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SCATTER FACTORS FOR DAMAGE TOLERANCE JUSTIFICATION

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Abstract

Nowadays, with structural optimization, fatigue dimensioning is used in the design of numerous aircraft parts. A scatter factor is applied to the life of these parts. It is, in fact, a generic factor which is not dependent on the type of the structure and then does not lead to an optimized damage tolerance design.

The aim of this study is to summarize the scatter factors used for damage tolerance justification of civil aircraft.

This article proposes:

- A summing up of factors currently in use,
- A factor determination method,
- A summing up of knowledge on each of the parameters used,
- A validation of current factors.

Introduction

Improving aircraft performance consists in increasing safe strength while reducing weight.

Current use of digital methods such as finite elements provides a good knowledge of structural strength to loads, at least where static is concerned.

However, such methods cannot disperse uncertainty as regards fatigue phenomena. In fact the real use of the aircraft can greatly modify its life. Scatter is also seen to exist on experimental results with apparently identical elements.

This is why we cannot assess the life of aircraft items but only calculate failure probability, which often entails major scatter.

However a certain level of security must be guaranteed, while retaining the longest possible life. This is why a scatter factor must be applied to the life obtained through calculations or tests. The current use of empirical factors has a major effect on aircraft design in terms of weights, but this effect is not always fully controlled.

The purpose of this study is to specify the factors in current use, to propose a general method of determination and to estimate the associated level of security. The reliability which corresponds to currently used factors is then calculated. Lastly, as a conclusion, a practical discussion is held on the use of these factors.

Scatter factor definition

Before defining the scatter factor, a difference must be made between the mean life of a structure and its "safe life" (associated with a certain level of reliability).

Mean life Nc is the mean number of cycles undergone upon failure by a given population.

Safe life Nd is the number of cycles undergone by a very large proportion of the population. This proportion is called reliability. To say that 90% of the population is to reach Nd means that there is to be a 90% reliability for these parts.

Scatter factor k is defined as the ratio between the mean life and the "safe life":

$$k = \frac{Nc}{Nd} .$$

Factors currently in use

Aerospatiale recommends the use of different scatter factors for justification, depending on the type of part. The following must be used:

$3 \leq k \leq 8$ for Safe Life structures
$2 \leq k \leq 5$ for damage-tolerant structures

The factor depends on our knowledge of the structure and loads.

This factor must normally cover scatter in item geometry, material and applied loads.

An over-rigid use of these empirical values poses a problem. They are not dependent on the type of geometry nor on the type of load; Sometimes they are too strict, and at other times not strict enough.

Another problem is to identify the real reliability associated with this factor k.

This study to rationalize the use of these factors was undertaken in an effort to overcome these problems.

Scatter factor determination method

During structure design, numerous causes for scatter can affect structure life.

Scatter arises from part production, mechanical factors and lastly the tests run. They can be classified into two major groups, "qualitative" scatter (which is difficult to estimate) and "quantitative" scatter. Some of these causes are summarized in the following table:

A sample of n elements from a normal distribution population of unknown mean μ and of known variance σ^2 is considered.

The reliability must be α at time Nd, that is to say $100\alpha\%$ of the population have to reach Nd.

To obtain the scatter factor k, we must calculate $\frac{Nc}{Nd}$.

The life logarithm for each part has a distribution $\mathcal{N}(\mu,\sigma)$.

Therefore we have Log(Nc) which has a distribution $\mathcal{N}(\mu,\frac{\sigma}{\sqrt{n}})$,

Log(Nd) which has a distribution $\mathcal{N}(\mu,\sigma)$.

Let us suppose

$Xc = Log(Nc)$ and $Xd = Log(Nd)$.

We must have $P(Xc > Xd) > \alpha$, or

$P(Xc - Xd > 0) > \alpha$.

SCATTER SOURCES	QUALITATIVE PARAMETER	QUANTITATIVE PARAMETER
Production	-Assembly joints mismatch	-Material and manufacturing processes associated
Mechanics	-Environment	-Use of aircraft / Stresses -Geometry
Test data	-Representativeness of test (specimen+simplified loads) -Analyses of test data	-Number of specimens

The assembly joints mismatch leads to a specific analysis, if necessary. As the aircraft are designed for every kind of environment, the one considered is a "medium" environment. Concerning the test data, their scatter is taken into account in the calculation.

