LARGELY BENT WING FLUTTER

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Abstract

At present on bodies of revolution wraparound fins (or largely bent wings), as a rule, of cylindrical type are mounted . The flutter console forms of such wings can be determining flutter forms of vehicles both in subsonic, and in supersonic flow.

Report is devoted to advanced analysis of wing spanwise curvature influence on bent slender and high aspect-ratio wings flutter characteristics, on efficiency of actions application for flutter prevention . Spanwise wing curvature in paper in a set of presented results is resolved as self-parameter, which defines wings aeroelastic behavior. Simplified wing elastic-inertial scheme is used for parametric calculations.

In the report it is shown, that for slender wings the flutter curve behavior and aeroelastic oscillations type is different from similar behavior for high-aspect-ratio wings at increase of wing curvature.

The conclusions, that help on designing of safe from flutter spanwise bent slender wings, were made on the calculations basis.

The opportunity valuations to use simplified aerodynamic loads theories for largely bent wings flutter characteristics analyses are obtained, the example for bent wing flutter characteristics fast calculations on practice is given.

Introduction

At present the task about flutter console forms of wraparound, largely bent (of cylindrical type) wings both in subsonic, and in supersonic flow is actual.

Bent wings flutter task decision has several features. First, design of wing is three-dimensional and, in difference from flat wing, at small oscillations the wing points can deflect not only in normal, but also in tangential and in plan directions relatively undeformed wing position. Second, for exact account of three-dimensional space position of wing different sections it is necessary to apply the advanced aerodynamic theory. Third, in manufacturing the significant differences of elastic and inertial characteristics of one wings type are observed, the numerous modifications of wings are developed. As a result, it is necessary to take into account the influence of large number of parameters on wing flutter characteristics.

Bent wing features are stipulated by the fact, that excepting known stiffness and inertial parameters of wing design the wing spanwise curvature, new, not encountered earlier parameter, will influence on bent wing flutter characteristics.

Report is devoted to advanced analysis of wing spanwise curvature influence on bent slender and high aspect-ratio wings flutter characteristics, on efficiency of actions application for flutter prevention .

For flutter analysis simplified bent wing mathematical model using polynomial method is used. To simulate unsteady aerodynamic loads, acting on bent wing in subsonic flow the distributed douplet method, advanced for system of thin lifting surfaces, located in different planes is used [(1)]. In supersonic flow the linear theory of thin body in compressible flow with correction of relative position of aerodynamic focus and lifting force derivative by the angle of attack in sections of wing is used [(2)]. The precision of elastic-inertial and aerodynamic wing model was sufficient to main made in paper conclusions.

Using modal analysis, flutter equation complex roots analysis in the report it is shown, that for slender wings the flutter curve behavior and aeroelastic oscillations type is different from similar behavior for high-aspect-ratio wings at increase of wing curvature.

On the calculations basis the conclusions-recommendations on designing of safe from flutter spanwise bent slender wings are given. The opportunity valuations to use simplified aerodynamic loads theories for largely bent wings flutter characteristics analyses are obtained, the example for bent wing flutter characteristics fast calculations in supersonic flow is given.

Flutter calculations are executed with use of specialized computer program, developed by report author to fast parametric calculations of largely bent wings aeroelastic characteristics .

Mathematical model

Elastic-inertial scheme

The wing is defined as a set of boxes, fig.1(a). Every box is absolutely rigid body. Boxes are connected by elastic elements (springs), which model elastic forces of press-stretch, shift, bending, torsion, fig.1(b). Local coordinate system $x_i\ y_i\ z_i\ o_i$ is linked to every i-th box. In the problem about oscillations dx_i, dy_i, dz_i, $d\theta_1^i$, $d\theta_2^i$, $d\theta_3^i$ are taken as generalized coordinates (degrees of freedom), where i=1,...,N and degrees of freedom are small displacements of boxes centers coordinates and rotation angles (N - number of boxes) relatively local linked to boxes coordinates systems.

Free wing oscillations are described by equation:

$$[A]\cdot\{\ddot{q}\} + [G]\cdot\{q\} = \{0\} \quad (1)$$

where [A],[G] - matrices of system inertia and stiffness. $\{q\}^T$ - the column of generalized coordinates, upper point denotes time derivative, and vector-string $\{q\}^T$ has form:

$$\{q\}^T=\{dx_1, dy_1, dz_1, d\theta_1^1, d\theta_2^1, d\theta_3^1, \ldots, dx_N, dy_N, dz_N, d\theta_1^N, d\theta_2^N, d\theta_3^N\}$$

The form of symmetrical matrices [A] and [G] is close to diagonal form, and therefor it is possible to use memory economical and time-economical numeric algorithms. To model low eigen frequencies of beam with changing cross-section the springs stiffnesses (diagonal elements of some $[\mathbf{S^i}]$-matrix for each from N boxes) it is necessary to take as follows:

$$S^i_{11} = (l_i)^{-2}\int EF dx,\quad S^i_{22} = 2S^i_{11},\quad S^i_{33} = 2S^i_{11},$$
$$S^i_{44} = (l_i)^{-2}\int GI_{kp}(1+1/2N)^2 dx,$$
$$S^i_{55} = (l_i)^{-2}\int EI_y(1+1/N)^2 dx, \quad (2)$$
$$S^i_{66} = (l_i)^{-2}\int EI_z(1+1/N)^2 dx$$

where EI_y, EI_z bending beam stiffnesses relatively y_i and z_i axes; GJ_{kp} - torsion beam stiffness, EF - press-stretch stiffness, N - number of boxes, $\boldsymbol{l_i}$ - the length of i-th box. Integration is executed over the beam part, which is substituted by the box. Comparison of eigen modes of symmetrical console beam using beam theory and using described above model shows, that at N > 5 the calculations using presented model gives good agreement in low bending and torsion frequencies with beam theory results. Six-components springs, which connect boxes, are located in boxes rigidity centers.

