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OPTIMAL MIDCOURSE TRAJECTORIES FOR PRECISION GUIDED MUNITIONS WITH COLLOCATION AND NONLINEAR PROGRAMMING

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Abstract

The tool developed to optimize the midcourse trajectories for precision guided munitions is described. The tool uses collocation technique together with a standard nonlinear optimization package. The missile model has pseudo three-degrees-of-freedom. It incorporates all the nonlinearities from dynamics and aerodynamics. The tool is employed to solve the maximum range midcourse trajectories of a hypothetical air-to-surface missile. Trajectories obtained for two separate missions are given and discussed.

Introduction

Missiles and guided weapons have dramatic effect upon the accomplishment of the goals of armed forces. Many of them are in the inventories of the armed forces. Among them, air-to-surface missiles, guided bomb units or precision guided munitions in general are becoming more and more important. This importance of precision guided munitions in a battle field theater is demonstrated during the recent Gulf War. It is stated that with their increased precision to hit the target, these weapons may be viewed as strategic weapons, since they can penetrate the enemy air defense and destroy strategic targets with pinpoint accuracy (1).

The success of a precision guided munition depends on it is pre-planned trajectory. For example, when planning the flight path of a cruise missile it may be important to avoid certain areas where there is effective enemy air defense. Similarly, it may be important to prevent the identification of the launching aircraft. Thus, indirect firing may be preferred. On the other hand final impact conditions such as impact velocity, and impact angle are important for the successful destruction of the target. Too much impact angle may cause ricochet, or too little impact velocity may prevent penetration. All these issues should be addressed in the planning of the missile trajectory. With the ongoing developments on the guidance computers, and the emergence of new generation navigation systems, such as GPS aided INS, now it is possible to pre-program the way points at precise intervals and require the missile to reach its target as planned.

Trajectory planning and trajectory optimization has long been used for various aerospace vehicles. Examples can be found on rockets, spacecraft and aircraft (2,3,4,5,6,7). However, to the authors knowledge no work on air to surface missiles or bombs has been reported in the open literature. The purpose of this work reported here was to develop a tool which can generate optimal trajectories for missiles and guided munitions.

The missile dynamics is quite nonlinear. As well known these nonlinearities come both from the aerodynamics and dynamics of the missile(8). Consequently, it is difficult if not impossible to solve trajectory problems using calculus of variation techniques(9). As a result direct techniques of optimization should be used. In this work, collocation with nonlinear programming technique is chosen. The advantages of this technique over other direct techniques is discussed and demonstrated in the literature(10).

In this paper trajectory optimization of an air to surface missile is presented. First the model developed to be used for trajectory optimization is given, the solution technique used is briefly described, and the hypothetical missile employed in this study is presented. Finally, maximum range midcourse trajectories obtained using the hypothetical missile for two different missions with corresponding requirements on impact conditions are given and discussed.

Formulation of the Problem

In this study a pseudo three-degrees-of-freedom missile model is used. Assuming that the motion is only in the vertical plane, and neglecting the pitch equation(8) the following equations are derived:

$$\dot{r} = V\cos\gamma \qquad (1)$$

$$\dot{h} = V\sin\gamma \qquad (2)$$

$$\dot{V} = \frac{1}{m}\left(T\cos\alpha - D\right) - g\sin\gamma \qquad (3)$$

$$\dot{\gamma} = \frac{1}{mV}\left(T\sin\alpha + L\right) - \frac{g\cos\gamma}{V} \qquad (4)$$

$$\theta = \gamma + \alpha \qquad (5)$$

Here, L is the lift, T thrust, D drag, V total velocity of the missile, γ flight path angle, θ pitch angle, α angle of attack, r downrange and h altitude. These variables are shown in Figure 1. In the above equations the α is the input to the missile instead of the usual fin deflection. This angle of attack input may be realized by an angle of attack autopilot.