We will look at a factor capable of integrating scatter due to quantitative parameter on

- Production,
- Mechanics.

First of all we will examine material/process scatter. Then we will integrate use of aircraft scatter into the model.

Material/process scatter

An initial estimate of $\frac{Nc}{Nd}$ can be made by considering material/process scatter.

(Xc - Xd) has a distribution $\mathcal{N}(0,\sigma\sqrt{\frac{n+1}{n}})$

$\frac{Xc - Xd}{\sigma\sqrt{\frac{n+1}{n}}}$ has a distribution $\mathcal{N}(0,1)$

So we can write $\frac{Xc - Xd}{\sigma\sqrt{\frac{n+1}{n}}} = k_\alpha$, where

$k_\alpha = \Phi^{-1}(\alpha)$

(Φ distribution function of $\mathcal{N}(0,1)$).

$$Xc = Xd + k_\alpha\,\sigma\sqrt{\frac{n+1}{n}}$$

To get back to lives, we obtain:

$$\frac{Nc}{Nd} = 10^{k_\alpha \sigma \sqrt{\frac{n+1}{n}}}$$

where Nc is the calculated life,
Nd safe life,
$k_\alpha = \Phi^{-1}(\alpha)$,
σ material/process standard deviation,
n size of the sample.

<u>Integration of scatter in aircraft use</u>

A second estimate of $\frac{Nc}{Nd}$, which involves scatter in aircraft use in addition to material/process scatter, is possible.

Let's take Nc to be the life of a feature subjected to the testing load spectrum, and only including material/process scatter. We assume that log(Nc) has distribution $\mathcal{N}(m, \sigma_1)$ where $\sigma_1 = \frac{\sigma}{\sqrt{n}}$ (n is the number of tests).

Let's take Nu to be the life an in-service aircraft would have if the fatigue phenomenon were only due to scatter on aircraft use. We assume that log(P) has distribution $\mathcal{N}(m', \sigma_2)$.

Let Nd be the real life of an aircraft, i.e. the number of flight hours the aircraft would perform if it were left to fly until failure. We assume that log(Nd) has distribution $\mathcal{N}(\mu, \Sigma)$.

We can therefore see that Nd is the life of the aircraft in the presence of two causes of scatter. We can assume that the distribution of Nd is the result of the superimposing of the random character of the material and the use of the in-service aircraft. We can deduce the value of Σ from this, as both these characters are separate:

$\Sigma^2 = \sigma^2 + \sigma_2^2$.

log(Nd) has a gaussian distribution $\mathcal{N}(\mu, \sqrt{\sigma^2 + \sigma_2^2})$, and log(Nc) $\mathcal{N}(\mu, \sigma_1)$, since μ=m when σ_2=0.

It is necessary, as in the previous case, that $P(Xc > Xd) > \alpha$, or else $P(Xc - Xd > 0) > \alpha$.

(Xc - Xd) has a gaussian distribution

$$\mathcal{N}\left(0, \sqrt{\frac{n+1}{n}\sigma^2 + \sigma_2^2}\right)$$

$$\frac{Xc - Xd}{\sqrt{\frac{n+1}{n}\sigma^2 + \sigma_2^2}}$$ has a gaussian distribution $\mathcal{N}(0,1)$

We can then write

$$\frac{Xc - Xd}{\sqrt{\frac{n+1}{n}\sigma^2 + \sigma_2^2}} = k_\alpha \text{, where}$$

$$k_\alpha = \Phi^{-1}(\alpha)$$

$$Xc = Xd + k_\alpha \sqrt{\frac{n+1}{n}\sigma^2 + \sigma_2^2}$$

Getting back to lives, we obtain:

$$\frac{Nc}{Nd} = 10^{k_\alpha \sqrt{\frac{n+1}{n}\sigma^2 + \sigma_2^2}}$$

where Nc is calculated life,
Nd safe life,
$k_\alpha = \Phi^{-1}(\alpha)$,
σ standard deviation in materials/processes,
σ_2 standard deviation due to in service use,
n size of the sample.

