THE NEW COMMON MATHEMATICAL MODEL FOR EXPRESSING FATIGUE CHARACTERISTICS OF THE TYPICAL AIRFRAME STRUCTURE ELEMENTS.

Zdeněk MALÉŘ, PhD., Moravan Inc.
765 81 Otrokovice, Czech Republic

Abstract

In this paper a new common mathematical model is proposed.This is able to describe fatigue characteristics in the whole necessary range by one equation only:

$\log N = A(R) + B(R) \cdot \log Sa$

where

$A(R) = A.R^2 + B.R + C$ and $B(R) = D.R^2 + E.R + F$

This model was verified by five sets of fatigue data taken from literature and by three additional author s original fatigue test sets.The fatigue data usually described the reason of $N=10^4$to $3.10^6$and stress ratio of R=-2 to 0.5.In all this cases the proposed model described fatigue results with small scatter. Studying this model following knowledge was obtained:
parameter R is a good physical characteristic; proposed model was able to describe very well the eight collections of fatigue test results by one equation only; the scatter of test results through the whole scope is only a little greater than that round the individual S/N curve; using this model while testing may reduce number of test samples and shorten test time; as the proposed model represents a common form of the S/N curve it may be used to the uniform objective fatigue life results processing which may enables mutual comparison of fatigue characteristics.

Introduction.

The present-used regulations for aircraft construction lay great emphasis upon structure fatigue life proof - see e.g. FAR 23.572. For this reason designer needs reliable fatigue characteristics that describe just his structure element. Renowned manufacturers use well tested structure elements for which they have their own fatigue characteristics and they also have a lot of operating experience with them. However, fatigue characteristics of new structure elements that are life important, must be obtained from literature or through tests. Fatigue tests are money and time consuming. When suggesting the tests attention must be paid to whether the structure element is used only once or whether its utilisation in next new model is expected. It is also judged whether the structure element will be loaded within narrow or wide loads ranges. E.g. wing structure elements of transport airplanes are usually loaded only within a narrow range of mean loads while the same elements of acrobatic aircraft are loaded within a wide range of amplitude and mean stress values.
These considerations decide on test range. From these it follows, whether a test with a single result is suitable or whether to perform a test the results of which will be of wider and more general application, having possibly a shape of Haigh diagram in its final form - see Fig.3.

As an example, I will mention here our company approach to solving the problem of fatigue life of sporting aircraft.
Moravan Inc. has been manufacturing sporting aircraft continuously since 1934. In the post-war period our company manufactured school and training aircraft, series ZLIN 126 through 526F (famous also as "TRAINER"), school and training aircraft ZLIN 42 through 242L, touring aircraft ZLIN 43 and 143L, acrobatic aircraft ZLIN 50L and agricultural aircraft ZLIN 37 (AGRO) and ZLIN 137T (AGRO TURBO). School aircraft were designed in acrobatic category and some of them were used very often as acrobatic competition.
It is typical for all the aircraft that the wing is single spar with auxiliary rear spar, riveted, made of aluminium alloys of 2024 type. Fuselage is welded, lattice-framework made of steel Cr-Mo tubes.
As during the safe life proof there were few reliable data on fatigue characteristics of the welded tube structure it was decided to carry out necessary fatigue tests. Fatigue tests of lattice welded structure were suggested and completed by its simple model. During these tests both stress amplitude and stress ratio R influences were studied. Further, our company paid attention to cables of small diameter (used to control the rudder), to models of general critical point of wing spar flange made of Al-alloy and to model of critical point of steel attachment. Preparation of the above mentioned fatigue tests led us to considerations about the final character of results, and then back to the test suggestion and the way of results processing. All this resulted in suggestion of a new mathematical model that is described further.

The Present Way of Fatigue Characteristics Expression

Sa amplitude and mean stress Sm of load cycle are the decisive and at the same time also geometrically transparent factors influencing structure elements life. Influence of these quantities upon life expressed by number of cycles until failure N is often represented in graphical way of the system of S/N curves(1) (see Fig.1) or by means of Haigh diagram(2) (see Fig.3).