When simulation low eigen modes of plate the formulas to calculate $[\mathbf{S^i}]$ -matrix elements are differed from similar formulas of thin-section beam. In flutter calculations of real bent slender wings, as a rule, it is necessary to take into account characteristics of two lowest eigen modes of wings: lowest bending mode and lowest torsion mode. It can be shown, that to simulate lowest eigen modes of rectangular plates-wings at small, close to one wings aspect-ratios the calculation of $[\mathbf{S^i}]$-matrix elements as follows:

$$S^i_{11} = (l_i)^{-2}\int Ehb dx,\ S^i_{22} = 2S^i_{11},\ S^i_{33} = 2S^i_{11},$$
$$S^i_{44} = (l_i)^{-2}\int (1/3)Gh^3(1+1/2N)^2(1-th(4\lambda^*)/(4\lambda^*))^{-2} b dx,$$
$$\lambda^* = l/b\sqrt{1{,}5(1-\mu)}, \quad (3)$$
$$S^i_{55} = (l_i)^{-2}\int (1/12)Eh^3(1+1/N)^2 h dx,$$
$$S^i_{66} = (l_i)^{-2}\int (1/12)Eh^3(1-\mu^2)^{-1}(1+1/N)^2 b dx$$

(***l, b, h*** - length, chord, thickness of plate-wing, E, G - Young's and shift modulus, μ - Poisson coefficient) results in satisfactory agreement of three low eigen frequencies with the respect to console plate eigen frequencies, but the form of oscillations remains as in beam, without wing chord deformation.

For slender wings, as it is shown in the control calculations, the difference in values of flutter ram and frequencies, calculated with use of polynomial method, taking into account chord deformations, and values, calculated using described above, simplified scheme underestimates flutter ram relatively more precise scheme [3]. This precision for parametric flutter calculations is satisfactory.

Aerodynamic loads

Using developed wing scheme as a system of rigid boxes, on the basis of aerodynamic loads to the system of flat surfaces, located in different planes (distributed douplet method) [1] , there are calculated so-called aerodynamic stiffness and damping matrices [$\underline{B}$], [$\underline{D}$]. At finite value of reduced frequency k calculation of [$\underline{B}$] and [$\underline{D}$] matrices in flutter problem is performed as follows:

$$[\underline{B}] = -0{,}5 \cdot \rho \upsilon^2 [F]^T \{[L_1][F_M'] + (k/b)[L_2][F_M]\},$$
$$[\underline{D}] = -0{,}5 \cdot \rho \upsilon [F]^T \{[L_1][F_M] - (b/k)[L_2][F_M']\}. \quad (4)$$

When using so-called harmonic theory ($k \to 0$) and quasisteady theory the formulas are more and more simplified. Adopted here notations are: ρ, υ - airflow density and speed. $[F]$,$[F_M]$,$[F_M']$ - matrices, depending upon wing form, douplets and boundary points location, deformations at low modes oscillations, b - typical chord length, $[L_1]$,$[L_2]$ - aerodynamic influence matrices.

Aerodynamic stiffness and damping matrices in subsonic (M<1) and supersonic (M>1) flow can be calculated also with use of plane sections theory [4]. At M<1 it is used quasisteady theory [4], at M>1 - harmonic theory [2] (for rigid wing in plane supersonic flow). In both cases it is possible to introduce experimental values of aerodynamic focus location in section "s" of wing $x_f = x_f(s)$ and values of lifting force coefficient derivative in section "s" of wing $C_y^\alpha = C_y^\alpha(s)$. Therefor, if in section "s" the distance from leading edge to rotation axes $x_0 = x_0(s)$, chord $b = b(s)$, rotation axe displacement $w_i = w_i(s)$ and angle of rotation $\theta_i = \theta_i(s)$ of section for i-th low eigen modes, are known, then [$\underline{B}$],[$\underline{D}$] matrices elements can be calculated as follows:

$$b_{kj} = -\frac{\rho \upsilon^2}{2} \int (\theta_j(s)\, w_k(s)\, Y^\theta + \theta_j(s)\, \theta_k(s)\, M^\theta) ds,$$
$$d_{kj} = -\frac{\rho \upsilon}{2} \int (\theta_j(s)\, w_k(s)\, Y^{\dot\theta} + w_j(s)\, w_k(s)\, Y^{\dot W} + \quad (5)$$
$$+ \theta_j(s)\, \theta_k(s)\, M^{\dot\theta} + w_j(s)\, \theta_k(s)\, M^{\dot W}) ds,$$

where $Y^\theta = C_y^\alpha b$, $Y^{\dot\theta} = -C_y^\alpha(-b(x_f - x_0) - m_1 b^2)$,

$Y^{\dot W} = -C_y^\alpha b$,

$M^\theta = C_y^\alpha b(x_0 - x_f)$, $M^{\dot W} = C_y^\alpha b(x_f - x_0)$,

$M^{\dot\theta} = -C_y^\alpha \{m_0 b^3 + m_1 b^2 (x_f - x_0) + b(x_0 - x_f)^2\}$;

at $M < 1$: $m_0 = \pi / (8 C_y^\alpha)$, $m_1 = 0{,}5$;

at $M > 1$: $m_0 = (M^2 - 2)/(12(M^2 - 1))$,

$m_1 = -0{,}5/(M^2 - 1)$

<u>Flutter equation</u>

The flutter equation form, as equation of wing disturbed motion, is shown below. Galerkin's method is used. Low wing eigen modes, orthonormalized to wing inertia matrix, are taken as prescribed forms.

$$\underline{\lambda}^2 [C]\{\underline{u}\} + \underline{\lambda}([H] + (\Delta/MV)[D])\{\underline{u}\} + ([K] + \Delta[B])\{\underline{u}\} = \{0\} \quad (6)$$

In equation (6) inertia matrix [C], structure stiffness matrix [K], structure damping matrix [H] are diagonal. The Δ parameter is value of critical ram, M -Mach number, V -sound speed,{$\underline{u}$}- reduced equation vector, [B] and [D] - aerodynamic stiffness and damping matrices without extracted values Δ and (Δ/MV). Instability in flow becomes possible, when some equation complex root $\underline{\lambda}$ ($\underline{\lambda} = \delta + i\omega$) changes real part sign from negative to positive. The ω value is therefor frequency of low bound of dynamic instability region. If $\omega \neq 0$, then instability has flutter type, if $\omega = 0$, then equation solution further will be called divergence.

