A PROBABILISTIC APPROACH TO FATIGUE DAMAGE ACCUMULATION FOR DAMAGE TOLERANCE AND DURABILITY OF AGEING AIRCRAFT

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1. Abstract

For damage tolerance and durability evaluation of ageing aircraft, a lap joint research project was started in the 1980s by the authors. A set of probabilistic analysis of fatigue damage accumulation has been conducted based upon two large sample size tests. From the test data, it is found that both crack initiation and crack growth have significant scatter even under identical loading and a laboratory environment. Therefore, both the probability of crack occurance and of crack growth are highlighted and predicted with different probabilistic models. A two–stage PFA+SCGA approach is developed to assess the distribution of total service time. Furthermore, combined with three statistical brittle fracture criteria, this approach is also applied to determine the probability of failure of ageing aircraft structure. Two large sample size tests with over 100 specimens each were completed using constant–amplitude loading and fighter spectrum loading (FALSTAFF). These test data provide important experimental confirmation and verfication to the probabilistic analysis of fatigue damage accumulation.

2. Introduction

With the passage of time, a new aircraft will gradually become an ageing aircraft. Following some accidents of ageing aircrafts, such as the Japan Airline accident of a Boeing 747 in 1985[1] and the Aloha accident of a Boeing 737 in 1988[2], etc., the tolerance and durability reevaluation of ageing aircraft were highlighted in aviation industries of the world. Some strong voluntary efforts are carried out reevaluating all the structural safety surrounding ageing aircraft. For example, the Airworthiness Assurance Working Group (AAWG) proposed an airplane evaluation process for 11 models of ageing aircraft[2–3]. Reevaluating ageing aircraft has led to some changes to the current concept of damage tolerance and durability analysis. As a consequence limited cracks are admitted in ageing aircraft structure on the condition that the structure has redundant resdiual strength and a catastrophic failure will not occur after fatigue failure of a structural element. Therefore, both the crack initiation and the crack growth are significant for reevaluating ageing aircraft structural safety, especially for crucial structures such as the main joints between wings and fuselage[4]. These structures are often difficult or uneconomic to be inspected and reparied. It is widely recongnized that fatigue damage accumulation is fundamentally a random phenomenon[3–4]. The probability of crack existence is particularly enhanced in ageing aircraft. For example, a probabilistic approach is often used to solve the key problem of ageing aircraft—MSD (Multiple Site Damage) / MED (Multiple Element Damage)[3–6], where the occurance of MSD / MED and the multi–crack growth are all random. At the present, a probabilistic approach is thought of a promising method of tolerance and durability analysis either for ageing aircraft or for new aircraft[7].

From the 1980s, our Fatigue and Fracture research group began studying a probabistical approach to fatigue damage accumulation for damage tolerance and durability of ageing aircraft. This project, had been supported by the National Natural Science Fund, Aeronautical Science Fund and contracts with several aircraft manufacturing companies in China, was started from a fatigue test for a multi–fastener, multi–layer lap joint (Figure 1.) in an ageing Boeing 707[8]. This joint was found to be a multi–crack source and have multi–crack growth and a multi–failure pattern. The main purposes of this project are:

○ On the basis of large sample tests, the statistical nature of fatigue damage accumulation is investigated. Then a set of probabilistic methods for fatigue crack initiation and growth of ageing aircraft is developed to carry out the probabilistic analysis of fatigue total life to a given crack length or to failure.

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○ Considering the influence of MSD / MED, these methods are applied to the probabilistic damage tolerance and durability analysis of the lap joint of ageing aircraft.

This paper focuses on the first part through theoretical research activity and has limited application for industrial assessment application.

3. Two Large Sample Size Test Programs

In 1979, Virlker et al. carried out the well known test series of 68 similar tests[9], as did Ghomen et al. in 1985[10]. However, all tests specimens of Virlker and Ghomen have precut crack and test data and include only the crack growth stage. Many existing results stated that the scatter of crack initiation is very different from the scatter of crack growth, since they are influenced by different internal and external factors[4].

In 1993[11] and 1989[12], the authors' research group finished two large sample size test programs under both constant–amplitude loading (R =−0.1) and fighter spectrum loading— the well known FALSTAFF (Fighter Aircraft Loading Standard For Fatigue). Dogbone plate specimens with a center hole was chosen for both test programs, and the specimen material was LY12 aluminuim alloy with the CZ heat treatment. None of the specimens in two test programs had precut cracks. An "Ageing fatigue crack" was initiated under cyclic loading. Optical reading microscope, replica technique and direct current electrical potential technique controlled by computer were used to monitor crack initiating and propagating. Post–mortem experimental fractographic examination by SEM was also performed on some specimens to reveal qualitatively the microstructural nature of random crack growth. 101 and 102 actual test specimens were used in the two test programs, respectively. From the test data, four results were obtained and are summarized below:

○ A large sample size of fatigue total life data of crack initiation, growth and fracture was obtained as shown in Figures 2–3. The division of fatigue total life is illustrated in Figure 4. The crack initiation life, t_i, is defined as the period from nondamage to an engineering inspectable crack length, a_0. The crack growth life t_p, is defined as the period from a_0 to a longer through crack even to fracture. Considerable scatter is found in the fatigue total life, t_T, as shown in Figures 2–3. The fatigue total life, t_T, can be fitted using normal distribution. The mean and standard deviation of t_T in the constant–amplitude test are:

$$\mu[t_T] = 100607 cycles,\ \sigma[t_T] = 20631 cycles \quad (1)$$

The mean and standard deviation of t_T in the FALSTAFF spectrum test are:

$$\mu[t_T] = 103899 cycles,\ \sigma[t_T] = 17635 cycles \quad (2)$$

From Figure 6, it is shown that macrocrack growth is very different from small crack growth. Therefore, more attentions must be paid when using macrocrack data to extrapolate small crack data or the IFQ (Initial Fatigue Quality) and the IFQ distribution[7].

○ A large sample size of crack initiation life data was also obtaine. From the above definition of initial crack length a_0, a_0 is equal to or greater than the upper limit of a small crack, meaning the crack initiation life must involve small crack growth life (Figure 4). For the constant–amplitude loading test, we take a_0 =0.5mm. The mean and standard deviation of $ln(t_i)$ are:

$$\mu[ln(t_i)] = 11.272,\ \sigma[ln(t_i)] = 0.2664 \quad (3)$$

For FALSTAFF spectrum test, a_0 =1.045mm and the mean and standard deviation are:

$$\mu[ln(t_i)] = 11.369,\ \sigma[ln(t_i)] = 0.2027 \quad (4)$$

Through the K–S test, is is found that crack initiation life, t_i, can be fitted very well using a lognormal distribution. Different lengths of a_0 do not have a big influence on the distribution of t_i, but can affect the standard deviation of t_i. For practical structure, a proper a_0 is determinated according to inspection methods and the corresponding inspection interval.