It can be noted that reliability depends on the number of tests. The scatter of the material does, in fact, depend on n. The more tests there are, the lower the scatter, which can be easily understood. In the case of a result obtained through calculation and supported by tests (on coupon, specimen and full scale), we can consider that the result obtained is reliable (since both the theory and the practice correspond). Moreover, the values used for the calculation are 95% confidence based. We assume there is an equivalence between this result and a result which would have been obtained after a large number of tests. We then have $\frac{n+1}{n}$ which tends to 1. To make writing this easier, we shall assume, after calculation supported by the results of a few tests, that the scatter factor equals:

$$\frac{Nc}{Nd} = 10^{k\alpha\sqrt{\sigma^2 + \sigma_2{}^2}} .$$

Study of integrated parameters

Calculating the scatter factor requires knowledge of the scatter in the materials/processes used and in the aircraft use .

Scatter concerning the material used

Numerous values are proposed in the documentation [1,2,3,4], and these are more or less consistent with each other. Benoy[1], for example, proposes the following values:

Material	Scatter factor
Aluminium	0.14
Steel <200 ksi	0.17
Titanium	0.19
Steel >200 ksi	0.25

Standard deviation

Dehaye[4], from an Aeronautical Tests Center, proposes values varying from 0.1 to 0.15.

A study conducted at Aerospatiale underlined some sources of inconsistency in determining material/process scatter. The size of the sample is not, in fact, a sufficient parameter to improve scatter accuracy.

Figure 1 clearly shows how important it is to choose a population with homogenous details.

If we consider all types of tests on aluminium alloys, (890 points), scatter at 10^5 cycles is 0.21.

If we consider a single Kt, but with different manufacturing processes, (600 points), scatter drops to 0.20.

With a homogenous population, i.e. with a given geometry and manufacturing process, (49 points), scatter is 0.15. This case is of course nearer to a real situation where evaluations are made for each given detail.

Moreover, scatter is a function of life. In particular, less scatter is expected for short lives.

The following practical conclusions can be drawn from these results:

- For a target life of 100,000 cycles, scatter relating to a given material/process is approximately 0.15.
- For shorter life (10,000 cycles or premature fatigue damage), scatter is lower at approximately 0.09.

Scatter due to in-service use

This scatter is mainly due to scatter in characteristic weights (Take Off Weight, Zero Fuel Weight, etc.) of the aircraft. Using aircraft take-off weight (data supplied by the operators), the differences in part life can be calculated, with respect to the weights considered during tests. These differences are used to find out scatter in loads in terms of life.

A study was conducted at Aerospatiale on different Airbus structural points on 300 aircraft of the same type in order to find out scatter due to in-service use, in terms of life. It is possible to obtain life variability factors using variations in the Zero Fuel Weight and in the Fuel At Landing.

This study indicates that the standard deviation due to in-service use lies between 0.048 and 0.084, depending on the structural points. These values are far lower than the 0.12 given by Benoy [1]. This value is perhaps quite different from ours because it takes into account the gust variability.

If we consider that the standard deviation due to aircraft use is 0.09, we can plot reliability charts for aluminium alloys (figures 2 and 3). We consequently directly obtain the factor to be applied depending on the reliability required for the structure.

Validation of currently applied factors

In this part an attempt is made to evaluate structure reliability according to the factors applied.

We have seen that the equation expressing the scatter factor with respect to the required reliability is expressed by:

$$k = 10^{k\alpha\sqrt{\frac{n+1}{n}\sigma^2 + \sigma_2{}^2}}$$

We obtain reliability by estimating $\Phi(k_\alpha)$, where $k_\alpha = \dfrac{\log(k)}{\sqrt{\frac{n+1}{n}\sigma^2 + \sigma_2^2}}$

Safe Life factor

We consider the case of an aluminium part, for which scatter due to the material/process is 0.14.

As far as scatter due to in-service use is concerned, we shall examine the case where loads are fully known (pressurization for example) and the case where loads are more difficult to apprehend. In the first case, $\sigma_2 = 0$, and in the second $\sigma_2 = 0.09$.