<u>Computer program</u>

To parametric investigation of spanwise largely bent wings, on the basis of algorithms, briefly described above, the specializes computer routine F1PC was developed. After analysis of really existing wings structures the choice of bent wings main parameters was made. From main parameters values all necessary inertia and stiffness wings data are calculated, mathematical model to eigen modes calculations is formed, in dependence from type of aerodynamic theory the wing loads are defined, mathematical model of wing in flow is calculated. Initial data separation to main and depending ones, calculated from main data, allows to fast and effectively execute eigen modes parametric investigations and investigations of wing in flow stability region bounds on the plane of every pair of main parameters of wing and flow. It need be noted, that wing and flow parameters are equivalent in that sense, that every of them can be or argument or function when instability bounds are calculated. This fact and possibility to be restricted by small number of boxes by which the wing is modeled, allow fast and effectively execute bent wings parametric investigations using relatively low-power computers, for example, of the type IBM PC AT-386.

Parametric flutter investigations

Some results of bent wings parametric investigations of eigen modes, frequencies, critical ram and flutter frequency in subsonic and supersonic flow will be presented further in the paper. Influence of wing balancing, embedding parameter, in-plane wing form variation, wing aspect-ratio to wing flutter characteristics is investigated. Main flutter study results present critical ram as a function of wing curvature radius. Wing curvature radius influence to wing flutter characteristics is investigated using typical in-plane rectangular bent wing.

Arc length S_o, chord length b, curvature radius R, the distance b_1 between root chord leading edge and location of six-components spring, which simulates wing attachment, balancing mass m_{gr} coordinates S_{gr}, Z_{gr}, thickness h, thickness parameter dh/ds and some other parameters are taken as main parameters, fig.2.

Some undimensional parameters as wing aspect-ratio $\lambda=S_o/b$, undimensional curvature radius $\underline{R}=R/S_o$ (or curvature angle $\varphi=S_o/R$), undimensional wing thickness $\underline{c}=h/b$ undimensional by coordinate "s" wing thickness derivative $\underline{dh/ds}=(dh/ds)\cdot(S_o/h)$, undimensional coordinates $\underline{S}_{gr}= S_{gr}/S_o$, $\underline{Z}_{gr}=Z_{gr}/b$, and balancing mass $\underline{m}_{gr}=m_{gr}/M_{wing}$, ($M_{wing}$- wing mass) undimensional wing attachment location $\underline{b}_1=b_1/b$ and some other parameters are taken to analysis.

Low eigen modes and frequencies as a functions of φ parameter (or $\underline{R}$ parameter) are investigated in paper for cantilever attached slender wing ($\lambda=1$, $\underline{c}=0{,}01$, $\underline{b}_1=0{,}5$). To used wraparound folding slender wings the $\underline{R}$ parameter must be limited as follows: $1/\pi<\underline{R}<\infty$ ($0 < \varphi < \pi$). For the $\underline{R}$ range that is given by first above statement the first, bending mode frequency, is increased and second, torsion mode frequency, is decreased at $\underline{R}$ parameter decreasing (φ increasing), fig.5. Such low frequencies behavior concerned with including to wing oscillating process not only normal to wing surface displacements v, but also tangential displacements u (along spanwise direction) and in-plane displacements w (along flow direction). Behavior of u, v, w displacements amplitude components for bent wing leading edge points for low eigen modes in dependence form φ parameter is shown on fig.3 and fig.4.

Eigen modes and frequencies variation at $\underline{R}$ diminishing causes to result, that at ($\underline{R}$, q) parameters plane the flutter region is closed, fig.6 (q - undimensional value, $q=\Delta/\Delta_o$, where Δ- bent wing flutter ram, Δ_o - plane wing flatter ram, span of plane wing is equal to arc length S_o of bent wing).

Figure 6 and following figures results are obtained at Mach number M=0,5 , using distributed douplet method. In calculations the wing was divided into 6 chordwise parts and 10 spanwise parts, therefor 60 panels were used. It was supposed, that damping in wing structure is absent. Flutter is caused by low modes in-flow interaction. At $\underline{R}$ diminishing the distinct q increasing is seen at some radius $\underline{R}_A$. This radius is marked by prominent in-plane wing motions including to wing oscillations. Flutter region closing is seen at $\underline{R}_B$ radius. This radius is marked by prominent tangential, spanwise wing motions including to wing oscillations.

Behavior consideration of undimensional values of aerodynamic stiffness b_{ik} and aerodynamic damping d_{ik} matrix coefficients and structure stiffness matrix coefficients k_{ii}, i,k=1,2 (fig.7) shows remarkable diminishing of b_{ik}, d_{ik} values in range $0{,}3<\underline{R}<1{,}3$ ($3{,}3>\varphi>0{,}79$) at $\underline{R}$ diminishing. Matrix coefficients are normalized to corresponding plane wing matrix coefficients ($\underline{R}=\infty$, $\varphi=0$). As a result, at $\underline{R}$ diminishing (φ increasing) the aerodynamic connectedness of oscillating modes in flow falls, "negative" aerodynamic stiffness is diminished, equation (6) roots trajectories on complex plane become "less dynamic", fig.8. The changing of real part of equation (6) unstable complex root, which is corresponded to torsion mode, becomes more slow at q increasing. fig.9. As a result the flatter phenomenon becomes impossible (flatter region is closed).

On the basis of stated above results the following conclusions can be made: it must be flutter ram q increasing (relatively plane wing flutter ram) at $\underline{R}<\underline{R}_A$ and remarkable flutter ram - increasing at $\underline{R}\approx\underline{R}_B$ for bent slender wings as a result of wing curvature increasing. This increasing is caused by prominent including of tangential and in-plane wing sections motion to wing oscillation process.

Influence of undimensional curvature radius $\underline{R}$ to bent wing critical ram in subsonic flow in dependence with aerodynamic loads type, using distributed douplet method is considered further. Calculations, executed using exact reduced frequency value k, and calculations, executed using harmonic theory, give the same results. This is explained by low reduced frequency value k=0,35 and small wing aspect-ratio value. It need be noted too, that calculations using quasisteady theory give lower flutter ram than calculations using harmonic theory. And this underrating is more large at low wing curvature radius. Body influence account in aerodynamic loads calculations results in shifting of bounds of flutter region closing to lower $\underline{R}$ value and in more difference between flutter ram q calculations results using quasisteady theory (curve 1, fig.10) and q calculations results using harmonic theory (curve 2, fig.10) in region of low $\underline{R}$. It is established, that usage of simplified, "flat" aerodynamic theory (curve 3, fig. 10) gives overrating of flutter ram in range of low $\underline{R}$ (relatively calculations with use of harmonic theory). In "flat" aerodynamic theory it takes into account the only bent wing aspect-ratio, the wing is considered flat, unbent, different wing sections orientation is not accounted.

