MODEL-INDEPENDENT FUZZY VIBRATION SUPPRESSION OF FLEXIBLE STRUCTURES USING NON-COLOCCATED TRANSDUCERS

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Abstract

The present investigation deals with an AFCA (Adaptive Fuzzy Control Algorithm) for an Euler-Bernoulli approximation of a cantilever beam-like orthogonal tetrahedral space truss, having a span of 150 m. Transient disturbances, modeled as a unit impulse, excite all the modes of the beam. The resulting transverse displacement and its corresponding rate at the free end are observed by sensors placed there, and active control of the beam is provided by a force actuator. A design methodology for the closed-loop control algorithm that is independent of an exact mathematical model of plant dynamics and based on fuzzy logic is presented. This approach results in relatively quick settling times, low overshoots and dying out of vibration within a few seconds. When the control force is turned off after a short while, almost all the vibrational energy is dissipated as the beam returns to its undisturbed state throughout its length. The sensitivity of the AFCA to non-collocation of actuator/sensor is examined. Results show that even though a marginal loss in settling time is obtained at this off-design point, the closed-loop performance, for the non-collocation is reasonably good.

Background

Future aerospace applications include concepts such as large space stations, high resolution radar and communication antennas, astronomical observatories and solar power stations. Qualifying as LFS (Large Flexible Structures) these facilities may generally comprise of repetitive latticed trusses, span large areas with a few intermediate supports, are light in weight and extremely flexible, and consequently are characterized by a large number of high density low frequency structural modes.

The LFS may be characterized by low inertia, light inherent damping, undamped rigid modes, low natural frequencies, high modal density, some joint non-linearity, and includes sensors, actuators and computers, on-board power and a laser pointing system. The dynamic characteristics of the above are poorly known and therefore make the analytical modeling of the structural dynamic problem of LFS cumbersome with substantial uncertainties. Hence, it would be advantageous to develop control design strategies that are independent of an exact analytical model (e.g. state-space model).

LFS may need to meet target tracking, slewing, stringent line-of-sight and jitter control, pointing accuracies, and microgravity acceleration requirements. However, when disturbed, the structure is likely to remain excited for some time because of its high structural modal density at low frequencies and possibly small damping. Therefore, it is vital to introduce means for either passive energy dissipation, or active control, or their combination to restrain the response of a given structure within an "inmission displacement-time allowables envelope" by using vibration control methods.

Introduction

Robustness of a LFS control in the presence of uncertainties has been an area of intense research. One such method, based on the positivity design assures that the closed loop system will be characterized by stability as well as an energy dissipation related to the input/output behavior. Hyland(1) describes this energy dissipative law that combines collocated actuator and sensor pairs as electromagnetically emulating passive structural damping.

Meyer, Burke and Hubbard(2) use velocity feedback with positive-definite feedback gain and collocated piezoelectric transducers in a control methodology integrating sliding mode control, distributed parameter systems theory, and fuzzy logic to develop vibration damping of a cantilever beam. A frequency-shaped LQR adaptive control scheme, based on a priori knowledge of the intervals of system parameter variations and fuzzy-logic, is applied to vibration suppression of a cantilever beam using collocated piezoceramic transducers(3).

Hollkamp and Starchville(4) present a piezoelectric actuator that emulates an inherently stable dynamic vibration absorber. This self-tuning piezoelectric absorber is made adaptive by tuning the electric resonance (i.e. by adjusting the shunt inductance and resistance). Within the past decade, experiments conducted on flexible...
structures validated the shunted piezoelectric models which predicted the optimal tuning parameters. In addition, real time tuning of the resistance and inductance of the shunted network has also been experimented on.

Cohen, Weller and Ben-Asher\(^{(5)}\) introduce an approach, based on fuzzy logic control, which provides continuous tuning of the damping parameter of the above described emulated absorber. The main advantages of using a fuzzy approach are the relative ease and simplicity of implementation and the robustness characteristics. The parameters of the above absorber may be adapted to provide fairly fast control for large deviations, of the measured state of the plant from the desired state, and a minor amount of control for small deviations. Thus, non-linear control actions, corresponding to a lightly damped absorber with a large mass ratio, which fully utilize the range of actuator displacements, send the plant state hurtling towards the desired state. On the other hand, in the vicinity of this desired state, the absorber is heavily damped, having a small mass ratio. Heuristic rules, based on basic "common sense" engineering insight, coupled with fuzzy reasoning provide crisp values for lightly, large, heavily, and small.

