A THEORETICAL APPROACH FOR THE DAMAGE BEHAVIOUR CHARACTERISATION OF COMPOSITE SHELLS

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Abstract

In the present paper a theoretical mechanical elastoplastic model have been developed, able to describe composite shells damage behaviour under multiaxial quasi-static loads, by introducing damage and elastoplastic parameters into layer constitutive equations. Particularly, a damage tensor has been introduced, able to simulate damage into the elastic field and to describe the new stress distribution into layers due to damage. Through this, it results possible to determine the active stresses upon the plastic deformation, and to express it as function of damage amount. The resulting set of equations requires numerical solution, since generally, the mechanical characteristics of the layers result function of their position into laminate shell, and depending of damage through the damaged stress tensor. The numerical results have been compared with experimental data, obtained for Glass-Epoxy composite shells under multiaxial loads.

Introduction

The damage behaviour and residual mechanical characteristics of composite materials is still an open issue. At today, the damage behaviour under simple load configurations has already been studied, but very few studies have been carried out about composite materials behaviour under multiaxial loads, as well as about their long time behaviour. (1,2) This paper is devoted to obtain a realistic behaviour for composite shells, with random stratification sequence, under multiaxial quasi-static loading. For this purpose, a non-linear damage-elastoplastic model must be applied for layers mechanical characterisation. (3,4)

In early modelling, the laminate has been considered as constituted of an orthotropic material and modelled directly. This model does not allow to take into account coupling effects that can occur between strain and bending, and is able to characterise only symmetrical laminate under traction and/or internal pressure loads. (5)

In the present paper, an alternative way is presented in order to model the layer damage-elastoplastic behaviour, and, subsequently, integrate this behaviour it into the laminate thickness through the extension of classical laminate theory. (6) In this way, it is possible to take into account multiaxial loading, such as traction, internal pressure with or without boundary effects, torsion and their combinations. Moreover, the stratification sequence is not any more a critical parameter since layers behaviour have been modelled directly.

Theoretical modelling

The total laminate deformation $\underline{\varepsilon}_t$, can be expressed through two internal variables, $\underline{\varepsilon}_e, \underline{\varepsilon}_p$, respectively elastic and plastic deformations, as following:

$$\underline{\varepsilon}_t = \underline{\varepsilon}_e + \underline{\varepsilon}_p \qquad (1)$$

Into the following analysis, micro-crack damage in the matrix, or debonding phenomena at the interface between fibre-matrix can be modelled. Delamination or fibre failure has not been considered, since these phenomena should be treated as macroscopic damage, related rather to the fracture mechanics than to a damage modelling.

Layer damage and elastic deformation

In order to introduce a damage parameter into layers constitutive equations, the effective stress tensor concept has been applied:

$$\underline{\tilde{\sigma}} = M(\underline{\underline{D}})\underline{\sigma} \qquad (2)$$

with $M(\underline{\underline{D}})$ damage operator, and $\underline{\tilde{\sigma}}$ effective stress tensor into the damaged layer. [7]

The concept of equivalence in strains between virgin and damage materials is used, such as:

$$\underline{\tilde{\varepsilon}} = \underline{\underline{S}}:\underline{\tilde{\sigma}} \text{ and } \underline{\varepsilon} = \underline{\underline{\tilde{S}}}:\underline{\sigma} \qquad (3)$$

which implies that $\underline{\tilde{\varepsilon}} = \underline{\varepsilon}$, with $\underline{\tilde{\varepsilon}}$ damaged material strain, $\underline{\underline{S}}$, $\underline{\underline{\tilde{S}}}$ respectively virgin and damaged material compliance matrixes.

The damage operator can be defined through the equation:

$$M(\underline{\underline{D}}) = (\underline{\underline{I}} - \underline{\underline{D}})^{-1} \qquad (4)$$

where $\underline{\underline{D}}$ represents the damage tensor. [1]

By using eq.(2),(3),(4), the damaged stiffness matrix of each layer $\underline{\underline{\tilde{C}}}$, can be expressed as:

$$\underline{\underline{\tilde{C}}} = M(\underline{\underline{D}})^{-1}\underline{\underline{C}} = (\underline{\underline{I}} - \underline{\underline{D}}):\underline{\underline{C}} \qquad (5)$$

Micro-cracks orientation in the layers has been considered parallel to the fibres, Fig.1, as the most representative damage direction.