As known, usage of antiflutter weight-balance is one of the main ways to wing flutter prevention. Critical ram q as a function of $\underline{R}$ for superbalanced, disbalanced wing and without balance wing is shown on fig.11. The wing, marked as 1, corresponds to bent wing with balance weight $\underline{m}_{gr}$=0,06 which has coordinates $\underline{S}_{gr}$=1,0, $\underline{Z}_{gr}$=1,0 (disbalancing), curve, marked as 2, corresponds to without balance wing, curve 3 corresponds to wing with balance $\underline{m}_{gr}$=0,06, which has coordinates $\underline{S}_{gr}$=1,0, $\underline{Z}_{gr}$=0,0 (superbalancing). As it is seen from presented results, superbalancing leads to the flutter region rising and closing at more high $\underline{R}$ values. Conclusion can be made, that for slender wings in subsonic flow superbalancing effectiveness at low $\underline{R}$ is increased.

Calculations with balance $\underline{m}_{gr}$=0,2, which is located in the middle of tip wing section ($\underline{S}_{gr}$=1,0, $\underline{Z}_{gr}$=0,5, curve 4, fig.11) show, that balance effectiveness is decreased at $\underline{R}$ diminishing. These calculations and analysis of low eigen frequencies behavior at $\underline{R}$ diminishing and corresponding analysis of main type of wing motion allow to make a conclusion, that additional including of in-plane wing motions to wing oscillating process results in bent slender wing flutter characteristics decline.

The influence of wing attachment location to critical ram q can be analyzed by the way of changing the point location of the first spring element, by which wing attachment is simulated, in mathematical model, that is variation of $\underline{b}_1$ parameter. As it follows from fig.12, the shift of bent wing attachment location point to the back sufficiently increases flutter ram, especially at range 0,4<$\underline{R}$<0,6, that is explained by zero normal displacement line shift in torsion mode to back, more closely to wing trailing edge and therefor by wing balancing effect.

For bent rectangular in plane form slender wing in subsonic flow there is one more way of console flutter ram increasing. This way is changing of in plane wing form by cut of wing tip sections trailing edge. Calculations were executed to evaluate cut efficiency. In calculations it was adopted, that wing is cut from trailing edge by rectangular parts, beginning from 0,75 span chord (S_2/S_o=0,75), the uncut wing part is characterized by b_2 parameter ($\underline{b}_2=b_2/b$), fig.2. As it follows from fig.13, the wing cutting acts more effectively to flutter ram in the range 0,4 <$\underline{R}$<0,7. The eigen modes and frequencies analysis shows, that flutter ram increasing is caused by balancing effect due to wing tip sections uncut parts. As a result, the following conclusions can be made: wing part cutting may be effective measure to prevent bent wing console flutter form, at low curvatures radiuses the cutting effectiveness is increased.

To evaluate curvature influence to bent wing flutter characteristics in supersonic flow the calculations were executed, in which pressure distribution in wing sections was taken from pressure distribution in tip sections of in plane rectangular flat wing at Mach number M=1,75. The behavior of flutter region bound in supersonic flow replicates bound behavior in subsonic flow, fig.14. Aerodynamic stiffness and damping matrices coefficients behavior at $\underline{R}$ variation in subsonic flow is similar to one in supersonic flow. Therefore the main conclusions about flutter prevention measures effectiveness are the same ones for bent slender wings both in subsonic and in supersonic flow.

Aspect ratio λ influence to bent wings flutter characteristics was analyzed using a set of thin wings: λ=1 ($\underline{c}$=0,01), λ=3 ($\underline{c}$=0,01), λ=7 ($\underline{c}$=0,033). It was recognized, that relatively larger diminishing of the torsion mode frequency at wing curvature increasing ($\underline{R}$ decreasing, φ increasing) is caused by larger addition of the in plane wing motion for high aspect ratio wings. As a result, at $\underline{R}$ diminishing there is falling of flutter ram in difference from slender wing ram, fig.15 (wing has λ=3). The wing aspect-ratio is larger, the flutter ram falling is stronger. For wings with λ<3,4 the flutter is caused by in flow interaction of two, lowest bending and torsion modes (first flutter form), fig.15. For high aspect-ratio wings, for example λ=7, at small wing curvature, the flutter is caused by in flow interaction of second bending and first torsion modes (second flutter form), fig.16. At large wing curvature the first flutter form is the main one again, fig.16. Such flutter bounds behavior, existence of second flutter form is related with eigen frequencies variation, which is shown at fig.17. Wing curvature increasing tends to torsion mode frequency falling lower than second bending mode frequency. Qualitative change of frequencies values tends to disappearing of the second flutter form.

Thus, at wings aspect-ratio increasing the wing curvature, as a rule, causes to flutter ram diminishing, therefor it is adverse factor.

Calculations and experimental results comparison

The console flutter form of cylindrical, bent, missile wings at Mach number M=1,75 is considered below. Analysis and comparison of the parametric calculations results and supersonic wind-tunnel tests are given. Four wings with parameters λ=0,62, ($\underline{c}$=0,0025÷0,01), $\underline{b}_1$=0,34, $\underline{dh/ds}$=-0,75, $\underline{R}$=0,6 and four shorter chord wings with parameters λ=0,83; $\underline{c}$=0,003÷0,013, $\underline{b}_1$=0,5, $\underline{dh/ds}$=-0,75, $\underline{R}$=0,6 were taken into consideration.

