obvious advantages of the modified model over the original. This model, however, is an intermediate development phase of another, more sophisticated model which would not be confined to a certain airfoil category but only to its geometry and airflow conditions. So far, this model can still be successfully used instead of the original in the design process of new airfoils, but correction for separated flow must be determined from the presented algorithms by placing the new airfoil in one of the existing categories according to its geometric characteristics. This might be a source of possible mistake for an not too experienced user and it should be done with caution.

The only problem encountered in the use of separation correction in the modified model at higher angles of attack is the tendency to shift $C_p$ curves slightly down compared with the experimental results. Thus lift and drag remain correct but $C_{n1/4}$ coefficient changes.

When modified model with separated flow correction is applied, in some cases it shows a tendency to translate the $C_p$ curves both for the upper and the lower camber of the airfoil just slightly down with
respect to the experimental results, when applied at higher angles of attack (figure 21.). This difference is generally very small and it does not affect results for \( C_L \) and \( C_D \), but it changes moment coefficient more remarkably. In that case, instead of \( C_{m_{1/4}} \) values (quarter chord moment coefficient), the \( C_{m_{a,c}} \) (moment coefficient about the aerodynamic center) should be recalculated from results for several smaller angles of attack where separated flow correction is not necessary, using modified boundary layer model. After that, the same \( C_{m_{a,c}} \) value can be used for aerodynamic moment calculations at higher angles of attack. The source of this could also be in the specific rectangular grid type used in computer program.

It should be kept in mind that described model for separated flow correction can be used for the analyses of airfoils at which moderate separation starts at the trailing edge. That means that it can not cover cases when larger separation bubble with reattachment is formed somewhere on the airfoil (for instance on thin laminar airfoils at higher angles of attack close to the leading edge, or on airfoils with concave upper camber in the trailing edge region). Even with this exception, it can be applied on most of the airfoils suitable for use on low speed general aviation aircraft, since Table 2. lists only some of the airfoils on which the successful use of modified model was proved. Although, if an attempt to apply this model on an airfoil from the exempted category is done for any reason, no unique solutions for \( C_L \) and \( C_D \) will be obtained and a user will not be drawn to wrong conclusions.

Unlike some existing modifications of this kind, (for instance as the one developed by well known American designer Burt Rutan for one of his canard-configured light aircraft where a completely new nose was formed), on GA(W)-NI10 the original nose section of GA(W)-1 was 'cut-off' and rotated nose-down for 10° about a certain point, and then blended with the rest of the contour of the original airfoil.

In case of GA(W)-M125 airfoil, a specific trailing edge modification of the original airfoil was done (figure 23.)

Actually, only approximately last quarter of the lower camber was replaced by a curve with the trailing edge angle of -25° instead of -11.96° of the original airfoil.

Results obtained by Tranpro with the modified turbulent boundary layer model with moderate separation correction are given in figures 24 - 27.

**Modified model in practice**

The presented modified turbulent boundary layer model with moderate separation correction was included in Tranpro computer program. Among the other cases, it was used for the analyses of two interesting GA(W)-1 airfoil derivatives, designated GA(W)-NI10 and GA(W)-M125, developed at the Belgrade Faculty of Mechanical Engineering.

These airfoils were designed for some practical project requirements with an intention to further improve the original airfoil's lift characteristics.

The airfoil GA(W)-NI10 was obtained by leading edge region modification of GA(W)-1 (figure 22.).

![Figure 22.](image)

![Figure 23.](image)

![Figure 24.](image)

![Figure 25.](image)

![Figure 26.](image)
In case of both of the new derivatives, a slightly altered GA(W)-1 modified model algorithm was applied.

The reason why these two airfoils were presented in this paper in such a detail requires an explanation. The primary task during the design of the two GA(W) derivatives was to further increase $C_{L_{\text{max}}}$ (although the original airfoil has very high value of maximum lift coefficient). As already mentioned, the described boundary layer model can not deal with massive separation i.e. with around-critical angles of attack. Still, at higher angles of attack with moderate separation, $C_{L}$ of both GA(W) derivatives have overcome the original airfoil's lift curve, while their polars show that their drag divergence region, corresponding the end of moderate and start of massive separation, is moved to higher $C_{L}$ values. Since the original nose shape was kept in both cases, no abrupt stall of this two derivatives is to be expected, and so an expectation of higher $C_{L_{\text{max}}}$ in both cases sounds very reasonably. It is strongly believed that some future experiments will prove this statement.

Thus modified model can, in some cases, be used to give 'reasonable estimates' of airfoil behavior beyond the model's range of application, besides fulfilling its primary role. Mentioned GA(W) derivatives are believed to be such an example.

**Conclusion**

The modified turbulent boundary layer model with moderate separation correction was derived from the original model, applied in computer programs Trandes and initially in Tranpro. It has shown remarkable advantages over the original model in cases of application at low Mach and moderate Reynolds numbers, on most of the airfoils suitable for use on low speed general aircraft. In all presented analyses transition was fixed to 6% chord, which corresponds to the airflow conditions most often encountered in flight, even on composite structure wings when polluted by insects, dirt or rain and on metal wings with leading edge flaps, leading edge structure caps, de-icer boots, round head rivets extending over the airfoil surface close to the leading edge, etc.

The lift coefficient range available for the analyses is spread by the modified model to most of the values that appear in flight, except very high $C_{L}$s during extreme maneuvers. Thus a computer program applying the modified model can be used in the airfoil selection or design phase of the aircraft design process for quick and efficient determination of optimal airfoil choice (but it certainly can not exclude the necessity for final experimental verifications).

So far, the modified model applies algorithms confined to airfoil categories and subcategories. The future aim is to connect them with airfoil geometry and airflow conditions only.

**References**

5. R.J. McQueen, W.D. Beasley: "Low Speed Aerodynamic Characteristics of a 17 Percent Thick Airfoil Section Designed to General Aviation Applications", NASA TN D-7428