It should not be forgotten that the justification factor used for Safe Life parts must lie between 3 and 8. We thus obtain the following failure probability tables depending on the number of specimens:

	$\sigma_2 = 0$	$\sigma_2 = 0.09$
k = 3	0.846%	1.611%
k = 5	0.021%	0.066%
k = 8	0.0002%	0.0016%

No. of specimens =1

	$\sigma_2 = 0$	$\sigma_2 = 0.09$
k = 3	0.274%	0.721%
k = 5	0.0023%	0.094%
k = 8	$6.9\ 10^{-6}$%	0.0001%

No. of specimens = 2

	$\sigma_2 = 0$	$\sigma_2 = 0.09$
k = 3	0.058%	0.284%
k = 5	$9.6\ 10^{-5}$%	0.0024%
k = 8	$3.8\ 10^{-8}$%	$7.8\ 10^{-6}$%

No. of specimens =10

	$\sigma_2 = 0$	$\sigma_2 = 0.09$
k = 3	0.032%	0.209%
k = 5	$2.9\ 10^{-5}$%	0.0013%
k = 8	$5.6\ 10^{-9}$%	$2.8\ 10^{-6}$%

calculation supported by tests

These tables and figure 4 indicate that quite quickly the increase in the scatter factor induces a very high level of reliability.

Damage tolerance factor

The factor used for damage-tolerant part justification must be between 2 and 5 .

The logic used to calculate the reliability associated with this factor is slightly different from that used for Safe Life parts.

Damage tolerant parts are inspected during their fatigue life, in order to verify their structural integrity. We will assume that every part is inspected.

The probability of failure must be the same, whatever the part is (Safe Life or damage tolerant part). But this probability does not increase in the same way for each kind of part. Indeed, it increases slowlier when there are inspections.

Let us consider a ratio R between the probability of failure with or without inspections, at Nd.

Since damage tolerant parts have a probability of failure R times lower than Safe Life ones, the scatter factor for damage tolerant parts can be evaluated as for Safe Life parts, but considering a probability of failure R times higher at Nd (this margin between the probabilities being set off by the inspections).

with R=100, the following results are then obtained:

	$\sigma_2 = 0$	$\sigma_2 = 0.09$
k = 2	0.29%	0.769%
k = 3	0.008%	0.016%
k = 4	0.001%	0.003%
k = 5	0.0002%	0.0006%

No. of specimens =1

	$\sigma_2 = 0$	$\sigma_2 = 0.09$
k = 2	0.079%	0.239%
k = 3	0.0027	0.007%
k = 4	0.0002%	0.0009%
k = 5	$2.3\ 10^{-5}$%	0.0001%

No. of specimens =2

	$\sigma_2 = 0$	$\sigma_2 = 0.09$
k= 2	0.025%	0.083%
k= 3	0.0006%	0.0028%
k= 4	$2.1\ 10^{-5}$%	0.0002%
k= 5	$9.7\ 10^{-7}$%	$2.5\ 10^{-5}$%

No. of specimens =10

	$\sigma_2 = 0$	$\sigma_2 = 0.09$
k= 2	0.0184%	0.0628%
k= 3	0.0003%	0.0021%
k= 4	$8.5\ 10^{-6}$%	0.0001%
k= 5	$2.9\ 10^{-7}$%	$1.3\ 10^{-5}$%

calculation supported by tests

Figure 5 reveals once more that reliability very quickly reaches a high level .

Scatter in propagation

The study conducted here takes the structure fatigue aspect into account. We could do the same for the crack propagation aspect. Very little data is available on propagation life variability. We can however consider a standard deviation of between 0.07 and 0.15 for aluminium alloys[5,6]. This scatter is approximately the same than that due to the phenomenon of fatigue. The scatter factors calculated above can be used to obtain roughly the same reliability.

Conclusion

This study indicates how to calculate scatter factors that take into account not only scatter on material but also scatter on aircraft use.

We have shown that the factors currently used provide a very high level of reliability. However, it must be noted that there are significant variations depending on scatter, as regards both the loads and the material used.

From the previous paragraphs, useful information can be highlighted for practical use.