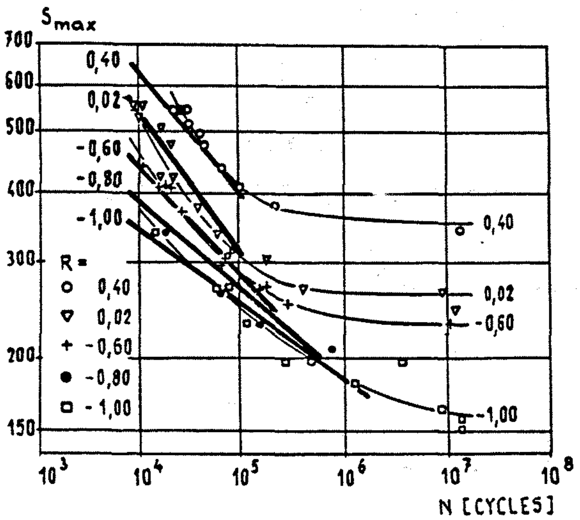

Fig.1. Fatigue data for axially loaded 7075-T6 Al-alloy samples at various stress ratios R - see [1]

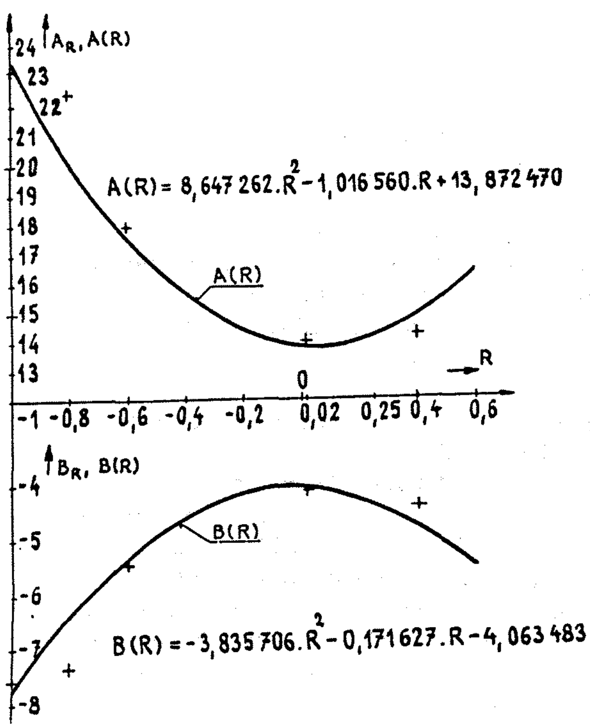

Fig.2. Comparison of AR and BR to functions A(R) and B(R)

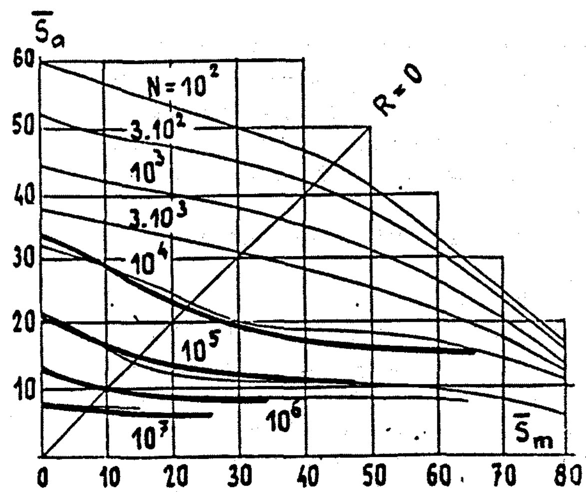

Fig.3. Haigh diagram for "Mustang" wing - see [2]

The advantage of graphical representation consists in its top transparency. At computer introduction the graphical representation of fatigue characteristics was not satisfactory any more and mathematical ways of expression were searched. An expression that was used to describe fatigue characteristics of wing and horizontal tail surfaces (3,4) belongs to one of the first models

$$\log N = a(Sm) + b(Sm) . \log (Sa - 0.835) \quad (1)$$

where a(Sm) and b(Sm) represent positions and slopes of S/N curve left branches. Parameters a(Sm) and b(Sm) are functionally dependent on mean stress value and they are described in four segments by means of polynoms of the first through the third degree (see Fig.4).