Elastic-inertial wings scheme was corrected using ground vibration tests results. Aerodynamic loads to vibrating wing were given according to formulas (5) at M>1. Spanwise lifting force characteristic $\underline{C}_y^\alpha=\underline{C}_y^\alpha(s)$ distribution was given following the pressure difference distribution in wing tip sections in and out Mach cone for in plane rectangular flat wing at Mach number M=1,75 and was corrected using "rigid" wings wind-tunnel tests as follows:

$$\mathbf{C_y}^\alpha\int\underline{C}_y^\alpha\cos(\beta)dF = C_y{}^\alpha{}_{ex}\int\cos(\beta)dF, \quad (7)$$

where $C_y{}^\alpha{}_{ex}$- experimental wing lift coefficient derivative, dF - undimensional wing area differential ($\int dF$=1,0), β - angle between current wing surface part normal vector **n** and average wing normal vector position **r**, fig.18. $\mathbf{C_y}^\alpha$- wing lift distribution scale. Aerodynamic focus position for all wing sections was taken the same and equal to x_f. It need be noted, that $\mathbf{C_y}^\alpha$=1,0 corresponds to $C_y{}^\alpha{}_{ex}$=2,8.

Despite of four wings similar type of the first group, they had some differences in bending f_1 and torsion f_2 eigen frequencies. In mathematical model these differences were taken into account by different bending and torsion wing attachment parameters K_1 and K_2. In wind-tunnel flutter tests at dynamic pressure increasing the N2 wing flutter was noted. Flutter and N2 wing parameters are: K_1=0,3, K_2=0,35, f_2/f_1=1,43, flutter frequency ϖ_2=1,0, ram q_2=1,0. Flutter ram was normalized to $\Delta=1,43\cdot10^5$ H/m^2, flutter frequency was normalized to ω=140Hz. Wing N4 (K_1=0,3, K_2=0,35, f_2/f_1=1,43) flutter appeared at q_4=1,13 and ϖ_4=0,99. Wing N3 (K_1=0,35, K_2=0,6, f_2/f_1=1,48) flutter appeared at q_3=1,25 ÷1,3 and ϖ_3=1,11. Wing N1 (K_1=0,25, K_2=0,6, f_2/f_1=1,72) did not subjected to flutter. Dynamic pressure range was up to q_{max}=1,51.

Wings flutter and flutter absence on wing N1 are correlated using single mathematical model and assumptions: wing parameters K_1 and K_2 are stable enough for in and out of flow wings, $\mathbf{C_y}^\alpha$ and x_f values for all wings have 0,1 scatter relatively similar wing N2 values.

According to assumptions q as a function of K_1 for all, four wings are shown at fig.19. Values of q_2, q_4, q_3 in correspondence with wings K_1 parameters are in calculated values q range, that corresponds to various $\mathbf{C_y}^\alpha$ and x_f combinations. Experimental flutter frequencies of 2-nd, 4-th, 3-rd wings are also in a good agreement with calculated ones. First wing calculated flutter ram are higher than maximum experimental dynamic pressure q_{max}. This is an explanation of flutter phenomenon absence for wing N1.

Using the same mathematical model, the flutter calculations of missile modified, with shorter chord , wings were executed. Wings mathematical model was corrected using ground vibration tests results too. Calculations shown, that these wings are flutter stable with sufficient safety margin, that was confirmed by additional supersonic wind-tunnel tests.

Therefore, developed wing mathematical model corrected by results of ground vibration tests and "rigid" wings wind-tunnel tests, allows to calculate flutter ram of cylindrical, bent, in-plane rectangular wraparound wings in supersonic flow with good accuracy for parametric investigations.

Conclusions

As a result of parametric calculations of curvature parameter influence to flutter characteristics of different aspect-ratio, largely bent wings it was ascertained, that for slender wings the spanwise wing curvature increasing causes the increasing flutter ram relatively unbent, flat wing both in subsonic and in supersonic flow. For high aspect-ratio wings in subsonic flow the spanwise wing curvature increasing causes the diminishing of wing flutter ram relatively flutter ram of unbent, flat wing.

Different console form flutter prevention measures were considered for largely bent folding slender wings. It was established, that wings weight balance effectiveness increases for small wings curvature radiuses. Wings flutter ram considerable increasing possibility is shown, when wings attachment location is shifted to trailing edge of wing root chord and also when tip sections of wings are cut at small wings curvature radiuses.

It was determined, that in subsonic flow underrating of flutter ram in calculations using quasisteady aerodynamic loads theory relatively calculations using unsteady aerodynamic loads theory is increased at small curvature radiuses for bent slender wings.

Calculations and experimental results for bent folding slender wings were compared. Good matching of console flutter form calculations results and wind tunnel flutter results was shown for bent folding wings in supersonic flow.

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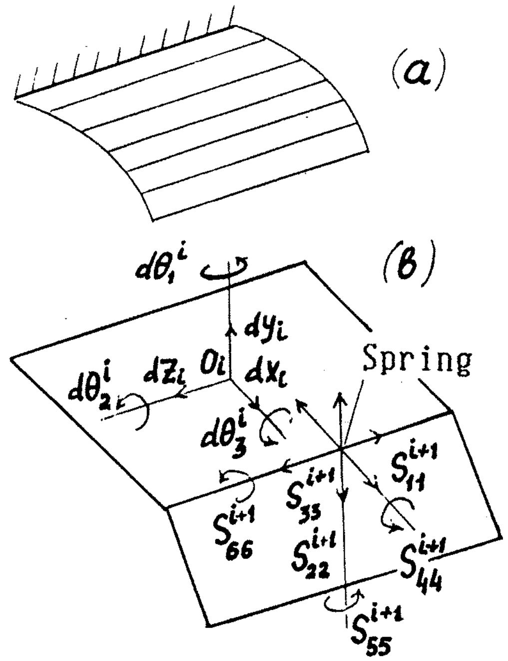