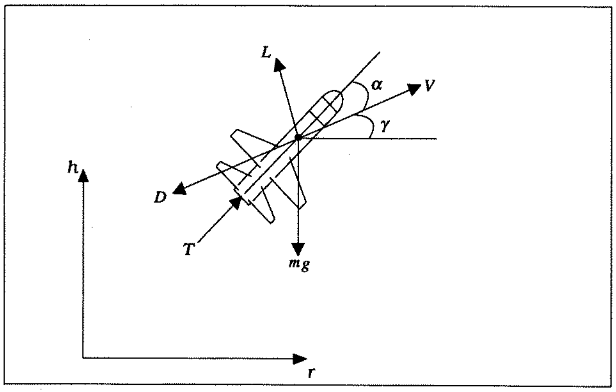

Figure 1. Pseudo three-degrees-of-freedom missile model

The missile mass changes linearly with time until burnout according to the following equation:

$$m = m_t - \left(\frac{t}{t_b}\right)m_p \qquad (6)$$

Here m_t is the total mass, m_p total propellant mass, t_b burnout time.

Method of Solution

In general an optimization problem may be formulated as follows

$$\underset{\boldsymbol{p}}{\text{maximize}} \quad J(\boldsymbol{p}) \qquad (7)$$

$$\text{subject to} \quad \boldsymbol{G}(\boldsymbol{p}) = \boldsymbol{0} \qquad (8)$$

$$\boldsymbol{h}_l \le \boldsymbol{H}(\boldsymbol{p}) \le \boldsymbol{h}_u \qquad (9)$$

Here J is the scalar cost function to be minimized (or maximized). $\boldsymbol{p}$ represents the vector containing various design variables. Eq. (8) indicates the vector of equality constraints. Eq. (9) represents the vector of inequality constraints.

Trajectory optimization problem is actually an optimum control problem(9). Fundamental difference between an optimal control and optimization problems are the differential equality constraints (i.e., equations of motion). These differential equality constraints may be converted into algebraic equality constraints in the form of Eq. (8) using various discretization techniques(10). Additionally, initial and terminal conditions on the state variables are also formulated as equality constraints. Inequality constraints are usually posed on inputs to the system, due to physical limitations. Other inequality constraints such as load factor or path constraints may also be specified in trajectory optimization problems. Consequently, vector $\boldsymbol{p}$ in the above equations include state, control and design variables. In a trajectory optimization problem, the cost function may be specified to maximize range, maximize impact velocity or in a missile design problem to minimize total weight.

The above equations of missile dynamics (1-4) may be written in state space form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \qquad (10)$$

where, $\boldsymbol{x} = \left[r, h, V, \gamma\right]^T$ and $\boldsymbol{u} = \left[\alpha\right]$. In this study these equations are discretized in time using collocation technique as described by Hargraves(10). The technique uses hermite cubic splines to calculate values of the states and their derivatives at the center of a section based on the neighboring left and right handside nodal values of the states and control inputs of the section. Values of these state derivatives, in general, will be different than the state derivatives calculated using $f(x,u)$. This difference is called defect (Δ_{ij}). These defects from each section are programmed in the optimization routine as equality constraints and forced to be equal to zero. Since the value of the control input at the middle of the section is not known, in this study, the control input at the middle of the section is taken as the average value of the inputs at the left and right handside neighbouring nodes of the section.

There are many off-the-shelf nonlinear optimization routines (11,12,13). In this study MatrixX Optimization routine is employed(13). All the routines are written in Xmath programming language and Systembuild(14). MatrixX Optimization module implements BFGS(15) algorithm to solve nonlinear programming problems. It has two loops: in the major iteration loop it converts the nonlinear problem into a linearly constrained nonlinear problem, in the minor iteration loop it converts the linearly constrained problem into a quadratic problem. The user should supply an Xmath cost function routine, a vector containing variables for equality and inequality constraints, and vectors containing limits on these constraints. User should also specify the values for a penalty parameter, number of major and minor iterations, and a tolerance on optimality or feasibility. The penalty parameter is updated automatically in the optimization code. Our experience indicated that penalty parameter should be chosen small initially. Otherwise convergence may not be achieved. Since the problem at hand is quite nonlinear, the number of minor iterations are also taken small. Large number of minor iterations did not offer any improvements.