○ A large sample size of crack growth data was obtained. From Figures 2–3, 102 constant–amplitude crack growth a vs. N curves (which are all start from a_0 =0.5mm), can be treated as shown in Figure 5; and 101 FALSTAFF spectrum crack growth a vs. N curves (which are all start from a_0 =1.045mm) are shown in Figure 6. From Figures 5–6, it is seen that the ensemble of a vs. N curves have nonnegligible scatter. Through the K–S test, the crack growth life, t_p, to reach different crack lengths can be fitted very well using a lognormal distribution. That the scatter of crack growth is smaller than the scatter of crack initiation.

○ From laboratory observation, the crack growth from a nondamaged hole, i.e., from an "Ageing Fatigue Crack", is much more irregular than from a pre–cracked hole. The former is often much closer to practical experience, especially for ageing aircraft service life has been very long. Furthermore, through post–mortem fractographic examination by SEM, the material intrinsic inhomogeneity in a polycrystalline material creats a random distribution of fatigue striation intervals. This random distribution behavior appears to substantiate at least qualitatively the hypothesis that the crack growth process is a stochastic process in space or time with a nonzero correlation distance or time.

4. Probabilistic Fatigue Analysis (PFA)— Calculating the Probability of Crack Occurance

Considering the influence of many internal and external factors, fatigue damage accumulation is in practice a very complicated stochastic process. Many cumulative fatigue damage rules have been developed. The most common rule is still the well known Miner's rule. Based upon the study of fatigue damage process mechanism D(t) and a probabilistic Miner's rule, the authors have developed a new cumulative fatigue damage dynamic interference statistical model to calculate the probability of crack occurance[13]. The main points are as follows:

○ The damage introduced by the ith cycle is defined phenomenologically as $D_i = 1/N_i$, where N_i is the crack initiation life at ith stress level; D_i and N_i are all random variables. Then,

$$\ln D_i = -\ln N_i \tag{5}$$

That is, the distribution of D_i is same as that of N_i for a corresponding stress cycle. Usually, the distribution of N_i is a lognormally distribution. The mean and variance of N_i are easily determined from a lot of existing constant–amplitude test data.

○ When a specimen is subjected to a number of n cycles loading at time t, fatigue damage accumulation at time t is given by the following conditional linear relation:

$$D(t) = \sum_{i=1}^{n} D_i = D_1 + D_2|_{D_1 = f_1(\triangle\varepsilon_1, R_1)} + \cdots\cdots + D_n|_{D_{n-1} = f_{n-1}(\triangle\varepsilon_{n-1}, R_{n-1})} \tag{6}$$

where $f_i(\cdot)$ represents a complex damage function, $\triangle\varepsilon_i$ is the cyclic strain range and R_i is the strain ratio. $f_i(\cdot)$ is calculated by a modified local stress–strain method in which the load sequence effect and the material cyclic hardening / softening effect are simulated by using the improved Neuber method coupled with transient cyclic stress–strain curve[12]. In fact, fatigue damage D_i in eq.(6) is nonlinear. From eq.(6), if n is large enough, the central limit theorem can be employed, and $D(t) = \sum_{i=1}^{n} D_i$ will be normally distributed. Here, D_1, D_2 $\cdots D_i \cdots$ are also independent of each other and have definite mean and variance; then

$$\begin{cases} \mu_{\Sigma D} = \sum_{i=1}^{n} \mu_{D_i} = \sum_{i=1}^{n} (1+\eta_{N_i}^2)/\mu_{N_i} \\ \sigma_{\Sigma D} = \sqrt{\sum_{i=1}^{n} \sigma_{D_i}^2} = \sqrt{\sum_{i=1}^{n} [(1+\eta_{N_i}^2)^2 \eta_{N_i}^2 / \mu_{N_i}^2]} \end{cases} \tag{7}$$

○ Critical cumulative damage, Dc, which is affected by material, loading and environment to ageing aircraft, is also a random variable. According to existing test data, it is shown that Dc can be assumed to be a lognormal distribution for most of metals under stationary loading, and $\mu_{Dc} \approx 1$, $\eta_{Dc} \approx \eta_N$ [12–15].

From the above points, a new model can be set up and combined with the stress–strength interference reliability model[1]. From this model, the probability of crack occurance at crack initiation time, t_i, corresponding to crack length a_0, $P_i(t_i)$, can be calculated by fitting t_i as a lognormal variable; then

$$\begin{aligned} P_i(t_i) &= F_i(t_i) = \int_0^{t_i} f_i(t|a_0)dt = P[D(t_i) \geqslant Dc] \\ &= P(\frac{Dc}{\sum D_i} \leqslant 1) = P(Y \leqslant 0) = \Phi\left[-\frac{\mu_Y}{\sigma_Y}\right] \end{aligned} \tag{8}$$

Where $f_i(t|a_0)$ is the probabilistic density function (pdf) of crack initiation life t_i, and $F_i(t_i)$ is the Probabilistic Distribution Function (PDF). And, $Y = ln(\frac{Dc}{\sum D_i})$ is a lognormal variable, μ_Y and σ_Y^2 are also easily calculated from eq.(7).

The comparison between the predicted results and test results is shown in Figures 7–8. The predicted results are seen to be very close to test results, either to constant–amplitude loading or to the FALSTAFF spectrum. Moreover, from Figures 7–8, the new model is seen to provide better predicted results than some existing models[12,15], especially for random spectrum loading.

5. Stochastic Crack Growth Analysis (SCGA) —Calculating the Probability of Crack Growth

In the crack growth stage, two important probabilistic distributions are important in SCGA. One is the distribution, $F_p(t_p)$, of crack growth life, t_p, to reach a given crack length, a_p^*. The other is the distribution $F_a(a_p)$ of crack length, a_p, at a given cycle (life or time) t_p^*.