By noticing that it these conditions, damage respects the orthotropic axis of the layer, it is possible to obtain for $\underline{\underline{D}}$ as follows:

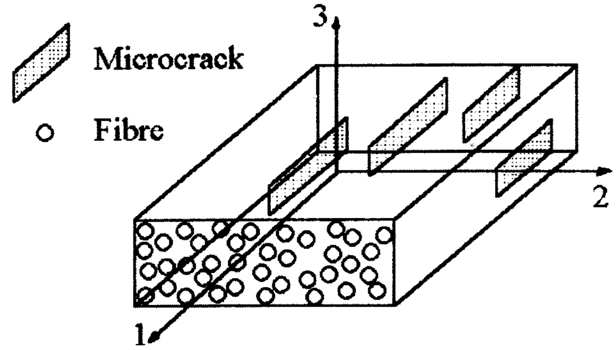


FIGURE 1 - Micro-cracks orientation.

$$\underline{\underline{D}} = \begin{bmatrix} 0 & \frac{C_{12}H_{22}}{1+C_{22}H_{22}} & 0 & 0 & 0 & 0 \\ 0 & \frac{C_{22}H_{22}}{1+C_{22}H_{22}} & 0 & 0 & \text{à} & 0 \\ 0 & \frac{C_{32}H_{22}}{1+C_{22}H_{22}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{44}H_{44}}{1+C_{44}H_{44}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{66}H_{66}}{1+C_{66}H_{66}} \end{bmatrix} \qquad (6)$$

with H_{22}, H_{44}, H_{66}, obtained through a method based upon the increase of elastic energy into the material, due to micro-cracks:

$$H_{22} = \frac{\pi d}{2\sqrt{2}}\left(S_{11}\left(S_{22}+H_{22}\right)\right)^{1/2} * \\ *\left[\left(\frac{S_{22}+H_{22}}{S_{11}}\right)^{1/2} + \frac{2S_{12}+S_{66}+H_{66}}{2S_{11}}\right]^{1/2} \qquad (7a)$$

$$H_{44} = \frac{\pi d}{4}\left(S_{55}\left(S_{44}+H_{44}\right)\right)^{1/2} \qquad (7b)$$

$$H_{66} = \frac{\pi d}{2\sqrt{2}} S_{11} * \\ *\left[\left(\frac{S_{22}+H_{22}}{S_{11}}\right)^{1/2} + \frac{2S_{12}+S_{66}+H_{66}}{2S_{11}}\right]^{1/2} \qquad (7c)$$

These last parameters result function of micro-cracks density d, given through the following equation:

$$d=\sum_{j=1}^{N}\frac{b_j a_j^2}{V} \quad (8)$$

with b_j, a_j, respectively thickness and length of the *jth* micro-crack, *V* volume of the layer, *N* number of micro-cracks in the layer. [1,5,6]

Through the above described equations it is possible to obtain the damage stiffness matrix $\underline{\underline{\tilde{C}}}$ as function of micro-crack density only.

However, since experimental measurement of *d* is rather a difficult operation, it should be wiser to use an other parameter to express damage stiffness matrix.

Indeed, by using as damage representative internal variable the variation of layer transversal Young modulus D_{22}, defined as * :

$$D_{22}=\frac{\Delta E_{22}}{E_{22}} \quad (9)$$

it is then possible, and easier, to evaluate the damage stiffness matrix $\underline{\underline{\tilde{C}}}$ and its evolution, as function of D_{22}. An approximation of $\underline{\underline{\tilde{C}}}$ can be written as:

$$\underline{\underline{\tilde{C}}}=\underline{\underline{C}}(D_{22}=0)+D_{22}\underline{\underline{C}}_1+D_{22}^2\underline{\underline{C}}_2 \quad (10)$$

with $\underline{\underline{C}}_1, \underline{\underline{C}}_2$, obtained numerically by solving the set of eq.(5),(6),(7),(9).

Through eq.(2),(3),(8), the effective stress tensor can be written as function of layer compliance matrix:

$$\underline{\tilde{\sigma}}=\left(\underline{\underline{I}}+D_{22}\underline{\underline{C}}_1:\underline{\underline{S}}+D_{22}^2\underline{\underline{C}}_2:\underline{\underline{S}}\right)^{-1}:\underline{\sigma} \quad (11)$$

In order to obtain the kinetic of D_{22}, Helmholtz free energy Ψ is given through the following equation:

$$\Psi=\frac{1}{2}\underline{\underline{C}}(D_{22})\underline{\varepsilon}_e:\underline{\varepsilon}_e+\Psi^* \quad (12)$$

with Ψ^* function of some other internal variables, like hardening variables etc.

A driving variable Y_{22} to the damage variable D_{22} can be defined:

$$Y_{22}=\frac{\partial\Psi}{\partial D_{22}}=\frac{1}{2}\left(2\underline{\underline{C}}_2 D_{22}+\underline{\underline{C}}_1\right):\underline{\varepsilon}_e:\underline{\varepsilon}_e \quad (13)$$

It is also necessary to introduce one more variable, R_D, representing an isotropic hardening, as:

$$R_D=\frac{\partial\Psi}{\partial\overline{D}_{22}} \quad (14)$$

with $\overline{D}_{22}$ an accumulate damage variable, which has the same value as D_{22}, but no the same meaning.