The flow chart of the program that finds optimal flight trajectories is given in Figure 2. Program listings are given in a departmental report

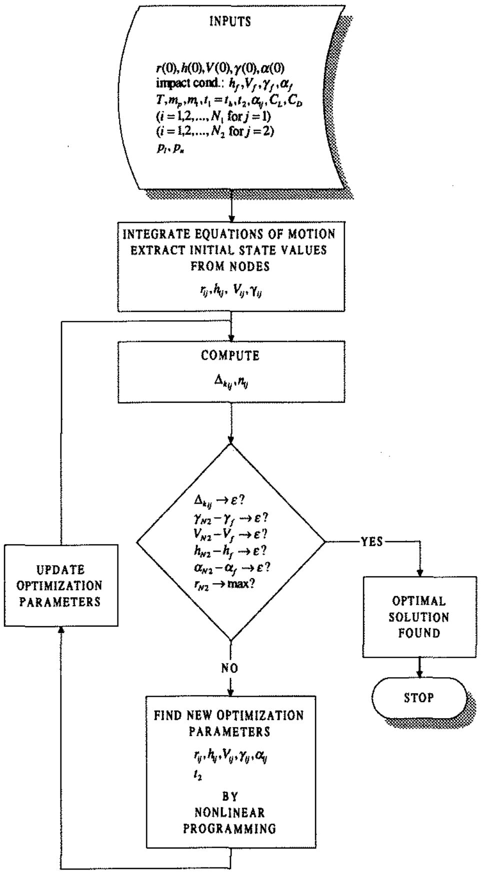

Figure 2. Flowchart of the trajectory optimization tool developed.

A Hypothetical Air to Surface Missile(16)

For this study a hypothetical missile configuration is selected. The configuration of this hypothetical missile is shown in Figure 3. Some parameters for the missile used in this study are listed in Table 1. The missile is assumed to have an end burning solid propellant rocket motor. Lift and drag coefficients of this missile are generated in a tabular form using Missile DATCOM(17) software and bicubicly interpolated and substituted to the equations during simulation according to the current angle of attack and Mach number. In the model atmospheric properties are also varied with altitude as obtained from Missile DATCOM. More complete description of the this missile is given in the departmental report(16).

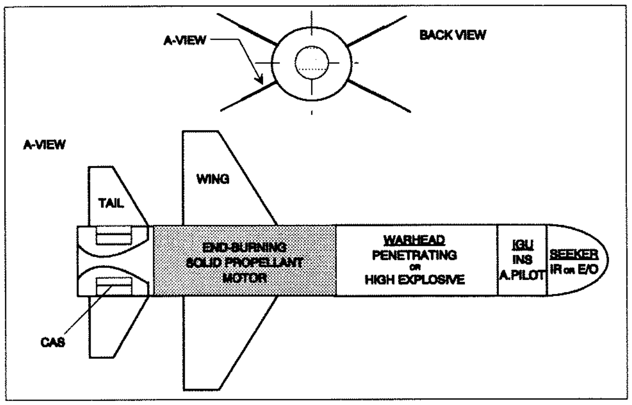

Figure 3. Configuration of the hypothetical missile used in this study

T	1450 lbf
t_b	130 sec
m_t	2580 lb
m_p	745 lb

Table 1. Major missile parameters used in the study

Results and Discussion

The missile trajectory optimization tool described above is used to find the maximum range trajectories of the hypothetical air to surface missile. For this purpose two separate missions are tested. It is assumed that the missile warhead can be changed according to the mission. The first mission requires a penetrating warhead (PW). Consequently, the end conditions should be suitable for the penetrating warhead. The second mission requires a high explosive warhead (HEW). For penetrating warhead the missile impact velocity and impact angle are taken to be larger than the HEW warhead. The initial conditions for both missions are the same and are listed in Table 2. According to the table missile is released at an altitude of 40 000 ft above sea level, with initial velocity of 0.9 Mach, and an angle of 5 degrees.