In 1993, the authors presented a CSCGP (Combined Stochastic Crack Growth Process model) model, in which fatigue crack growth is randomized by a spacial stochastic process X(a). The theoretical work of the CSCGP model is adopted from the Paris law.

$$\frac{da}{dN} = c(\Delta K)^m X(a) \tag{9}$$

In eq.(9), parameter c is taken as a lognormal random variable to represent the ensemble variation of crack growth rate; and parameter m is taken as a constant.$X(a)$ is defined as a positive spacial stochastic process taking values around unity to respresent the individual irregularvariation of crack growth rate in each test. Through test data verfication, $Z(a)= logX(a)$ can be taken to be a zero mean stationary Gaussian process[16]. Integrating eq.(9) from the crack initiation length a_0 to any given crack length a, the corresponding life $N(a_0,a)$ is equal to:

$$\begin{aligned} N(a_0,a) &= \frac{1}{c}\int_{a_0}^{a} \frac{1}{(\triangle K)^m} \cdot \frac{1}{X(a)} da \\ &= \frac{1}{c} N_0(a) \cdot N_x(a) = \frac{1}{c} N(a_0,a)^* \end{aligned} \tag{10}$$

where,

$$N_0(a) = \int_{a_0}^{a} \frac{1}{(\triangle K)^m} da \tag{11}$$

$$N(a_0,a)^* = N_0(a) \cdot N_x(a) \tag{12}$$

$$N_x(a) = N(a_0,a)^* / N_0(a) \tag{13}$$

and, $N_0(a)$ is the deterministic component of $N(a_0,a)^*$, $N_x(a)$ is the random component of $N(a_0,a)^*$, which reflects the random noise of $X(a)$. The mean and variance functions of $N(a_0,a)^*$ can be deduced from eqs.(10–12)

$$\mu[N(a_0,a)^*] = \mu_x \int_{a_0}^{a} \frac{da}{(\triangle K)^m} = \mu_x \cdot N_0(a) \tag{14}$$

$$\sigma^2[N(a_0,a)^*] = \sigma_x^2 \int_{a_0}^{a} \int_{a_0}^{a} \frac{\rho_x(t-s)}{[\triangle K(t) \cdot \triangle K(S)]^m} dtds \tag{15}$$

then,

$$\mu[N_x(a)] = \frac{\mu[N(a_0,a)]^*}{N_0(a)} = \mu_x \tag{16}$$

$$\sigma^2[N_x(a)] = \sigma_x^2 \cdot \frac{\int_{a_0}^{a} \int_{a_0}^{a} \frac{\rho_x(t-s)}{[\triangle K(t) \cdot \triangle K(s)]^m} dtds}{\left[\int_{a_0}^{a} \frac{ds}{[\triangle K(s)]^m}\right]^2} \tag{17}$$

In eqs.(14–17), μ_x and σ_x^2 are the mean and variance functions of $X(a)$, respectively, which can be calculated with μ_Z and σ_Z^2 through equation $Z(a) = logX(a)$. $Z(a)$ is a zero mean stationary Gaussian process. Then, by taking a first order approximation of $Z(a)$ and using an exponential function to fit $\rho_Z(\tau)$[16], the approximate auto–correlation function of $X(a)$ is

$$\rho_x(\tau) \approx \rho_Z(\tau) = exp(-\xi \cdot |\tau|) \tag{18}$$

where, ξ^{-1} is called the correlation length (distance). Through a time series analysis of the large sample size of constant–amplitude crack growth data mentioned in Section 3, an optimum value $\xi^{-1} = 0.122mm$ is calculated for LY12–CZ aluminium alloy plate which is similar to existing results[16] (for mild steel specimens $\xi^{-1} = 0.125mm$, and for Virklers' data, $\xi^{-1} = 0.12mm$). Furthermore, taking the logarithm of eq.(10), we have:

$$\log N(a_0,a) = \log N_0(a) + \log N_x(a) - \log(c) \tag{19}$$

where, $logN_0(a)$ is a definite value for a given a_0, $logN_x(a)$ could be also fitted using a normal distribution according to test results. Since, $N(a_0,a)$ is obviously a lognormal random variable, then,

$$F_N[N(a_0,a)] = \Phi\left[\frac{\log N - \mu(\log N)}{\sigma(\log N)}\right] \tag{20}$$

where

$$\mu[\log N] = \log N_0(a) + \mu[\log N_x(a)] - \mu[\log(c)] \quad (21)$$

$$\sigma^2[\log N] = \sigma^2 \log N_x(a)] + \sigma^2[\log(c)] \quad (22)$$

In the same analysis, the distribution function of crack length $a(t)$ at a given time t can also be deduced from the above procedure.

The application of the CSCGP model is verified with the large sample size of constant–amplitude crack growth data mentioned in Section 3. Table 1. summarizes the model parameters for the constant–amplitude data. The predicted curves of crack growth life distribution $F_N(N|a_0, a)$ to reach different crack lengths, shown in Figure 9, are very close to the test results. Figure 10 gives the comparison between the predicted means of $\log N(a_0, a)$ and the test results. The predicted results agree very well with the test results. Therefore, the CSCGP model is also able to forecast accurately the median fatigue life curve.

Table 1. Ensemble statistics for constant–amplitude test data (a= 0.5—9.0mm)

Parameter	Mean Value	Standard Deviation
$\mathrm{Log}_{10}(c)$	−11.0390	0.0960
m	4.415	/
σ_Z^2	0.014959	0.04980
ξ^{-1}	0.1220mm	/

Several square root variance curves predicted using different probabilistic models are given in Figure 11, in which the test results are also presented. From Figure 11, it is shown that the CSCGP model provides the closest fit to test data among the RCGL (1) model[16] (Random Crack Growth Law model, in which the parameter c and m in Paris law are both taken as random variables), the RCGL(2) model[16] (in which only the parameter c in Paris law is taken as a random variable and m is taken as constant) and the Yang et al. model[7,16], especially for very short crack length increments. The important advantage of the CSCGP model is that a proper correlation distance has been determined from the large sample size of constant–amplitude test data.

Therefore, the distribution of $F_p(t_p)$ of crack t_p to reach a given crack length a_p^* is given by:

$$F_p(t_p) = F_p(t_p|a_p^*) = \int_0^{t_p} f_p(t|a_p^*)dt$$
$$= F_N[N(a_0, a_p^*)] \quad (23)$$

where $f_p(t|a_p^*)$ is the pdf of crack growth life t_p to reach a given crack length a_p^*.