$$\log N = a - b(\log Sa - 0.838)$$

INTERVAL STŘEDNÍHO NAPĚTÍ S_{m_i} [MPa]	ÚSEK NA OSE LOG N_i $a_i = f_1(S_m)$	SMĚRNICE (SKLON) $b_i = f_2(S_m)$
$0 \leq S_{m_1} < 14$	$a_1 = 10{,}0556 - 0{,}07438 . S_{m_1}$	$b_1 = 4{,}8023 - 0{,}0638 . S_{m_1}$
$14 \leq S_{m_2} < 103$	$a_2 = 9{,}4598 - 0{,}0345 . S_{m_2} + 2{,}5.10^{-4} . S_{m_2}^2 - 8.10^{-7} . S_{m_2}^3$	$b_2 = 3{,}96687 - 0{,}00310 . S_{m_2} - 1.10^{-8} . S_{m_2}^2 + 2.10^{-8} . S_{m_2}^3$
$103 \leq S_{m_3} < 140$	$a_3 = 8{,}5278 - 0{,}0080 . S_{m_3}$	$b_3 = 3{,}9947 - 0{,}00419 . S_{m_3}$
$140 \leq S_{m_4} \leq 210$	$a_4 = 8{,}3050 - 0{,}00639 . S_{m_4}$	$b_4 = 3{,}9237 - 0{,}00368 . S_{m_4}$

Fig.4. Mathematical description of fatigue characteristics of wing and horizontal tail surfaces - see [3,4].

This way of expression is very accurate and it enables computer usage. Breadth of segments and mathematical functions are chosen so that to replace the results of fatigue tests as accurately as possible. However, selection of the segments width and suitable polynoms lead to the loss of the general character. The system of polynoms describes accurately the geometrical form of results but this description has no physical sense. The system does not describe physical relations among Sa, Sm and N.

After having supplied the existing results with another group of results or at processing another set of test results it is necessary to choose new segments width and a new system of polynoms to reach an accurate presentation of test results. Sections width and polynoms are chosen by way of trial.
Another mathematical model is represented by the relationship

$$Sa = A(N) + b(N) . Sm \quad (2)$$

here a(N) and b(N) are parameters of straight lines of N = const. in Haigh diagram. The application of this model is not so wide as that of the previous model because in Haigh diagram it is usually not possible to use straight line for presentation of constant live curves. For this reason here it is also necessary to express the dependency among SA-Sm-N by means of segments and to choose suitable polynoms. For the above reasons neither of the mentioned models is generally valid and cannot be used at fatigue tests design or during fatigue tests to control them.
Requirements laid upon the new mathematical model can be expressed as follows. The model must be able:
1. to describe results of any fatigue tests trough one mathematical function within their whole range
2. the function should be always the same for any tested structure or element loaded with arbitrary, accurately defined load, only its parameters should be variable
3. the function should be valid within a certain physical area, e.g. within high-cycle area
4. change of equation parameters must depend only upon test results. If the test results are homogenous and there is no deflection from physical limits, equation parameters cannot change considerably at test range extension (at increasing the number of test samples).

Design of New Mathematical Model

The load cycle can be described in different ways, e.g. by means of pairs (Sa,Sm), (Sa,R), (Smax,Smin), (2.Sa,Sm) etc.- see Fig.5.