- In the case of structural details whose manufacturing processes are very similar and verified by an adaquate quality control, and where a structural analysis is available, a scatter of 3 is suffisant to ensure 99,96% reliability and 95% confidence for safe life components.

- For early in-service findings, the life being small, the scatter is also small. The application of design scatter factors, based on higher lives, is not adequat. Reduced scatter factors might be used.

- In general, few full scale components are tested. By themselves, they are not aimed at statistically justifying a value. It is the combination of a calculation (taking into account the vast amount of experience) and of a few tests that ensures the confidence in the statistical analysis.

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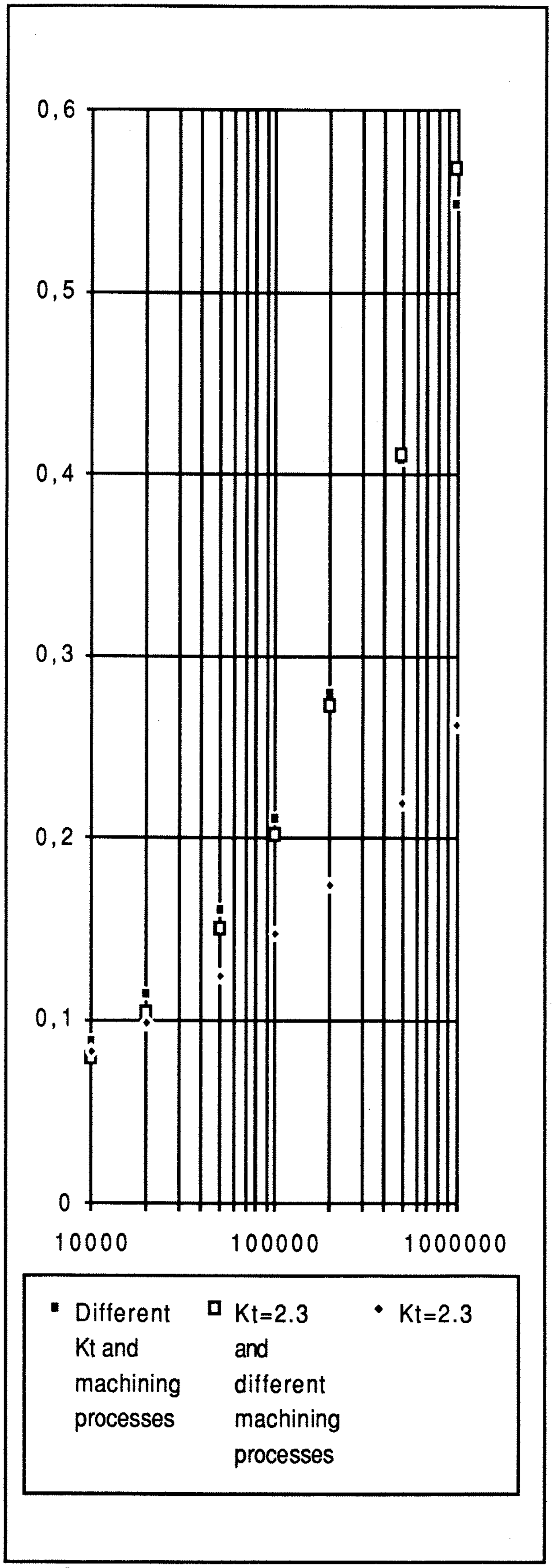