6. Two Stage PFA+SCGA Description of Fatigue Total Life

6.1 The Probabilistic Distribution of Fatigue Total Life

Development of this method is intended to solve the following two problems: Because of the influence of many internal and external factors, the total service time, t_T, of ageing aircraft when a crack propagates to a given length, a_T, is a random variable. Refering to Section 3 (Figure 4), t_T includes the crack initiation time t_i and the crack growth time t_p, i.e. $t_T = t_i + t_p$. The first problem is to determine the distribution of t_T; if the total service time is taken as a given time t_T^*, then a_T must be a random variable. The second problem is to determine the distribution of a_T or the probability of a crack exceeding a given length a_T^* ($>a_0$) at a given time t_T^*.
From Sections 4–5, the pdf of crack initiation life t_i at a given length a_0 : $f_i(t_i|a_0)$, and the pdf of crack growth life t_p to reach a given length a_T–a_0 : $f_p(t_p|a_T - a_0)$, can be obtained from eqs.(8) and (23). The different internal and external factors influencing crack initiation and crack growth, t_i and t_p can be assumed statistically independent from each other. So the PDF of total life t_T can be inferred to be:

$$F_T(t_T|a_T) = \int_0^{t_T} f_T(t_T|a_T)dt_T$$
$$= \int_0^{t_T}\int_0^{t_T} f_i(t_T - t_p|a_0)\cdot f_p(t_p|a_T - a_0)dt_p dt_T$$
$$= \int_0^{t_T} F_i(t_T - t_p|a_0)\cdot f_p(t_p|a_T - a_0)dt_p$$
$$= \int_0^{t_T} f_i(t_i|a_0)\cdot F_p(t_T - t_i|a_T - a_0)dt_i \quad (24)$$

This function is in fact obtained from an ideal spare system in reliability theory. Its geometric meaning is shown in Figure 12. From Sections 4–5, $f_i(t_i|a_0)$ and $f_p(t_p|a_T - a_0)$ usually can be fitted with log–normal distributions. According to classic probabilistic theory, the distribution of crack length $a_T(> a_0)$ at a given total service time, t_T^*, can be given by:

$$F_a(a_T|t_T^*)= P[a\leqslant a_T,t= t_T^*]=$$
$$P[E_1(t\leqslant t_i,a= a_0)\bigcap E_2(a\leqslant a_T - a_0,\ t= t_T^* - t_i)]$$
$$= \int_0^{t_T^*} f_i(t_i|a_0)\cdot F_p(a_T - a_0|t_T^* - t_i)dt_i \tag{25}$$

Moreover, the probability of a crack exceeding a given crack length $a_T^*(> a_0)$ at a given total service time, t_T^*, is:

$$P_T(a_T^*|t_T^*)= P[a> a^*,\ t= t_T^*]$$
$$= 1- F_a(a_T^*|t_T^*) \tag{26}$$

After deriving the probability of crack exceedance $P_T(a_T^*|t_T^*)$ at a given service time t_T^*, it is possible evaluate a group of structural details (as a whole) at any total service time, determining how many cracks in these details have already exceeded the given crack length a_T^*.

6. 2 The Probability of Failure of Fatigue Total Life

The development of this analysis is intended to predict the Probability of Failure (POF) of ageing aircraft structure at any total service time t_T. Fracture toughness, K_{IC}, critical crack length, a_c, and residual strength, $R(a)$, are used to assess structural static failure by fatigue in this paper:

$$K_{IC}\ \ Criterion:\ K_I= \sigma\sqrt{\pi a}\,B(a)\geqslant K_{IC} \tag{27}$$
$$B(a)-\ Geometric\ \ Factor$$

$$a_c\ \ Criterion:\ a\geqslant a_c= f(K_{IC}) \tag{28}$$

$$R(a)\ Criterion:\ \sigma\leqslant R(a)= \frac{K_{IC}}{\sqrt{\pi a}\cdot B(a)} \tag{29}$$

Due to the material intrinsic inhomogeneity and random deviation in manufacturing, these fracture critical values (K_{IC}, a_c, $R(a)$) of brittle materials also display a great amount of scatter. Considering K_{IC}, a_c, and $R(a)$ as random variables, respectively, the POF of ageing aircraft structure at any service time is deduced as follows.

○ Fracture Toughness Criterion— At any service time, t_T, with a given crack length, a, the random fluctuation of loading will cause variability of stress intensity factor K_I. The pdf of K_I can be given by:

$$f_{K_I}(K_I)= f_\sigma(\frac{K_I}{g(a)})\cdot \frac{1}{g(a)} \tag{30}$$

In eq.(30), $g(a)= \sqrt{\pi a}\cdot B(a)$ and $f_\sigma(x)$ is the pdf of stress level σ. By applying the stress–strength interference reliability model, the POF of a structure with a given crack length a at any service time t_T is obtained:

$$P_f(t_T|a)= P[K_I\geqslant K_{IC}]$$
$$= \int_0^\infty [\int_{K_{IC}}^\infty f_{K_I}(K_I)dK_I]f_{K_{IC}}(K_{IC})dK_{IC} \tag{31}$$

where, $f_{K_{IC}}(K_{IC})$ is the pdf of fracture toughness K_{IC}. Considering the stochastic nature of crack growth caused by material intrinsic inhomogeneity and all possible crack length values at service time t_T, the POF of the structure is given by:

$$P_f(t_T)= \int_{a_0}^\infty \int_0^\infty [\int_{K_{IC}}^\infty f_{K_I}(K_I)dK_I]f_{K_{IC}}(K_{IC})$$
$$\cdot f(a|t_T)\cdot dK_{IC}da \tag{32}$$

Substituting eq.(25) and (30) into (32), the POF of the structure at any total service time t_T is given by:

$$P_f(t_T)= \int_{a_0}^\infty \int_0^\infty [\int_{K_{IC}}^\infty f_\sigma\left(\frac{K_I}{g(a)}\right)\frac{1}{g(a)}dK_I]f_{K_{IC}}(K_{IC})$$
$$\cdot \int_0^{t_T} f_p(a- a_0|t_T - t_i)\cdot f_i(t_i|a_0)dt_i dK_{IC}da$$
$$= \int_{a_0}^\infty \int_0^{t_T} f_p(a- a_0|t_T - t_i)\cdot f_i(t_i|a_0)\int_0^\infty f_{K_{IC}}(K_{IC})$$
$$\cdot \int_{K_{IC}}^\infty f_\sigma(\frac{K_I}{g(a)})\frac{1}{g(a)}dK_I dK_{IC}dt_i da \tag{33}$$

○ Critical Crack Length Criterion: Starting from eq.(30), and using the same procedures mentioned above, the POF of a structure at any total service time t_T is given by:

$$P_f(t_T)= \int_0^\infty f_\sigma(\sigma)\int_{a_0}^\infty \int_0^{t_T} f_i(t_i|a_0)\cdot f_p(a- a_0|t_T$$
$$- t_i)\cdot \int_{a_0}^a f_{K_{IC}}[\sigma g(a_c)]\frac{1}{g'(a_c)\sigma}da_c dt_i da d\sigma \tag{34}$$