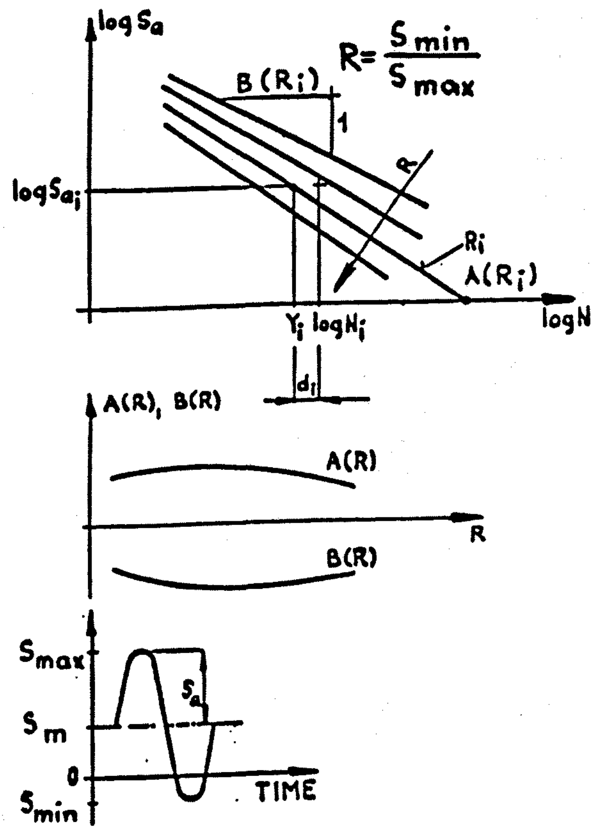

Fig.5. Graphical demonstration of the new mathematical model (3) Sa-R-N dependence

At model design we come out from the following knowledge. It was found out, see e.g.[6], that at description of the mean stress influence, or better to say, at description of load cycle position influence upon crack growth the more suitable parameter is the stress ratio R than the mean stress Sm. For this reason parameter R was used instead of Sm at new model design. Relation among SA, R, N is chosen in the shape

$$Y = A(R) + B(R) . \log Sa \quad (3a)$$

This function describes the system of S/N curve branches that correspond to different values of R = const. in area of high-cycles fatigue, see Fig 5. Position and slope parameters of S/N curve branches, in co-ordinate system logSa - logN, depend on R value. Dependencies between A(R) = f(R) and B(R) = f(R) are expressed by means of following functions:

$$A(R) = A.R^2+B.R+C \quad (3b)$$

$$B(R) = D.R^2+E.R+F \quad (3c)$$

Relations for A,B,C,...,F parameters calculation are determined through the method of smallest squares of deviations di between results Ni and the model function Yi:

$$q = \sum_{i=1}^{n} di^2 = \sum_{i=1}^{n} (\log Ni - Yi)^2 \text{--- MINIMUM} \quad (4)$$

$$\frac{\delta q}{\delta A} = 0, \frac{\delta q}{\delta B} = 0, \frac{\delta q}{\delta C} = 0, \ldots, \frac{\delta q}{\delta F} = 0 \quad (5)$$

As a result we obtain a system of six normal equations. They enable to establish six parameters A,B,C,..,F of model function Sa-R-N.
As input values for calculation of model function parameters serve n triples of values (Sai,Ri,Ni).
Determination of A,B,...,F parameters is easy and so it makes no difficulty to process the fatigue tests results gradually during testing and thus to control this test gradually in its course. It means to alter the test plan on the base of the obtained results and thus to reach reduction of the number of test samples and to shorten the time of testing.
To be able to describe fatigue characteristics of the tested part by means of the model function, it is necessary to carry out fatigue tests at least at three stress ratios of R, while at each R it is necessary to perform the test at least at two Sa levels. It means that for complex evaluation of Sa-R-N diagram it is necessary to obtain S/N curve for three value of R = const. On this base it is possible to complete Sa-R-N diagram by doing other tests only in regions with information shortage.
At the test of Al-alloy elements at which theS/N curve has two straight line sections of different slopes, each part of the curve must be solved separately.
For describing the test results scatter round the model function (3) the following expression is used

$$s_F = \frac{1}{n-6} \sum_{i=1}^{n} (\log Ni - Yi)^2 \quad (6)$$

where n is the number of test results described by n-triples (Sa,R,N), Yi is theoretical number of load cycles given by model function (3) - Yi value corresponds to probability of failure of p= 50%.