FIGURE 1 - Scatter in Aluminium Alloys

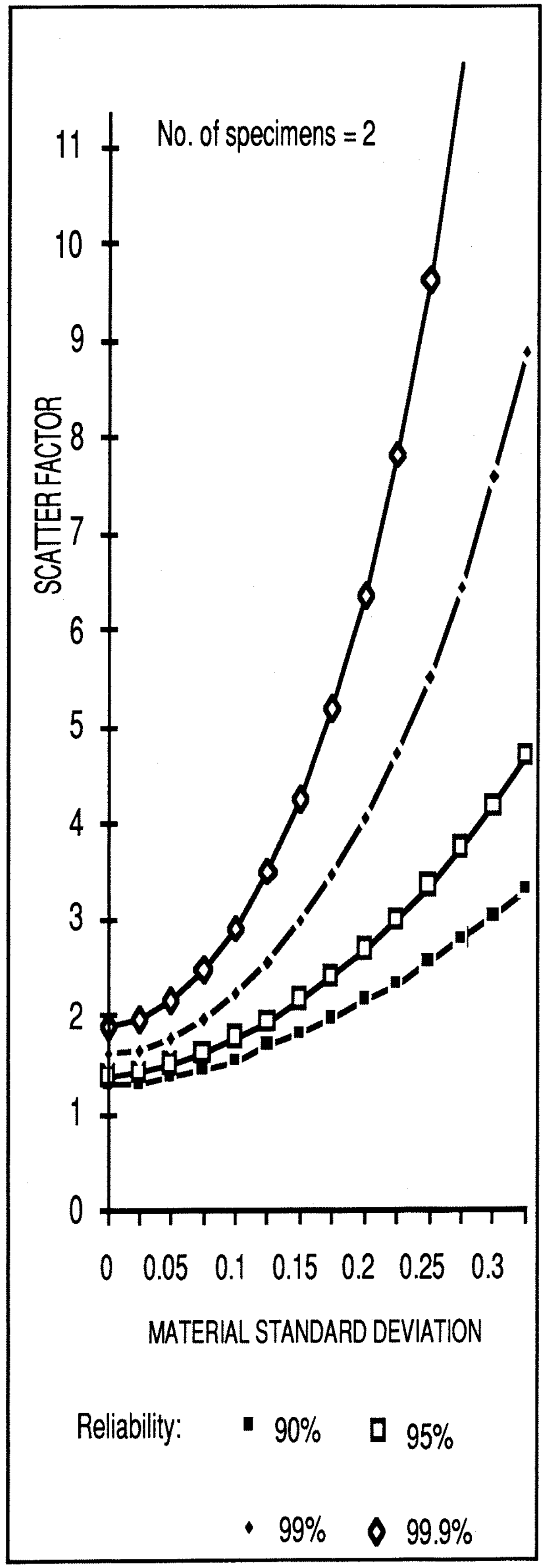

FIGURE 2 - Reliability Chart

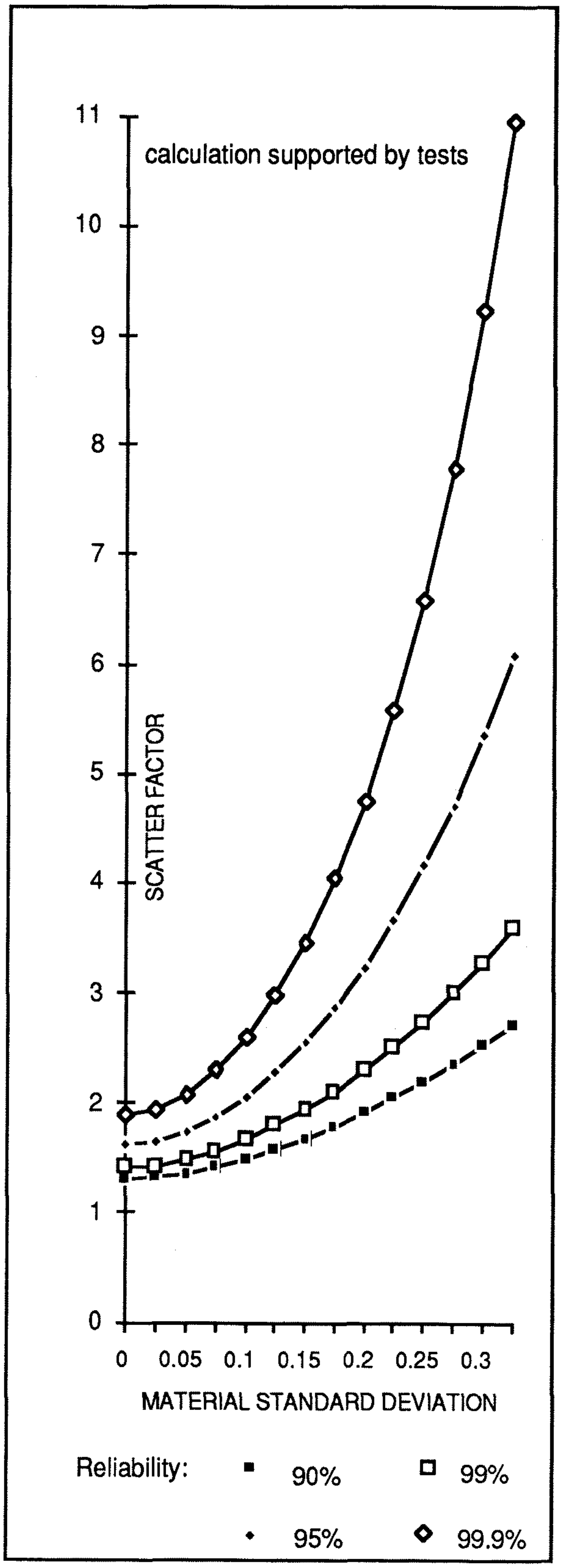

FIGURE 3 - Reliability Chart