An experimental damage criterion has been proposed, through a damage representative function f_D, as:

$$f_D=\langle -Y_{22}\rangle-R_D-Y_2^c\leq 0 \quad (15)$$

with Y_2^c yield point of damage, and $\langle a\rangle=a$ if $a\geq 0$, or $\langle a\rangle=0$ if $a<0$.

At this point, knowing that damage grows if $f_D=0$ and $\frac{\partial f_D}{\partial Y_{22}}dY_{22}\geq 0$, the rate of damage could be obtained by using the rule of normality to the criterion:

$$\dot{D}_{22}=-\dot{\lambda}_D\frac{\partial f_D}{\partial Y_{22}} \quad (16a)$$

$$\dot{\overline{D}}_{22}=-\dot{\lambda}_D\frac{\partial f_D}{\partial R_D} \quad (16b)$$

Finally, using an experimental identification, relating R_D to $\overline{D}_{22}$, as:

$$R_D=\alpha_D\overline{D}_{22}^{P_D} \quad (17)$$

with α_D, P_D, experimental constants, it is possible to obtain $\dot{\lambda}_D$ through the consistency equation.

In this way, the evolution of $\underline{\tilde{\sigma}}$ is completely defined, and the elastic damage deformation can be expressed as:

$$\underline{\varepsilon}_e=\underline{\underline{S}}:\underline{\tilde{\sigma}} \quad (18)$$

* Notice that damage internal variable D_{22}, is not a component of damage tensor $\underline{\underline{D}}$.

Layer plastic deformation

The effective stress tensor $\underline{\tilde{\sigma}}$, imposes its non-linearity upon the elastic strain $\underline{\varepsilon}_e$, through eq. (18). Since $\underline{\varepsilon}_e$ represents reversible phenomena, this parameter is null after a load/unload cycle. Through experimental investigation, permanent deformations have been detected in test specimens, under repeated progressively increasing applied load; it results necessary to introduce a plastic deformation term $\underline{\varepsilon}_p$, which takes into account the non-reversible phenomena that occur during loading cycles.

With the hypothesis that micro-cracks are parallel to the fibre direction, some components of effective stress tensor $\underline{\tilde{\sigma}}$, are not active upon plastic strain. Into the following, the method proposed by A.J. Spencer [8] for the determination of the active stress components upon plastic deformation will be applied, in order to obtain a plastic criterion and the plastic strain component.

Supposing that an elastic behaviour is achieved into fibres direction, and that plastic strain is due mainly to friction between micro-cracks surfaces, it is possible to suppose that only shear stress components should be active upon plastic strain. Consequently, a plastic criterion can be proposed into the following form:

$$f_p(\underline{\tilde{\sigma}}) = \sqrt{\beta_1 \tilde{\sigma}_6^2 + \beta_2 \tilde{\sigma}_4^2} \leq 0 \qquad (19)$$

with β_1, β_2 material constants.

This criterion, eq.(19), can be written into a matrix form, as:

$$f_p(\underline{\tilde{\sigma}}) = \left[\underline{\tilde{\sigma}}:\underline{\underline{M}}:\underline{\tilde{\sigma}}\right]^{1/2} - S_6^c \qquad (20)$$

with $\underline{\underline{M}}$ anisotropic matrix of the material, given as:

$$\underline{\underline{M}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\dfrac{S_6^c}{S_4^c}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (21)$$

where S_6^c, S_4^c, are respectively shear elastic yield points. [7,8]

In order to fully model the hysteresis loop of Glass-Epoxy shells, appears necessary the introduction of kinematic hardening variables, which modify the criterion as follows:

$$f_p(\underline{\tilde{\sigma}}) = \overline{\underline{\tilde{\sigma}} - \underline{X}} - S_6^c \qquad (22)$$

with $\overline{\underline{\tilde{\sigma}} - \underline{X}} = \left[(\underline{\tilde{\sigma}} - \underline{X}):\underline{\underline{M}}:(\underline{\tilde{\sigma}} - \underline{X})\right]^{1/2}$, $\underline{X}$ sum of associate variables to kinematic hardening, as:

$$\underline{X} = \underline{X}_1 + \underline{X}_2 \qquad (23)$$

where $\underline{X}_1$ represents a non-linear kinetic hardening as described by the Armstrong-Frederick equations, while $\underline{X}_2$ is linear. These last two parameters are given through:

$$\dot{\underline{X}}_1 = \delta_1 \dot{\underline{\varepsilon}}_p - \gamma_1 \underline{X}_1 \dot{\bar{\varepsilon}}_p \qquad (24a)$$

$$\dot{\underline{X}}_2 = \delta_2 \dot{\underline{\varepsilon}}_p \qquad (24b)$$

with $\delta_1, \delta_2, \gamma_1$, material parameters, $\dot{\bar{\varepsilon}}_p$ cumulative plastic deformation velocity.