Examples of judgement of validity and accuracy of Sa-R-N model

Suggested model (3) Sa-R-N was judged on five sets of fatigue results taken from literature and were used at processing of three own sets of test results. Validity and accuracy were judged by: - comparing of origin and Sa-R-N diagrams
- comparing of S/N curve parameters (ARi,BRi) and model functions A(R),B(R)
- value of standard deviation s_F.

Example 1.

In the Fig.1 there are shown S/N curves (thin curves), including test results and Sa-R-N model curves (thick lines) describing fatigue data for axially loaded 7075-T6 Al-alloy specimens that were tested at various stress ratio R. It is seen the new model fits the test results well. Comparison of the partially S/N curve parameters (ARi,BRi) and model parameters A(R),B(R) fit well, too -see Fig.2. Value of standard deviation is $s_F = 0.186$.

Example 2.

In the Fig.3 there are compared original curves (thin curves) and curves established by new model (3) (thick curves). Model curves were calculated in the range of N = 10^4 to 10^7cycles. As a test results were taken points which were read from the original N = const curves for R = = -1, -0.8, -0.6,..., 0.6. The S_F value represents the difference between new model and original curves. Value of standard deviation is $s_F = 0.107$.

Example 3.

Test results of the tube welded structure model were processed in the form of Sa-R-N diagram. Test pieces -see Fig.6- were made of Cr-Mo steel, Czech Standard 15130 that is equal to U.S.A. steel AISI 430 (SAE 4130). Test pieces were heat treated to the strength of Rm = 680 to 780 MPa.

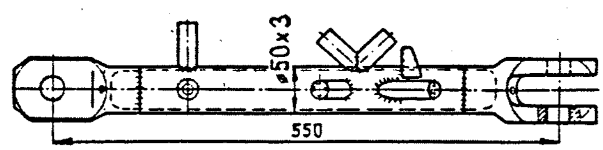

Fig.6. Schema of the tube welded structure model

Test specimens were axially loaded at various R = -2, -1.5, -1, -0.5, 0, 0.23, 0.42. Obtained equation describing model Sa-R-N is

$$\log N = A(R) + B(R) . \log Sa$$
$$A(R) = -0.7805.R^2 - 2.8118.R + 17.59264$$
$$B(R) = +0.2884.R^2 + 0.8923.R - 5.77036$$

Equation is valid in the range of R = -2 through 0.42 and N = 10^4 to 10^6. Value of standard deviation is $s_F = 0.192$.

Example 4.

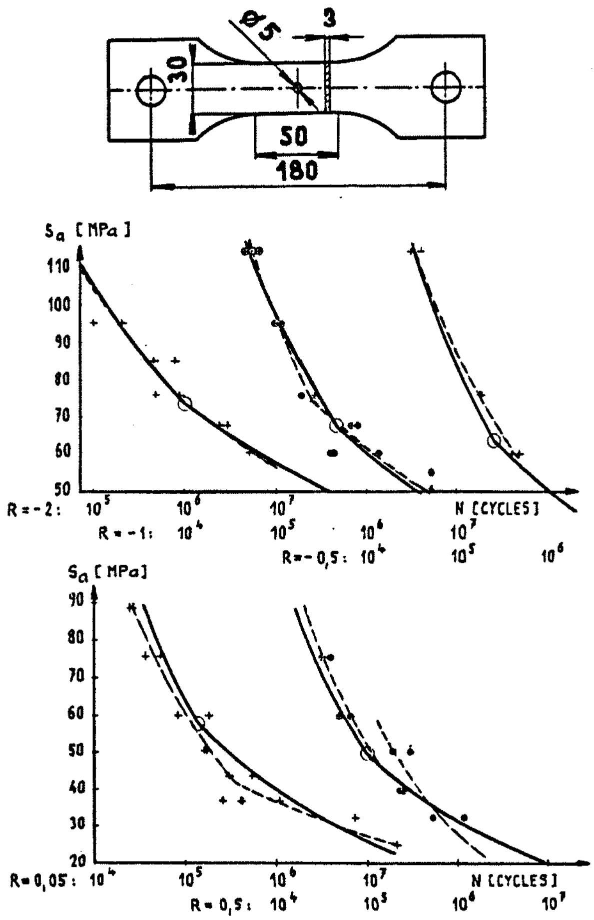

Fig. 7. Test results, SIN curve for R = const. (dashed line) and Sa-R-N model curves for Al-alloy samples.