The input to the control law is based on sensor readings of displacement/distortion, and their corresponding rates. The control law, based on emulated dynamic vibration absorbers, requires that the positioning of the sensors be collocated with that of the actuator. However, in many real-life situations involving the control of LFS, because of physical placement and hardware limitations, absolute collocation is often impossible\(^{(6)}\). Hence, the performance sensitivity of the AFCA to non-collocation of actuator/sensor needs to be examined.

### Problem Description

The LFS studied is typical of large space structures\(^{(9)}\). The discrete, transducers applied to actively control the above structure emulate the behavior of a dynamic vibration absorber. Figure 1 describes the 2-D beam-like lattice with a virtual absorber attached. The first five modes of the structure are considered in the MATLAB simulation. The respective natural frequencies, given in [rad/sec] and corresponding to the beam model, are displayed in Table 1 (Cohen and Weller\(^{(10)}\)):

<table>
<thead>
<tr>
<th>Mode</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
<th>Fifth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1.59</td>
<td>9.98</td>
<td>27.95</td>
<td>54.77</td>
<td>90.53</td>
</tr>
</tbody>
</table>

Table 1- Natural frequencies of the structure in Figure 1 [rad/sec]

The structure, subjected to an initial condition unit impulse, provides similar transient disturbance to each of the first five bending modes, thereby exciting all of them. The closed-loop controller is applied at the lapse of a second, at which time the beam is in the vicinity of the maximum open-loop amplitude (see Figure 2). The control law is fed with sensor readings of displacement and velocity, placed at a distance 1 (m=1) from the fixed end. An additional sensor, positioned midway between the fixed and the free ends, at l/2, observes the transverse displacement-time history. Nominaly, the actuator and the sensor are collocated. The closed-loop performance will be examined for a case where the sensor is placed at a distance of at least 10% of beam length from the actuator.

### Proposed Approach

The fuzzy adaptation strategy developed selects the most appropriate damping factor for the virtual absorber. The mass ratio of this absorber is a function of the damping factor as described by Cohen et. al.\(^{(11)}\), whereby a lightly damped absorber corresponds to a large mass ratio and vice-versa. Based on this mass ratio, the absorber is then tuned to the fundamental frequency of the beam, obtained from the open-loop transverse displacement sensor output (see Figure 2). After each sensor reading, the damping and the mass ratios of the absorber adapt themselves using a fuzzy decision-making process. The above adaptation leads to the calculation of the actuation force by solving the 2nd order differential equations concerning the equations of motion of the emulated absorber. This approach, not only assures inherent stability associated with passive absorbers, but also circumvents the phenomenon of modal spillover.
Fuzzy Adaptation Strategy

Fuzzy logic, on which the fuzzy control is based, is a convenient way to map an input space into an output space. The major mechanisms of FLC (Fuzzy Logic Control) are: a set of if-then statements called linguistic control rules; and a fuzzy inference system that interprets the values in the input vector and, based on the linguistic rules, assigns values to the output vector. The experience of the past decade, with the successful marketing of a wide variety of products based on the FLC [12], has shown that for certain applications FLC provides superior results to those obtained by other conventional means.

Fuzzy Membership Functions

The first step in building the fuzzy part of the controller involves the Fuzzification of the input/output parameters. Here, the transverse displacement \( y(m,t) \) and the transverse velocity \( \dot{y}(m,t) \) of the beam are normalized to yield the dimensionless variables \( y_N \) and \( \dot{y}_N \), respectively, using the following relations:

\[
\begin{align*}
  y_N &= N_Y \cdot y(m,t), \\
  \dot{y}_N &= N_{\dot{y}} \cdot \dot{y}(m,t)
\end{align*}
\]  

(1)

where \( N_Y \) and \( N_{\dot{y}} \) function as tuning parameters for the arbitrarily chosen membership functions illustrated in Figures 3 and 4, respectively. The use of these "tuning knobs" substantially cuts the degrees of freedom involved in reaching the required membership functions. In addition, once \( N_Y \) and \( N_{\dot{y}} \) are found, the sensitivity of individual fuzzy sets to the closed-loop performance is examined. Since no improvement is obtained, no additional changes are made to the arbitrarily selected membership functions of \( y_N \) and \( \dot{y}_N \).