Considering that plastic strain $\underline{\varepsilon}_p$, occurs if $f_p = 0$ and $\dfrac{\partial f_p}{\partial \underline{\sigma}}:d\underline{\sigma} > 0$, and that $\dot{\underline{\varepsilon}}_p$ and $\dot{\bar{\varepsilon}}_p$ are given through:

$$\dot{\underline{\varepsilon}}_p = \dot{\lambda}_p \frac{\partial f_p}{\partial \underline{\sigma}} \qquad (25)$$

$$\dot{\underline{\varepsilon}}_p:(\underline{\sigma} - \underline{X}) = \overline{(\underline{\sigma} - \underline{X})}\,\dot{\bar{\varepsilon}}_p \qquad (26)$$

with the previously described kinetics, and the consistency equations applied to the plastic criterion f_p, it is possible to obtain $\dot{\lambda}_p$.

The plastic strain component is now completely defined. Into plastic behaviour, the damage is taken into account by replacing $\underline{\sigma}$ with $\underline{\tilde{\sigma}}$.

Now the model is completely written at the mesoscopic scale. It rests the integration of layer behaviour into laminate thickness.

Numerical implementation

Through Classical Laminate Theory (applied into elasticity field), it is possible to obtain the well known equations for laminate shells:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & F \end{bmatrix} \begin{bmatrix} e_0 \\ \rho \end{bmatrix} \qquad (27)$$

with *N,M,* respectively applied forces and moments, e_0, ρ mid plane deformation and bending, and:

$$A = \sum_{k=1}^{n} \overline{Q}_k \left(z_k - z_{k-1} \right) \qquad (28a)$$

$$B = \frac{1}{2} \sum_{k=1}^{n} \overline{Q}_k \left(z_k^2 - z_{k-1}^2 \right) \qquad (28b)$$

$$F = \frac{1}{3} \sum_{k=1}^{n} \overline{Q}_k \left(z_k^3 - z_{k-1}^3 \right) \qquad (28c)$$

where n is the number of layers, $\overline{Q}_k$ layer stiffness matrix written into laminate axis, z co-ordinate through laminate thickness.
The integration of eq.(28) can be obtained analytically, since $\overline{Q}_k$ is not function of $\underline{\sigma}$ or z.
In the elastoplastic field, layers behaviour can be described through the following incremental equation:

$$d\underline{\sigma} = \left(\underline{\underline{\overline{\overline{S}}}}_e + \underline{\underline{\overline{\overline{S}}}}_p \right)^{-1} : d\underline{\varepsilon} \qquad (29)$$

By integration of eq.(29), through the laminate thickness it is possible to obtain an equation similar to eq.(27), written into damage elastoplastic field, as:

$$\begin{bmatrix} dN \\ dM \end{bmatrix} = \begin{bmatrix} \tilde{A}_{ep} & \tilde{B}_{ep} \\ \tilde{B}_{ep} & \tilde{F}_{ep} \end{bmatrix} \begin{bmatrix} de_0 \\ d\rho \end{bmatrix} \qquad (30)$$

with

$$\tilde{A}_{ep} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\underline{\underline{\overline{\overline{S}}}}_e + \underline{\underline{\overline{\overline{S}}}}_p \right)^{-1} dz \qquad (31a)$$

$$\tilde{B}_{ep} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\underline{\underline{\overline{\overline{S}}}}_e + \underline{\underline{\overline{\overline{S}}}}_p \right)^{-1} z\, dz \qquad (31b)$$

$$\tilde{F}_{ep} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\underline{\underline{\overline{\overline{S}}}}_e + \underline{\underline{\overline{\overline{S}}}}_p \right)^{-1} z^2\, dz \qquad (31c)$$

The analytical integration of these last equations is not possible, since stiffness matrix is function of $\underline{\sigma}$, either through damage behaviour or hardening. In the case of a flexion applied load, or even in the case of a non-symmetrical stratification sequence, stress in the layers results function of the layer position within the laminate shell. [6]

Resolution principle

Into damage and plasticity behaviour, non-reversible phenomena occur, resulting function of the path. Consequently, it is necessary to fix a starting point and, step by step, following the loading cycle, solve numerically the non-linear set of equations.

In the case that an external load is applied to the composite shell, the stress field in each layer is not directly accessible. It is necessary to start the resolution cycle with an evaluation of mid plane strain and bending. Then, it is possible to determine strain in each layer, and through it, layers stress field, and finally, calculate the external load by layers stress field integration through laminate thickness. At this point, a comparison between applied and calculate external loads, allows to proceed with a new evaluation of mid plane strain and bending. This interactive cycle must be repeated until calculated loads are close to external applied loads unless of a fixed tolerance.

Into the following Fig.2, it is possible to observe a flowchart of the resolution method.