In the figure 7 are shown test results of the axially loaded test specimen. The specimen are made of the Czech Al-alloy Czech Standard 424203 that is equal to U.S.A. Al-alloy 2024. Tensile strength is Rm = 410 to 440 MPa. Test results had to be divided into two groups.
General equation describing both group is

$\log N = A(R) + B(R) . \log Sa$

Upper group of test results, valid for Sa Sa_{BREAK}, is described by $A(R) = +1.0884.R^2 - 1.3899.R + 10.77131$

$B(R) = - 0.6247.R^2 + 0.3544.R - 3.17756$

Bottom group of test results, valid for $Sa < Sa_{BREAK}$, is described by $A(R) = +0.7919.R^2 - 2.9015.R - 14.67800$

$B(R) = - 0.4742.R^2 + 1.0906.R - 5.39006$

The value of Sa_{BREAK} that describes points where the S/N changes their slopes is described by

$Sa_{BREAK} = -2.6400.R^2 - 12.9937.R + 57.69910.$

Equation is valid in the range of R = -2 to 0.5 and N = 2.10^4 to 10^7.

Partial conclusion

In all eight checked fatigue sets the model function Sa-R-N (3) described tests result well.

Conclusion

The following conclusions can be made from comparing the original characteristics and characteristics obtained at using the designed mathematical model (3):
1. The designed model function (3) was able to describe, with sufficient accuracy and reliability, results of five different structures and elements within the range of $N=10^4$ through 10^6 and R=-1 through 0.6, processed before in some way (graphical or mathematical).
2. At processing the original results of fatigue tests, the S/N curves determined through model function (3), e.g.Sa-R-N were nearly identical with partial S/N curves calculated for different values of R = const. The range of test results calculated from the whole set was not much wider than that around individual S/N curves.
3. Parameter R (together with Sa) proved to be a suitable physical parameter for load cycle description.

From this follows further knowledge:

4. The designed model Sa-R-N can be considered as a suitable, generally valid mathematical description of fatigue characteristics within the high-cycle fatigue area and it is applicable to description of large elements and details, too.
5. For this reason the designed model (3) enables economical design of fatigue tests to obtain dependence Sa-R-N because at gradual processing of fatigue tests results by means of this model it is possible, after the basic information have been obtained, to supply information in unclear areas only. This enables considerable reduction of the number of samples and of the testing time.
6. As the designed model is only a generalised form of a commonly used S/N curve, it is a base of a uniform, comparable, objective and accurate processing of fatigue tests results.
7. Instead of Sa, Smax can be used ,too.

Literature

Fatigue in Aircraft Structures, Proceedings of the International Conference - Columbia University, 1956:
(1) Schleicher, R.L.: Practical Aspects of Fatigue in Aircraft Structures - Fig. on page 379.
(2) Johnstone, W.W.- Payne, A.O.: Aircraft Structural Fatigue Research in Australia - Fig. on page 436.

(3) Engineering Science Data Unit, E.02.01.: Endurance for Complete Wing and Tailplanes. Royal Aeronautical Society, 1958.

(4) Hangartner, R.: Correlation of Fatigue Data for Aluminium Aircraft Wing and Tail Structures. National Aeronautical Establishment, Ottawa, 1974, Aeronautical Report LR-582.

(5) Maléř, Z.: Basis for Calculation Estimate of Safe Life of Small Airplane Primary Structure. Doctor thesis, Military Academy, 1990, Brno, Czech Republic.

(6) Hudson, C.M.: Effect of Stress Ratio on Fatigue Crack Growth. NASA Res. Center, Bethlehem, 1967.

(7) Maléř, Z.: Fatigue Characteristics of the Airplane Critical Point Made of Al-alloy. Moravan Inc. internal report, Structure department No.2117,1995.