○Residual Strength Criterion: Starting from eq.(31), and using the same procedures mentioned above, the POF of a structure at any total service time t_T is also given by:

$$P_f(t_T)=\int_{a_0}^{\infty}\int_0^{t_T} f_i(t_i|a_0)\cdot f_p(a-a_0|t_T-t_i)$$
$$\cdot\int_0^{\infty} f_{K_{IC}}[g(a)R(a)]g(a)\cdot\int_{R(a)}^{\infty} f_\sigma(\sigma)d\sigma dR dt_i da \quad (35)$$

In eqs.(32)–(34), the POF of a structure is established for any total service time corresponding to the K_{IC}, a_c and $R(a)$ criteria, respectively. Obviously, it is nearly impossible to get the analytic solutions of these integral equations. In engineering application, usually only part of the main scatter factors will be accounted for, yielding much more simplified expressions. Numerical methods such as importance sampling and the multi–layer sampling method in the Monte–Carlo technique[16] are often very efficient to determine numerical solutions of the POF of ageing aircraft structure.

6.3 Application Examples

Again, two large sample size tests are employed to verify the two–stage PFA+SCGA description of fatigue total life.

○ The PDF of Fatigue Total Life—Combined with the above analysis methods, the distributions of fatigue total life, t_T, at a given crack length, a_T =2.5mm, have been calculated with constant–amplitude test data and FALSTAFF spectrum test data, respectively. The comparison between the predicted results and test results are given in Figures 13–14. It is shown that the predicted results are very close to the test results, either constant–amplitude loading or random spectrum loading. Through the K–S test to the test data and the predicted results, fatigue total life, t_T, at longer crack lengths are found to be fitted very well with a normal distribution. At some short crack lengths, t_T, lognormal distributions provide a better fit. The variable characteristics of the distribution of t_T have also been noted in existing research results[17].

○ The POF of Fatigue Total Life—Here, the critical crack length a_c criterion is used to calculate the POF of fatigue total life t_T. First, because of the identical cyclic loading and a laboratory environment, the random fluctuation of loading has been minimized so that the scatter of stress level σ can be ignored. Second, from existing test data[18], a normal distribution is used to fit the distribution of fracture toughness K_{IC}, and $\mu(K_{IC})= 91.70Mpa\sqrt{m}$ $\sigma(K_{IC})= 2.07$ for Ly12–cz aluminium plate of 3mm thick. Third, the scatter of crack initiation and growth caused by material intrinsic inhomogeneity is taken as the main random factor in two large sample size test. Then, from eq.(34), the POF of the specimen at any total service time t_T is simplified as:

$$P_f(t_T)=\int_{a_0}^{\infty}\int_0^{t_T} f_i(t_i|a_0)\cdot f_p(a-a_0|t_T-t_i)$$
$$\cdot\int_{a_0}^{a} f_{K_{IC}}[\sigma g(a_c)]\frac{1}{g'(a_c)\sigma}da_c dt_i da \qquad (36)$$

In this paper, the Monte–Carlo technique coupled with the multi–layer sampling method is used to solve eq.(36). The predicted results and test results are given in Figures 15–16. It is shown the predicted results fit very well with the test results. Furthermore, through the K–S test to Monte–Carlo simulated results, it is also found that the normal distribution is more suitable to the fatigue total life, which involves the crack initiation life and crack growth life.

7. Conclusion

As the first step of the authors' project, a series of probabilistic description are used to determine the fatigue damage accumulation for the damage tolerance and durability evaluation of ageing aircraft. Some useful conclusions are obtained.

○ On the basis of two large sample size test, the statistical nature of fatigue damage accumulation from nondamage to failure are obtained. An "Ageing Fatigue Crack" is enhanced and applied in tests. Although the crack growth data is also found to have smaller scatter than the crack initiation data, both the scatter during crack initiation and propagation process must be accounted for in the probabilistic analysis of total service time of ageing aircraft. Fractographic observations provide an important physical understanding for developing probabilistic models.

○ A new cumulative fatigue damage dynamic interference statistical model is applied in the crack initiation stage to calculate the probability of crack occurance. And the CSCGP model is developed in the crack growth stage to predict the statistical distributions of crack growth.

○ A two–stage PFA+SCGA approach, presented in this paper proves to be reasonable and efficient for at least the probabilistic analysis of a simple fatigue specimen. Combined with three statistical brittle fracture criteria, it is shown that the two–stage PFA+SCGA approach could also be used to predict the POF of ageing aircraft structure.

This work provides some theoretical and application basis for advanced damage tolerance and durability analysis an ageing aircraft, or even a new type of aircraft.

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8. References

(1) Suresh, S., "Fatigue of Materials", Cambridge University Press, 1991.

(2) Jean–Yves BEAUFILS, et al., The 19th ICAS Proceedings, pp.2652–2662, U.S.A., 1994.

(3) Peter HORST, et al., The 19th ICAS Proceedings, pp.1773–1783, U.S.A., 1994.

(4) Schijve, J., Fatigue Fract. Engng Mater. Struct., 17(4), pp.381–396, 1994.

(5) Partl,O. and Schijve,J., Int.J.Fatigue, 15(4), pp.293–299, 1993.

(6) Wang,G.S., Fatigue Fract. Engng Mater. Struct., 18(2), pp.257–272, 1995

(7) Yang,J.N. and Manning,S.D., J. of Aircraft, 31(2), pp.419–425, 1994.

(8) Liao Min, The 4th International Conference On Fatigue and Fatigue Threshold, "Fatigue 90", pp.2335–2340, U.S.A. 1990.

(9) Virkler, D.A., et al., J. Engng. Mat. Tech., ASME, 101, pp.148–153, 1979

(10) Ghonem, M. and Dore, S., Engng. Fract. Mech., 22(6), pp.1151–1168, 1985.

(11) Liao Min, et al.,Int.J.Fatigue, 18(2), pp.87–94, 1996

(12) Wang Ping, Master dissertation, Northwestern Polytechnical University, 1989

(13) Liao Min, et al., Int.J.Fatigue, 17(8), pp.559–566, 1995.

(14) Tanaka, S., et al., Int.J.Fatigue, 2(4), pp.159–167, 1980

(15) An Jun–ping, Ph.D. Thesis, Beijing University of Aeronautics and Astronautics, 1992

(16) Provan,J.W., "Probabilitic Fracture Mechanics and Reliabilty", Martinus Nijhoff Publishers, Dordrecht, 1987

(17) Ochi,Y., et al., Fatigue Fract. Engng Mater. Struct., 8(4), pp.327–339, 1985.