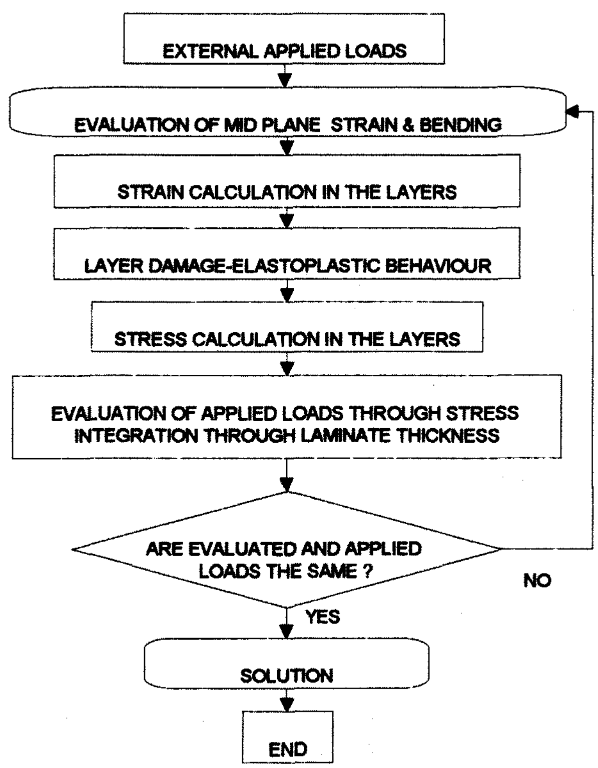


FIGURE 2 - Flowchart of the resolution method.

Finally, the resolution cycle can be sub-divided in three main phases:
- Progressive and optimal evaluation of mid plain strain and bending;
- Resolution of layers damage elastoplastic model imposing the deformations;
- Integration of layers stress field into laminate thickness.

Mid plain strain and bending evaluation. Newton Raphson method has been applied to solve eq.(30). The first iteration starts with an elastic evaluation of mid plane strain and bending. The test of convergence results as follows:

$$\frac{\left|\begin{bmatrix} dN \\ dM \end{bmatrix}_{calculated}\right|}{\left|\begin{bmatrix} dN \\ dM \end{bmatrix}_{applied}\right|} \leq \text{Tolerance} \tag{32}$$

If eq.(32) is satisfied, the evaluated parameters are updated ($X^{p+1} = X^{p} + dX$, with X a vector containing the parameters evaluated step by step), and the method is repeated for the next step.

Resolution of layers behaviour model. The layers damage elastoplastic model, as described in the first part of the present paper, can be represented through a system of differential equations written in the following form:

$$F\,Y = X \text{ or } Y = F^{-1}X \tag{33}$$

with X vector containing problem parameters, Y vector containing the increments of these parameters, F representative matrix of the set of equations.
Such a system, can be solved through Runge-Kutta method.

Integration of layers stress field into laminate thickness. In general loading conditions and for a random stratification sequence, the stress tensor is not constant through laminate thickness. In such conditions, into Newton-Raphson method, also Jacobean matrix elements are not constant.
The Gauss-Legendre method has been applied in this case. It is necessary to consider a certain number m of points into the layers (sub-dividing them in sub-layers), wisely and not equally distributed. In these points, both, function to be integrated and its values, are known.

Finally, it is possible to obtain the following set of equations:

$$\tilde{A}_{ep} = \sum_{k=1}^{n} \frac{h_k}{2}\left[\sum_{l=1}^{m} w_l \left(\underline{\underline{\tilde{S}}}_e + \underline{\underline{\tilde{S}}}_p\right)^{-1}_{k,l}\right] \tag{34a}$$

$$\tilde{B}_{ep} = \sum_{k=1}^{n} \frac{h_k}{2}\left[\sum_{l=1}^{m} w_l \left(\underline{\underline{\tilde{S}}}_e + \underline{\underline{\tilde{S}}}_p\right)^{-1}_{k,l} z_{k,l}\right] \tag{34b}$$

$$\tilde{F}_{ep} = \sum_{k=1}^{n} \frac{h_k}{2}\left[\sum_{l=1}^{m} w_l \left(\underline{\underline{\tilde{S}}}_e + \underline{\underline{\tilde{S}}}_p\right)^{-1}_{k,l} z^2_{k,l}\right] \tag{34c}$$

$$dN = \sum_{k=1}^{n} \frac{h_k}{2}\left[\sum_{l=1}^{m} w_l \, d\underline{\sigma}_{k,l}\right] \tag{34d}$$

$$dM = \sum_{k=1}^{n} \frac{h_k}{2}\left[\sum_{l=1}^{m} w_l \, d\underline{\sigma}_{k,l} \, z_{k,l}\right] \tag{34e}$$

with h layer thickness, k, l, respectively layers and sub-layers, n number of layers, m number of sub-layers, w_l Gauss-Legendre integration factors, z co-ordinate of layers and sub-layers through laminate thickness.