Figure 1

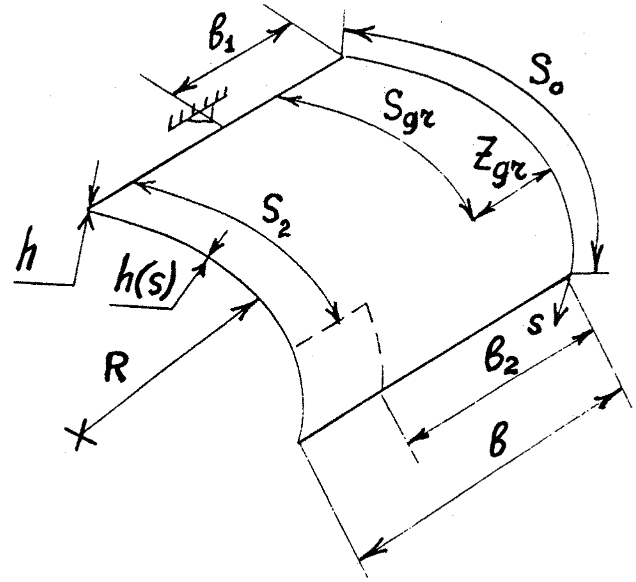


Figure 2

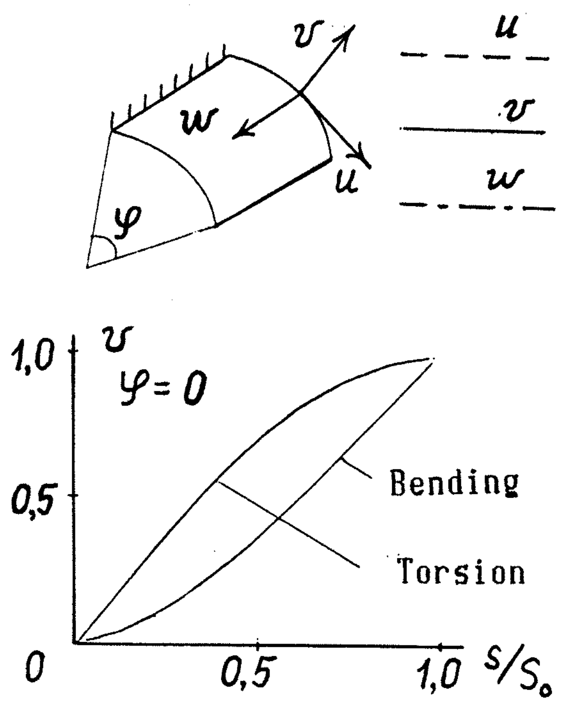
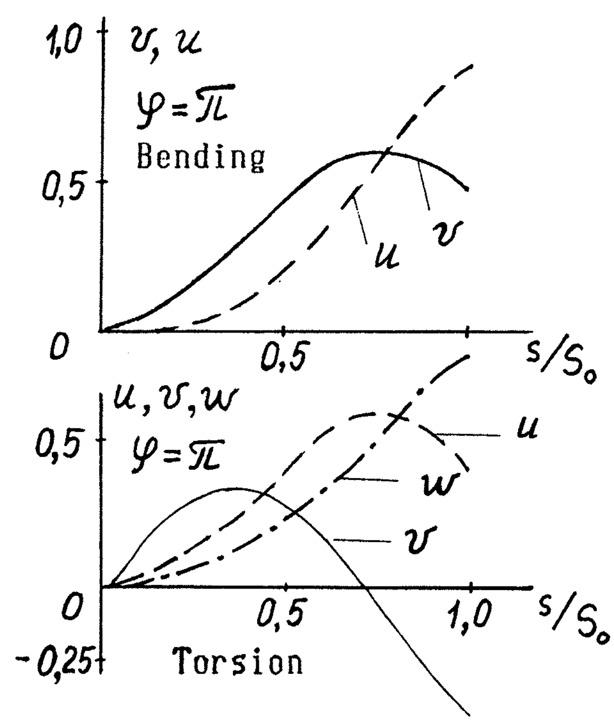


Figure 3



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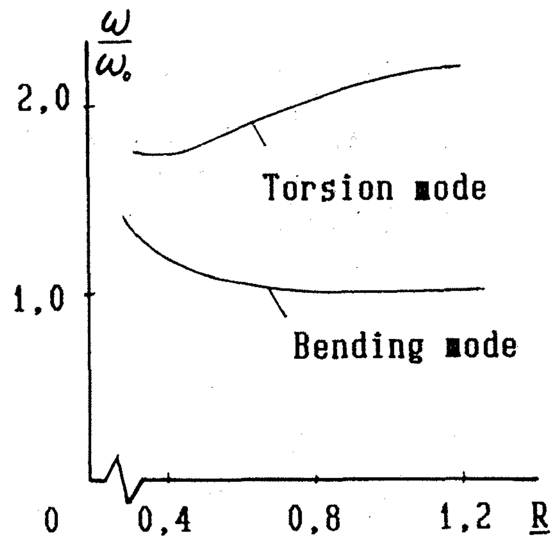

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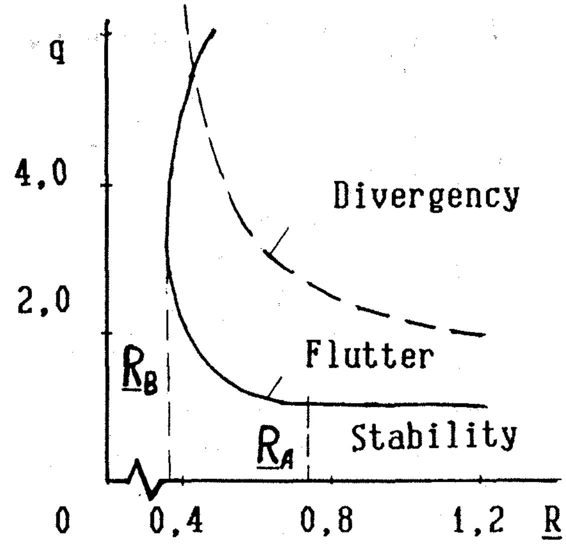

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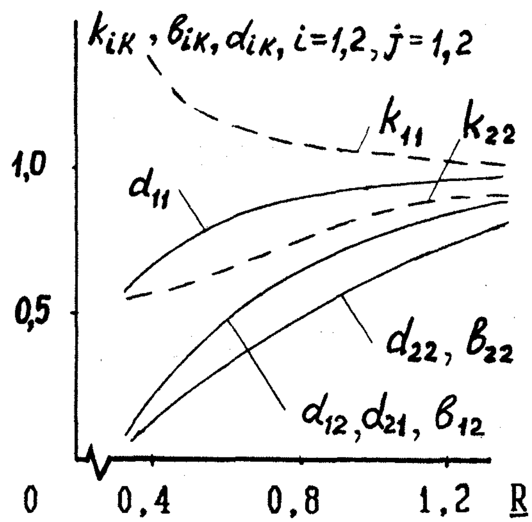