(18) Bruckner,A., et al. In the Proceedings of Advances in Probabilistic Fracture Mechanics Conference, pp.113–131, San Antonio, U.S.A., 1984.

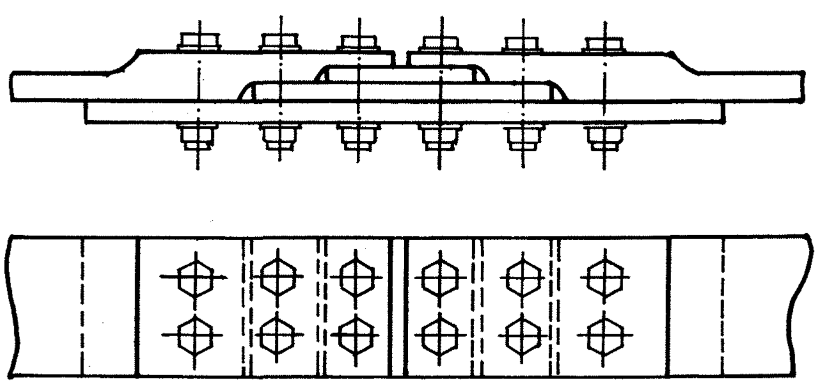

Figure 1 A lap joint of an ageing Boeing 707

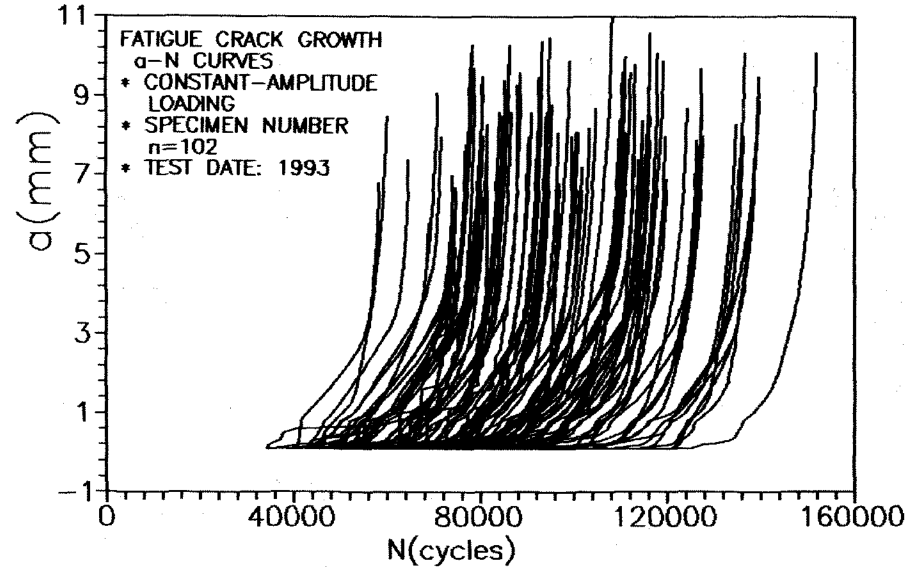

Figure 2 Fatigue crack initiation and crack growth a–N test data

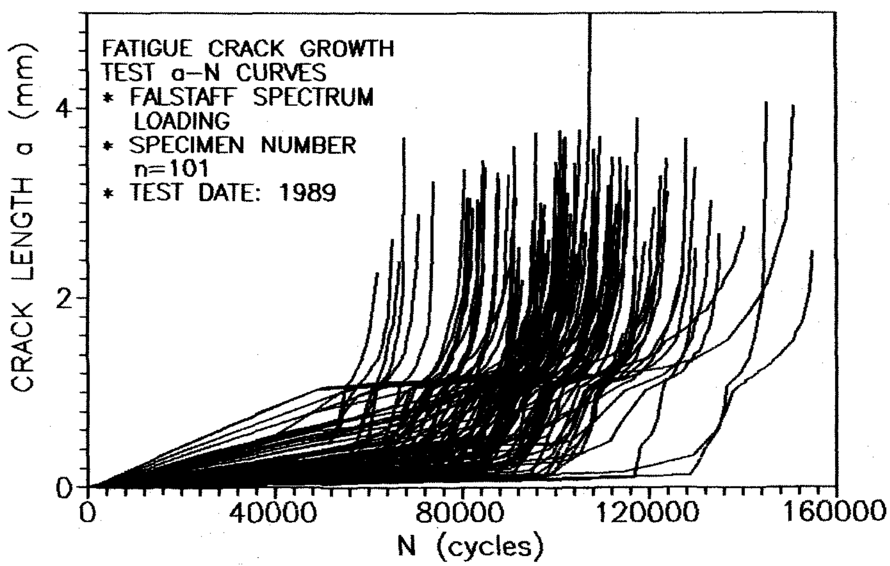

Figure 3 Fatigue crack initiation and crack growth a–N test data

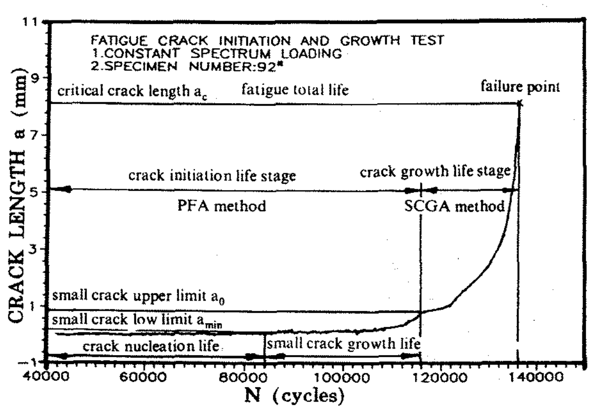

Figure 4 The division of fatigue total life