Numerical and experimental results

The theoretical damage elastoplastic model has been validated through comparison with experimental data. For this purpose, cylindrical Glass-Epoxy specimens have been used, length 300mm, diameter 60mm, thickness 2mm, achieved by winding 6 layers with a winding angle ±55°. Repeated, progressively increasing loads have been applied in quasi-static conditions, uniaxially and multiaxially, respectively through traction or pure internal pressure loads, Fig.5,6, and internal pressure load with boundary effects, Fig.7.

In Fig.3, it is possible to observe a specimen under traction loads, with some details about anchoring system. It is also possible to observe that circumferential and axial strains have been achieved through extensometers wisely positioned on the central zone of the specimen. Particularly, for axial strain measurement, two extensometers have been used, in order to monitor possible flexional effects.
In Fig.4, it is visible the special form of the anchoring system, for multiaxial internal pressure tests, with the internal rubber vessel, specially constructed for this purpose.

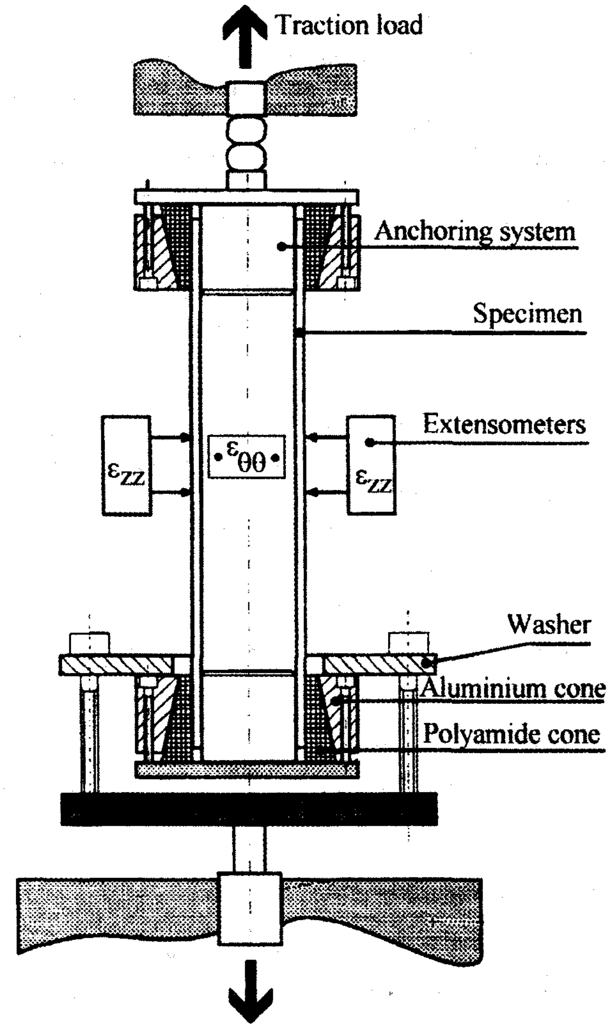


FIGURE 3 - Measurement and anchoring systems for experimental tests.

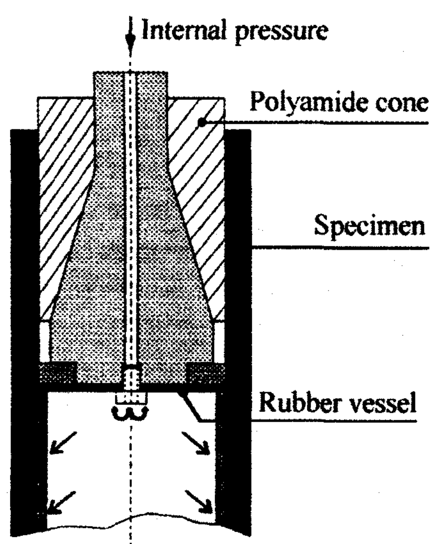


FIGURE 4 - Anchoring system, and special rubber vessel for internal pressure tests.

Into the following Fig.5,6,7, it is possible to observe the obtained theoretical/numerical results and their comparison with experimental data.