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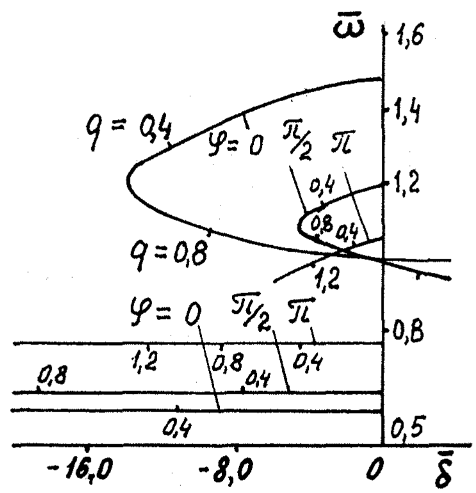

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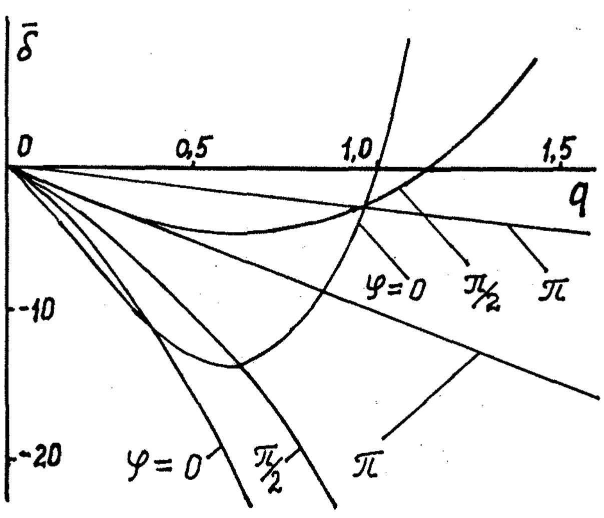


Figure 9

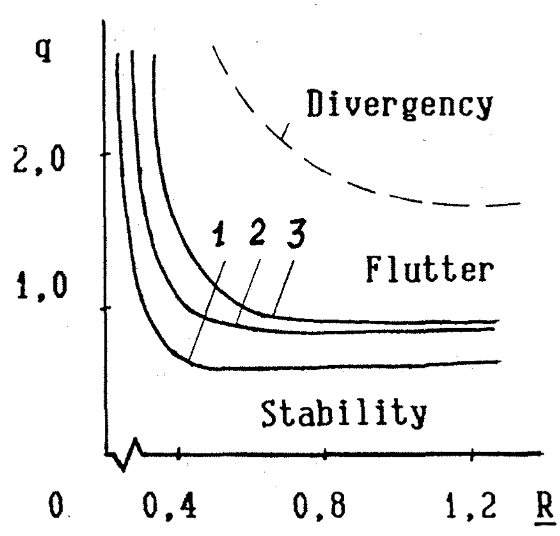
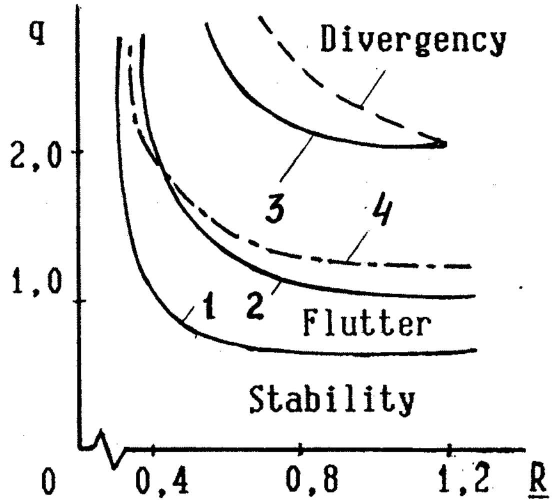


Figure 10



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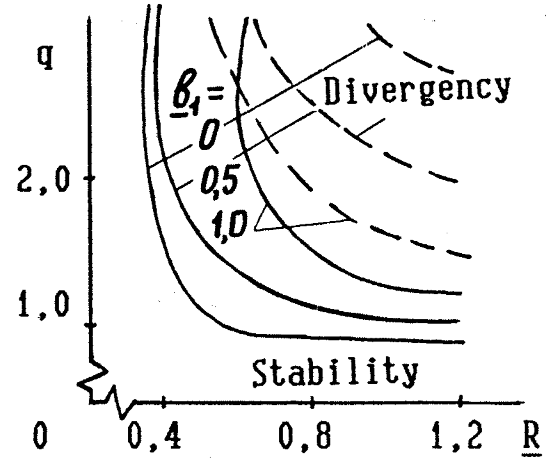