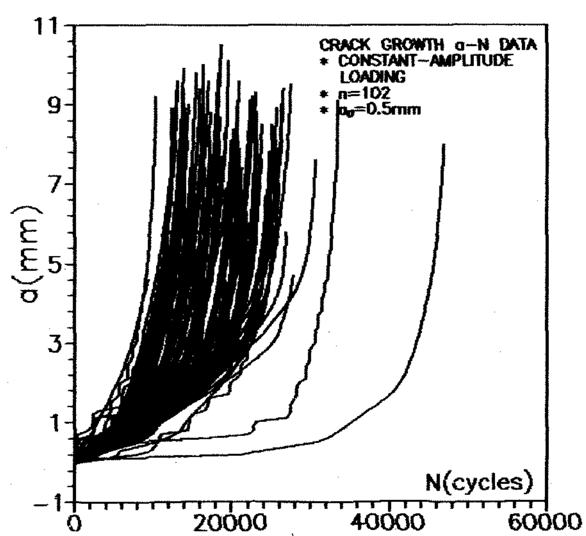

Figure 5 Fatigue crack growth a–N test data

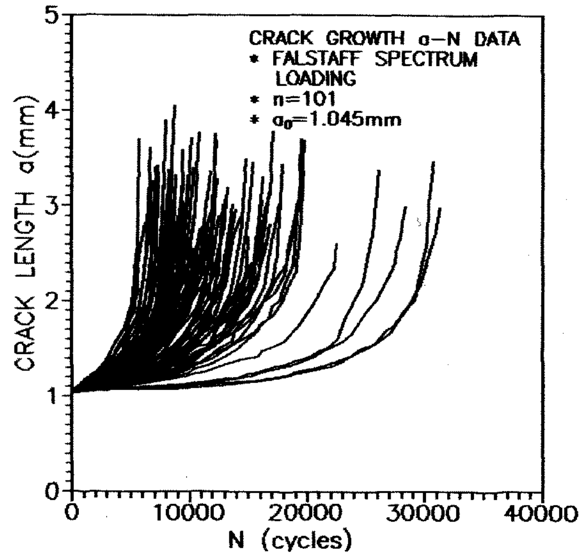

Figure 6 Fatigue crack growth a–N test data

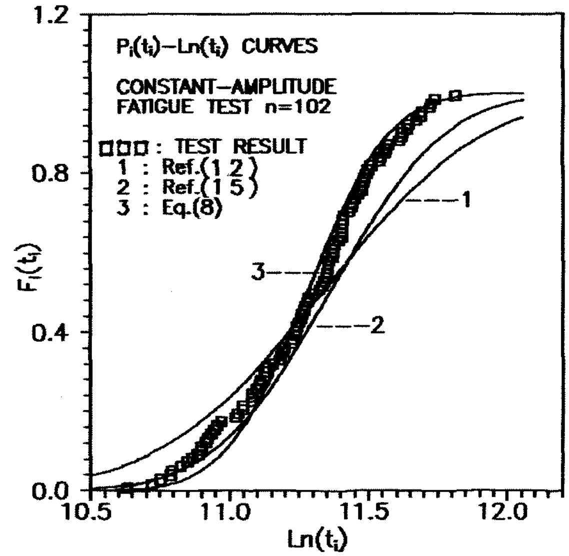

Figure 7 The probability of crack initiation

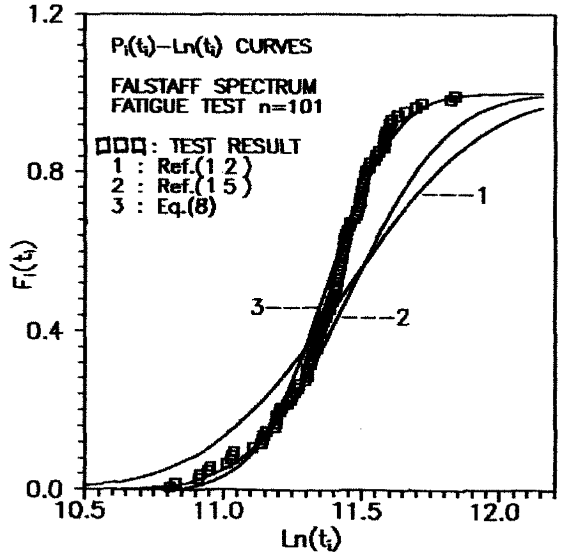

Figure 8 The probability of crack initiation

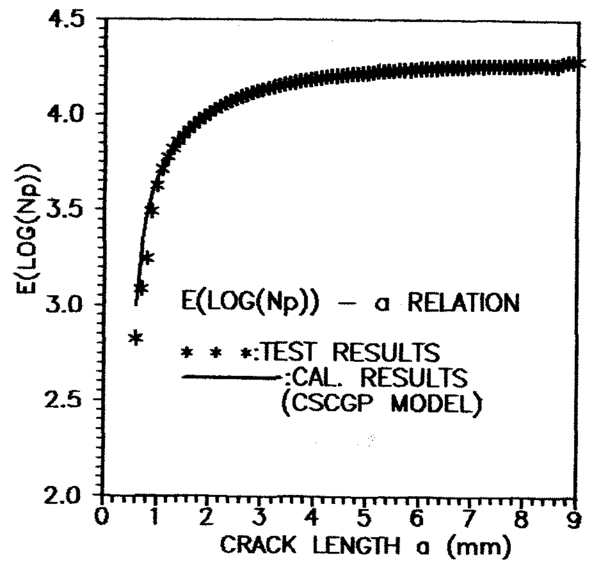

Figure 10 The mean of $\log N(a_0,a)$

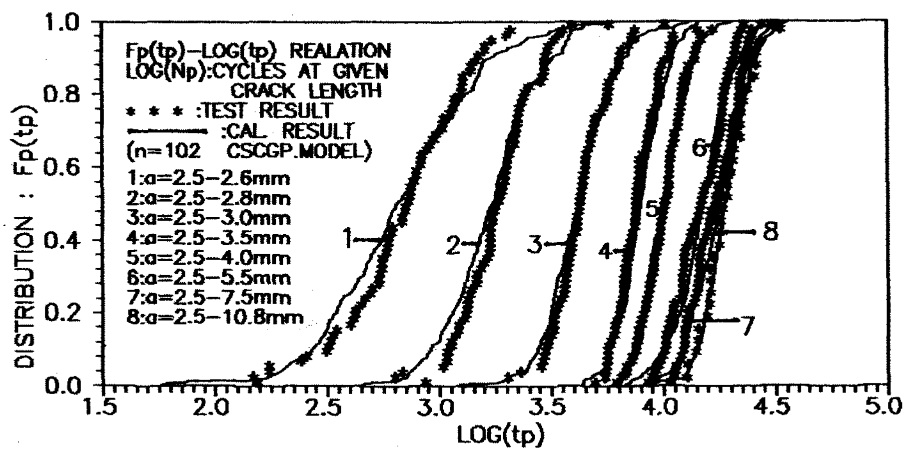

Figure 9 The distribution of crack life