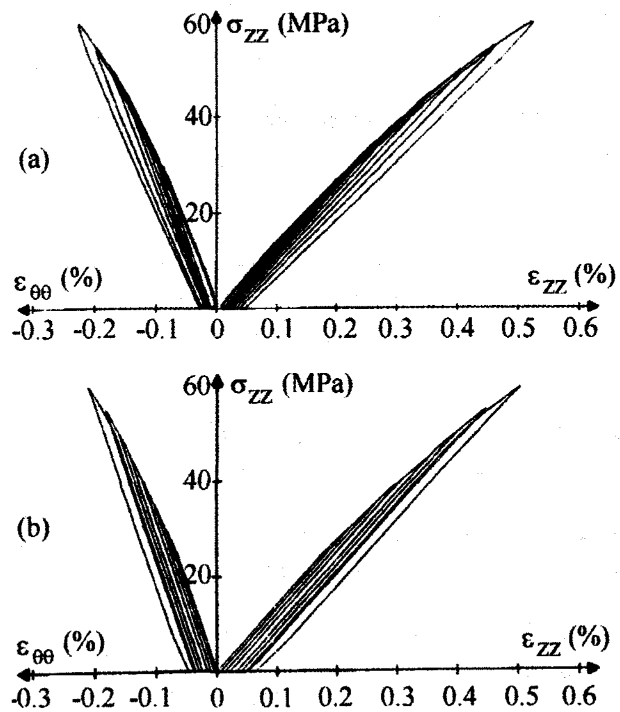
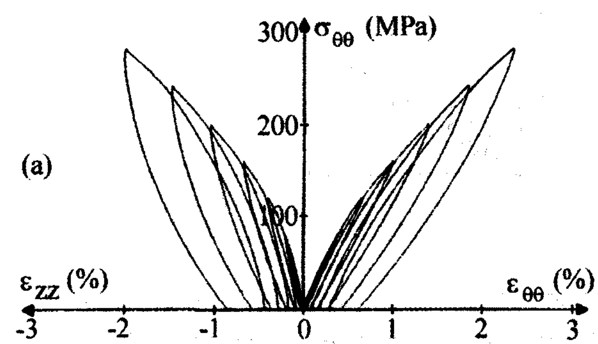


FIGURE 5 - Cylindrical composite shells under repeated progressively increasing traction loads, (a) experimental, (b) numerical.

In this first test under repeated progressively increasing traction loads, it is possible to observe that the numerical results are in good accordance with experimental data. Form and shape of both numerical and experimental curves are similar, specially for the first loading cycles, having only light strain over estimation during the last loading cycle.



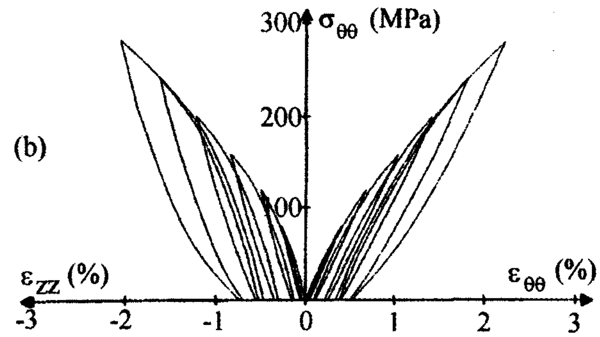


FIGURE 6 - Cylindrical composite shells under repeated progressively increasing pure internal pressure loads, (a) experimental, (b) numerical.

In the case of pure internal pressure, Fig.6, it is possible to observe that the numerical approach simulates correctly both circumferential and axial strains for all loading cycles, having quite the same residual strain components in both experimental analysis and numerical calculation.

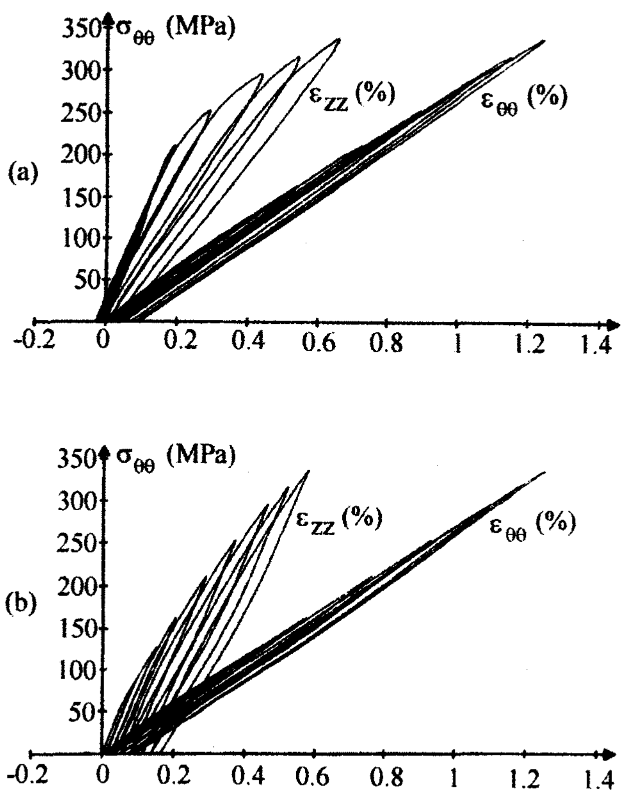


FIGURE 7 - Cylindrical composite shells under repeated progressively increasing internal pressure loads with boundary effects, (a) experimental, (b) numerical.

In the case of internal pressure with boundary effects, Fig.7, the present formulation of the damage and plasticity criteria are able to simulate correctly the phenomena, giving to the numerical curves a particular physiognomy for both circumferential and axial strains. By careful investigation, it is possible to notice that the damage variable has not the desired evolution, resulting a little smaller. This phenomenon occurs since in the present case the leading parameter is plastic deformation , and consequently, a very small amount of elastic deformation (directly connected to damage parameters) is not able to create the correct quantity of damage. However, it is possible to observe that circumferential strain is represented correctly, while an over estimation occurs during the evaluation of axial stain.