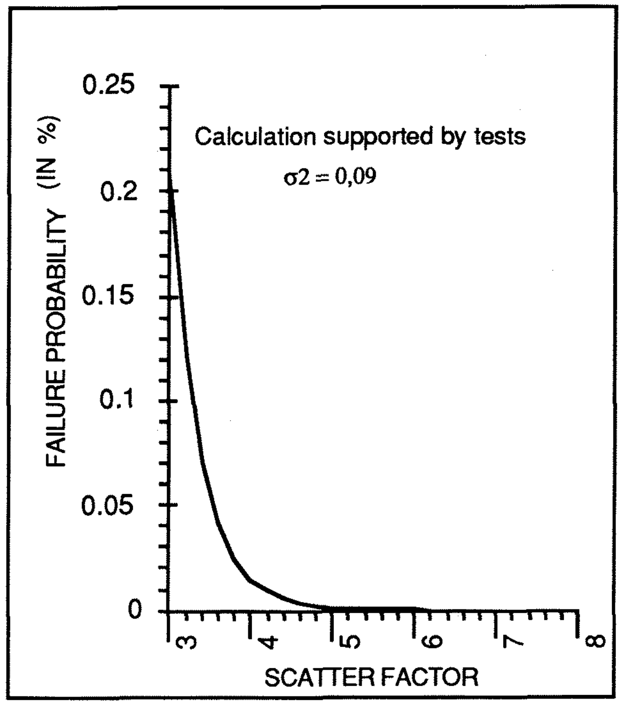

FIGURE 4 - Reliability vs Factor
(Safe Life Part)

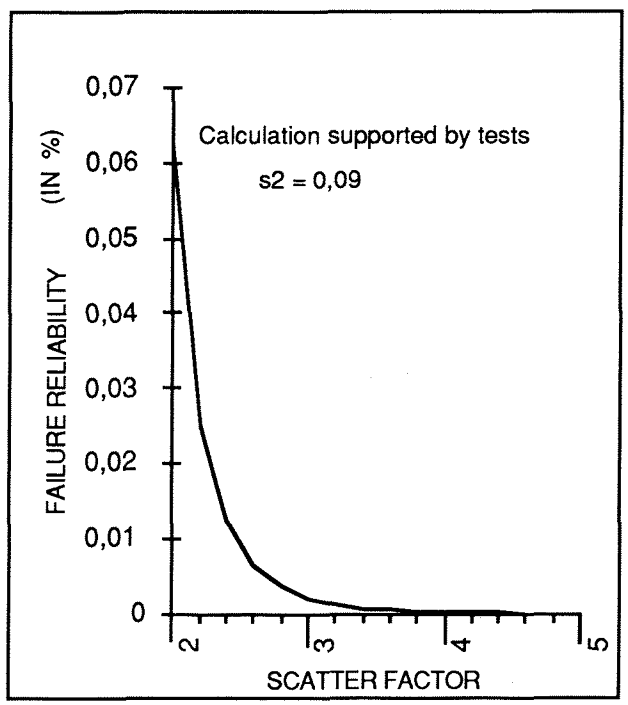

FIGURE 5 - Reliability vs Factor
(Damage tolerant Part)