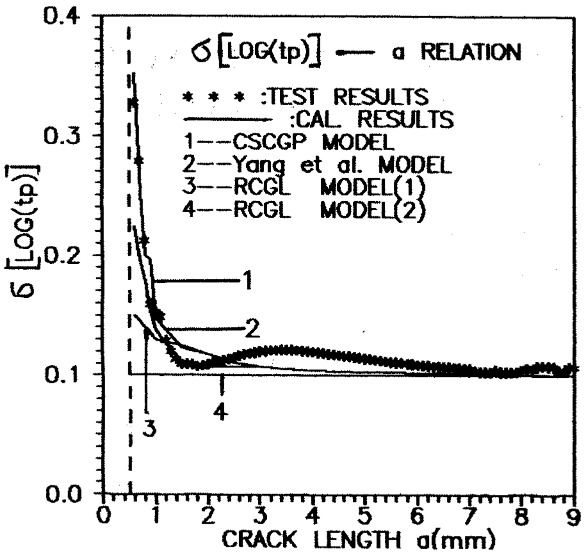

Figure 11 The square root variance of $\log N(a_0, a)$

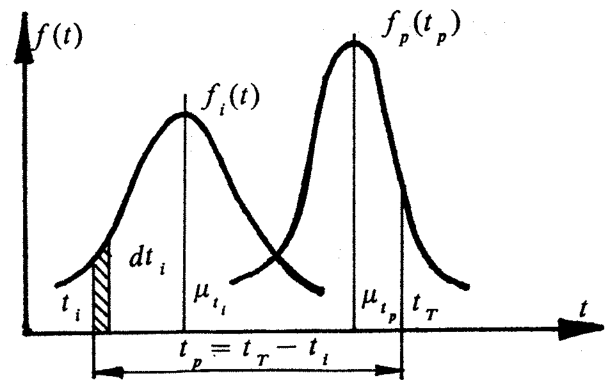

Figure 12 The pdf of total life t_T

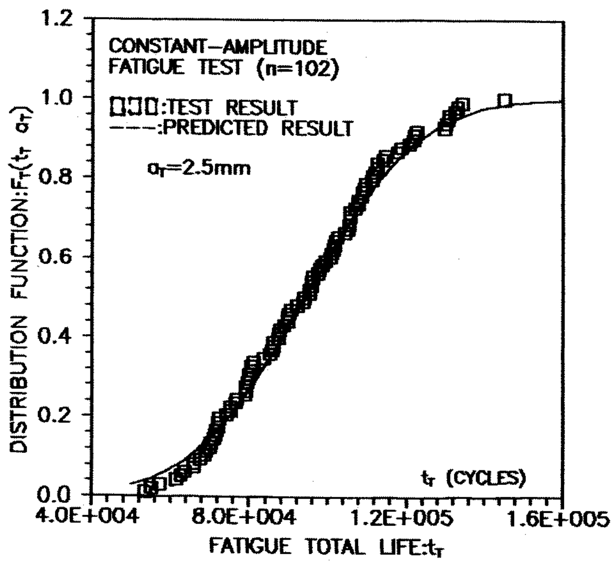

Figure 13 The distribution of total life t_T

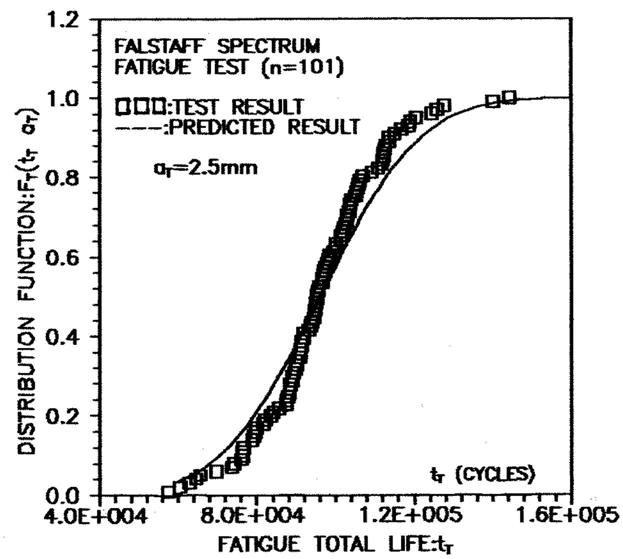

Figure 14 The distribution of total life t_T

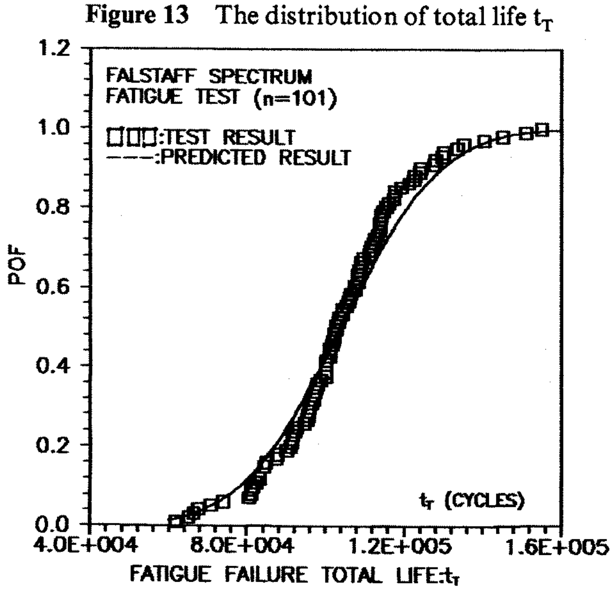

Figure 15 The probability of failure of t_T

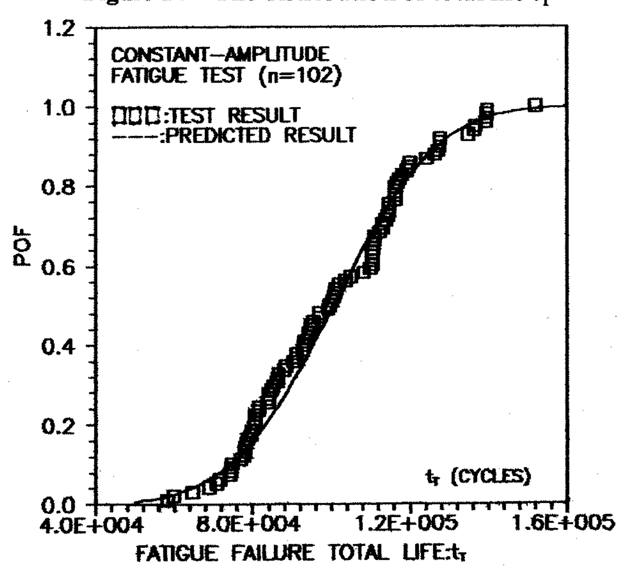

Figure 16 The probability of failure of t_T