In internal pressure loading with boundary effects (that corresponds to a biaxial load of internal pressure and traction) ε^p_{zz} is negative during the first loading cycles. In order to take into account this phenomenon, it is necessary to introduce $\tilde{\sigma}_2$ stress component into anisotropy matrix M, defined into eq.(21), and to optimise the ratio $\frac{\tilde{\sigma}_6}{\tilde{\sigma}_2}$. In these conditions shear stress component should be the leading variable during the first loading cycles, conducing to $\varepsilon^p_{zz} < 0$. Into the next loading cycles, the presence of damage should be able to exploit the influence of $\tilde{\sigma}_2$ stress component upon ε^p_{zz}, resulting $\varepsilon^p_{zz} > 0$.

Conclusions

Nowadays, composite material behaviour is still the object of several studies. A very large amount of composite materials , as combination of two distinct solid phases, renders impossible the elaboration of an *universal model*. For this reason, researchers generally focus their attention upon one type of composite material. In the present paper Glass-Epoxy composite shells damage behaviour has been analysed.

The presented modelling has been obtained through a theoretical\numerical approach, and the evaluation of particular characteristics of the selected material has been carried out through experimental procedures. The resulting non-linear set of equations of the theoretical model has been implemented into a numerical code, using numerical approaches, like Newton-Raphson, Runge-Kutta, and Gauss-Legendre methods.

The model has been validated through comparison of the results with experimental data, in quasi static conditions and under multiaxial loads, such as traction, internal pressure with or without boundary effects and their combinations. It is necessary to notice that the generality of the theoretical approach, achieved through the layer damage-elastoplastic modelling, allows to handle also torsion and flexion loads.

Taking into account the present encouraging results, to complete the model should be necessary to introduce viscoelastic and viscplastic behaviour in order to handle also dynamic and creep loads.

Nomenclature

a:	micro-crack length.
b:	micro-crack thickness.
d:	micro-crack density.
e_0:	laminate mid plain deformation.
f_D:	damage criterion function.
f_p:	criterion of plasticity function.
h:	layer thickness.
m:	number of sub-layers.
n:	number of laminate layers.
w_l:	Gauss-Legendre integration factors
z:	co-ordinate through laminate thickness.
$\underline{\underline{A}}$:	classical laminate theory matrix.
$\underline{\underline{B}}$:	classical laminate theory matrix.
$\underline{\underline{C}}$:	layer stiffness matrix.
$\underline{\underline{C}}_1, \underline{\underline{C}}_2$:	numerical evaluation of layers stiffness matrix.
$\underline{\underline{D}}$:	damage tensor.
D_{22}:	variation of layer transversal Young modulus.
$\overline{D}_{22}$:	cumulative damage variable.
E_{22}:	layer transversal Young modulus.
$\underline{\underline{F}}$:	classical laminate theory matrix.
H:	energetic method parameters.
$\underline{\underline{I}}$:	identity tensor.
M:	applied moments.
$\underline{\underline{M}}$:	plasticity anisotropic matrix..
$M(\underline{\underline{D}})$:	damage operator.
N:	applied forces.
P_D:	damage experimental constant.
$\underline{\underline{\overline{Q}}}_k$:	layer stiffness matrix into laminate axis.
R_D:	associate variable to damage isotropic hardening.
$\underline{\underline{S}}$:	layer compliance matrix.
S_4^c, S_6^c:	shear elastic yield points.
Y_{22}:	associate variable to damage.
Y_2^c:	yield point of damage.
$\underline{\underline{X}}$:	sum of associate variables to kinematic hardening in plasticity.
$\underline{\underline{X}}_1$:	non-linear kinematic hardening.
$\underline{\underline{X}}_2$:	linear kinematic hardening.
V:	layer volume.
α_D:	damage experimental constant.
β_1, β_2:	plasticity material constants.
γ_1:	plasticity material parameter.
δ_1, δ_2:	plasticity material parameters.
$\underline{\varepsilon}$:	strain.
λ_D:	Lagrange factor in damage.
λ_p:	Lagrange factor in plasticity.
ρ:	laminate mid plain bending.
$\underline{\sigma}$:	stress tensor.
Ψ:	Helmholtz free energy.
Ψ*:	internal variables function free energy.
e:	indicates elastic components.

ep: indicates elastoplastic components.

p: indicates plastic components.

t: indicates total components.

~ : indicates the presence of damage.

_ : indicates a vector.

$\underline{\underline{\ }}$: indicates tensor.

< >: $\langle a\rangle = a$ if $a \geq 0$, or $\langle a\rangle = 0$ if $a < 0$.

• : indicates an increment or velocity.

¯ : indicates a cumulative variable or parameters written into laminate axis.

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