Fuzzy sets for the normalized transverse displacement, \( y_N \), are characterized by membership functions \( \mu_{N}, \mu_{NS}, \mu_{Z}, \mu_{PS}, \mu_{P} \) that map elements of the universe of discourse, \( y_N \), into the closed interval \([0,1]\) as follows:

\[
\mu_L = y_N \rightarrow [0,1] \quad \text{for} \quad L=\text{N,N,S,Z,PS,P}
\]

(2)

where \( L \) stands for one of the linguistic terms used in this effort to categorize \( y_N \) i.e. N(Negative), NS(Negative small), Z(Zero), PS(Positive Small) and P(Positive). The membership functions given in Equation (2) express the degree to which \( y_N \) belongs to some category \( L \). These fuzzy sets may be viewed by plotting \( y_N \) versus \( \mu_L \) as shown in Figure 3. For example, for the normalized displacement, \( y_N \), a crisp value of 2.5 corresponds to 50 percent Positive Small and 50 percent Zero.

Similarly, fuzzy sets for the normalized transverse velocity, \( \dot{y}_N \), are characterized by the membership functions \( \mu_{Q} \), for \( Q = N, Z \) and \( P \) (where \( N \) (Negative), \( Z \) (Zero) and \( P \) (Positive)). In Figure 4, \( \dot{y}_N \) is mapped onto the characteristic fuzzy sets. In this case, it was sufficient to describe the mapping by using only three membership functions.

The fuzzy sets for the damping factor, \( \delta \), presented in Figure 5, are characterized by four membership functions \( \mu_{S}, \mu_{M}, \mu_{L} \) and \( \mu_{EL} \) (where \( S \) (Small), \( M \) (Medium), \( L \) (Large) and \( EL \) (Extra Large)). This mapping was initially based on insight and previous work [10]. However, some fine-tuning, of the medium and large fuzzy sets, was required to further reduce settling times.

Fuzzy Rule-Base and Inference

The fuzzy adaptation strategy, presented in this effort, is based on rules inspired by "common sense" engineering reasoning whereby large values of the inputs require a lightly damped absorber, which would provide quick rise times. However, when the plant state is in the vicinity of the desired state the damping factor is large to reduce the overshoot and steady state error. The resulting rule-base that converts fuzzified inputs into a fuzzy output is presented in Table 2. For example, the rule described by the first row, first column, in Table 2, reads "if \( y_N \) is Negative AND \( \dot{y}_N \) is Positive, THEN the damping factor, \( \delta \), is Small".

As observed in Table 2, and is common practice in fuzzy logic control, the rule-base contains quite a few rules relating to the same output variable. Therefore, to obtain an overall output in the fuzzy state, an inference method is applied. First, the degree of fulfillment of each and every rule is found by applying the fuzzy "AND" operation. Let us represent the individual elements of the rule-base "matrix", presented in Table 2, as \( \delta_{ij} \) (for \( i=1,3; j=1,5 \)), where:

\[
\delta_{ij} = \text{Minimum}(\mu_{Q}, \mu_{L})
\]

(3)

for \( Q = \text{Positive, Zero and Negative} \).

L= Negative, Negative Small, Zero, Positive Small and Positive.
<table>
<thead>
<tr>
<th>( \dot{y}_N ) Positive</th>
<th>( y_N ) Negative SMALL</th>
<th>( y_N ) Zero</th>
<th>( y_N ) Positive SMALL</th>
<th>( y_N ) Positive</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMALL</td>
<td>MEDIUM</td>
<td>LARGE</td>
<td>MEDIUM</td>
<td>SMALL</td>
</tr>
<tr>
<td>MEDIUM</td>
<td>LARGE</td>
<td>EXTRA LARGE</td>
<td>LARGE</td>
<td>MEDIUM</td>
</tr>
<tr>
<td>SMALL</td>
<td>MEDIUM</td>
<td>LARGE</td>
<td>MEDIUM</td>
<td>SMALL</td>
</tr>
</tbody>
</table>

Table 2 - Rule-Base for computing the viscous damping factor (6)

Then, from Table 2, the union of the fuzzy sets for the same output variable is taken to reach the membership functions of the output, described in Figure 5, as follows:

\[
\mu_S = \delta_{11} + \delta_{12} + \delta_{31} + \delta_{32} + \delta_{35}; \\
\mu_M = \delta_{12} + \delta_{14} + \delta_{21} + \delta_{25} + \delta_{32} + \delta_{34}; \\
\mu_L = \delta_{13} + \delta_{22} + \delta_{24} + \delta_{33}; \\
\mu_{EL} = \delta_{23}; \\
\]

(4)

The rule-base, which was not made to be part of the tuning process, underwent only a single closed iteration to rid inactive rules. However, the sensitivity of closed-loop performance to changes in the rule-base was examined to verify the continued use of these heuristic rules.