Figure 12

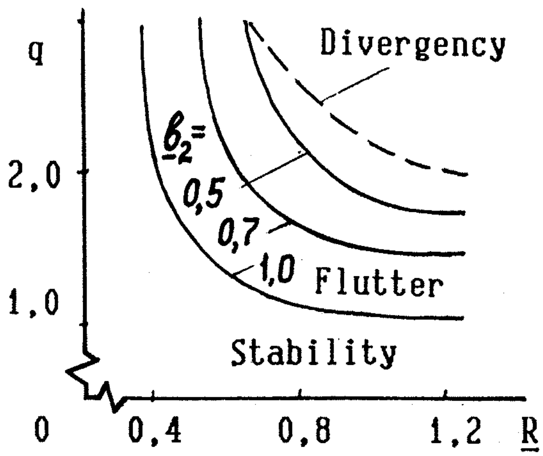

Figure 13

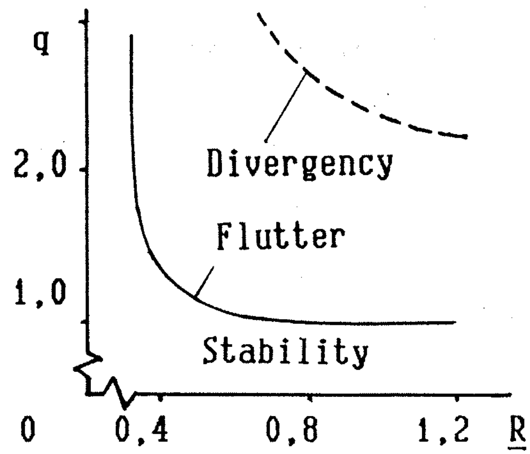

Figure 14

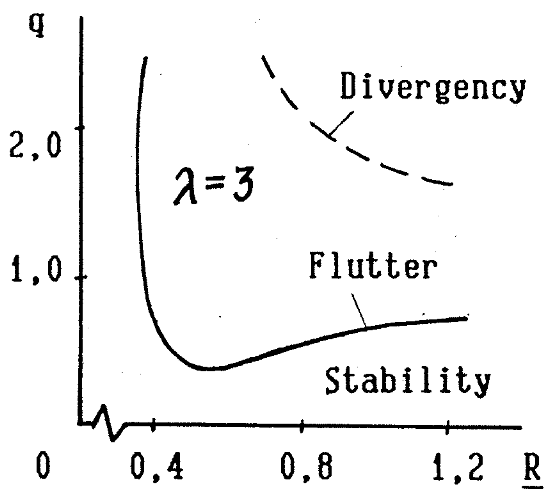

Figure 15

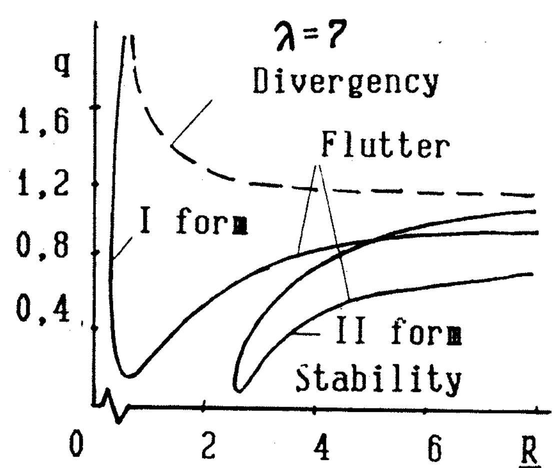

Figure 16

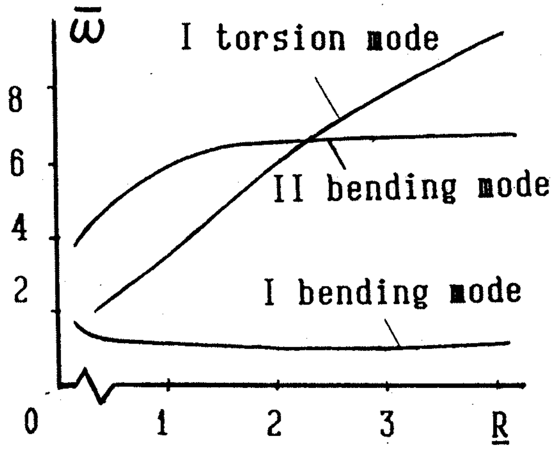

Figure 17

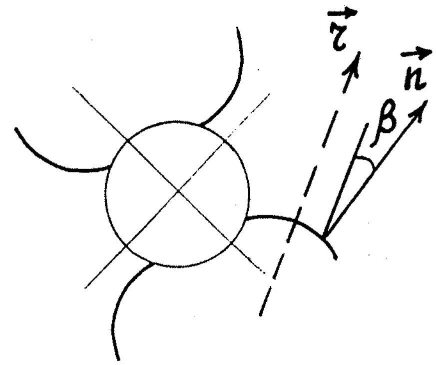

Figure 18

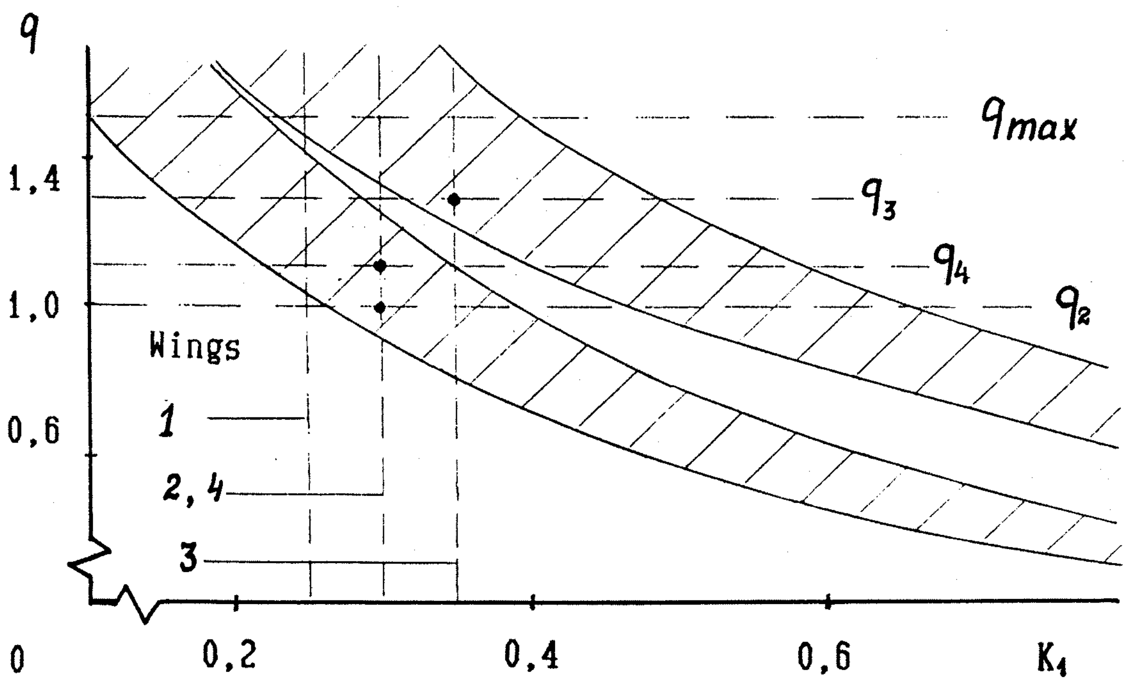

Figure 19