**Defuzzification**

Finally, in order to reach a practical controller a control action comprising of a single numerical value is required. Therefore, the space of the fuzzy damping factor, obtained using the method described in the previous section, is mapped into a non-fuzzy space (crisp) by defuzzification.

There are various strategies aimed at producing a crisp value. Some of the commonly used strategies are the center of area (COA), the mean of maximum and the max. criterion(13). Since, there is no accepted systematic methodology for selecting a defuzzification strategy, herein, the COA scheme is adapted. This strategy was found to yield better steady-state performance when compared to the other above mentioned strategies. The COA method projects the centroid of the output membership function \( \mu_R \) (for \( R=S,M,L \) and EL), defined in Equation (4) as the crisp value of the output viscous damping factor, \( \delta \):

\[
\delta = \frac{\sum_{R} \mu_R \cdot A_R \cdot c_R}{\sum_{R} \mu_R \cdot A_R} \\
\]

(5)

where \( A_R \) - Area under the "R"th fuzzy set defined in Figure 5.

\( c_R \) - Centroid of the area \( A_R \).

\( R \) - Small, Medium, Large and Extra Large (See Figure 5).

**Results**

The closed-loop fuzzy based adaptive controller, developed in Phase One, is now applied to the nominal plant. After some tuning, to a variety of initial conditions, the values of the tuning parameters, \( N_y \) and \( N_y \) are frozen:

\[
N_y = 15 \ [1/m]; \quad N_y = 0.5[sec/m]; \\
\]

(6)

The closed-loop displacement impulse response is presented in Figure 6. The application of the fuzzy based control law results in relatively quick settling times, low overshoots and dying out of vibration within a few seconds. When the control force is turned off after 16 seconds (about 10 seconds after settling time), almost all the vibrational energy is dissipated as the beam returns to its undisturbed state throughout its length. In addition, the performance of the control algorithm is insensitive to varying initial conditions.
To demonstrate the robustness of the control system to changes in the temporal dynamics of the cantilever beam, the transient disturbance response to a considerably perturbed plant is simulated. The Young's modulus of the beam was raised as well as lowered by 60%, substantially perturbing the natural frequencies of vibration. Results of this robustness test, described in Cohen et. al.\(^{(11)}\), illustrate nearly similar settling times and rates of vibrational energy dissipation when compared to the nominal structure.

Next, the sensitivity of the AFCA to non-collocation of actuator/sensor is examined. Results presented in Figure 7 show that even though a marginal loss in settling time is obtained at this off-design point, the closed-loop performance, for the non-collocation, is reasonably good.

**Conclusions and Recommendations**

- The present effort describes the application of an adaptive fuzzy control algorithm (AFCA), based on adaptive dynamic vibration absorbers, on a large flexible beam-like structure

- The controller is applied to an Euler-Bernoulli model of a cantilever beam which is subjected to an initial unit impulse disturbance.

- Numerical results, based on MATLAB simulations, of the closed-loop transient response demonstrate quick settling times, a high rate of vibrational energy dissipation and no control spillover to the higher modes.

- The performance of the AFCA is not very sensitive to deviations (about 10% of beam length) concerning the collocation of actuator/sensor.

- The controller presented may further be developed and tested for the vibration suppression of structures such as lattice trusses having very high modal densities and which include coupling between the bending and twisting modes.

- Future research plans should include experimental validation, a stability analyses of the AFCA and performance comparisons with other classical control laws.

**References**


**Figures**

![Diagram](image_url)

**Figure 1**: A 2-D version of a beam-like lattice with an absorber attached
Figure 2: Response of the open-loop system
Figure 3: Membership Functions for the Fuzzy Sets Characterizing $y_N$.

Figure 4: Membership Functions for the Fuzzy Sets Characterizing $\dot{y}_N$.

Figure 5: Membership Functions for the Fuzzy Sets Characterizing $\delta$. 
Figure 6: Response of closed-loop system using collocated transducers
Figure 7: Response of closed-loop system using non-collocated transducers