V_o	0.9 Mach
γ_o	5°
h_o	40000 ft
r_o	0 Nmi

Table 2. Missile launch conditions from the aircraft

To start optimization initial state and control variables are required. For this purpose the missile flight is divided into two stages. First stage is the powered flight. It lasts 130 seconds. This stage is divided into 20 segments. Initial angle of attack at this stage is given as + 7 degrees. The second stage is the flight after burnout. This stage is divided into 30 segments. Initial angle of attack is taken as -1 degrees in this stage. The flight time is assumed to be 70 seconds after burnout. To obtain the values of state variables at each node (r_{0i}, h_{0i}, V_{0i}, γ_{0i}) a simulation is conducted using the above values of the initial control variables, and flight times. The simulation results, for two of the four state variables, altitude versus range is given in Figure 4. Missiles final impact velocity is 1.18 Mach, and flight path angle at impact is -60 degrees.

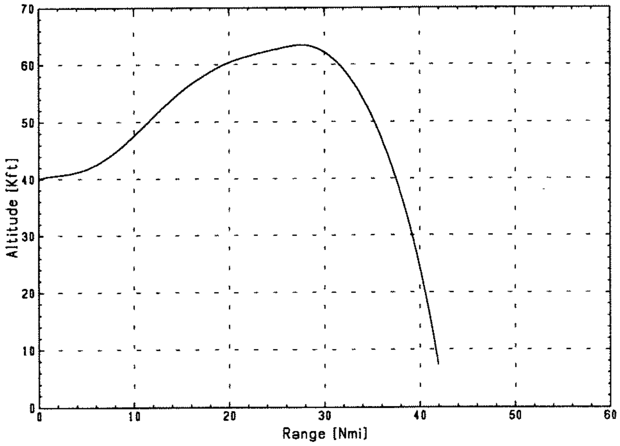

Figure 4. Initial altitude and range values as supplied to optimization

Required impact conditions for mission 1 which carries a PW configuration are listed in Table 3. The resulting optimum impact conditions calculated by the optimization tool are given in Table 4.

V_f	1.15 Mach
γ_f	-70°
α_f	0°
h_f	3200 ft

Table 3. Impact conditions specified for mission 1

r_{N2}	39.7 Nmi
V_{N2}	1.15 Mach
γ_{N2}	-70 °
α_{N2}	0°
h_{N2}	3200 ft
t_2	72.8 sec

Table 4. Impact conditions obtained for maximum range PW configuration missile

Convergence of the trajectory to the optimum one is shown in Figure 5. The figure legend indicates the major iteration number.

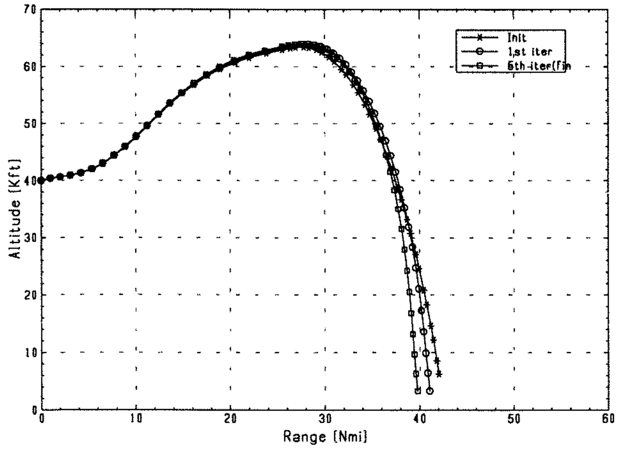

Figure 5. Convergence to the final trajectory for PW missile.

Comparing Tables 3 and 4 show that the impact conditions on optimized trajectory exactly matches the constraints. The final range is 39.7 Nmi, and flight time after burnout is calculated to be 72.8 s. The optimum angle of attack history is plotted in Figure 6. Since, the required impact angle is larger than the initial impact angle the optimized range for PW configuration missile is smaller than the initial guess. Also, it should be remembered that the flight for initial guess is terminated at 6000 ft, whereas the requirement for mission 1 was 3200 ft.

Since, the optimization routine incorporates the equality constraints, including the defects, in the cost function, it is advisable to compare the resulting state variables obtained from optimization with actual simulation results. This comparison is given in Figure 7. It can be observed from the figure that an excellant match between the optimization results (i.e collocation, in the figure), and actual integration of the equations using the optimum angle of attack history given in Figure 6. Similarly, excellent match is obtained for other state variables such as mach number and flight path angle$^{(18)}$.

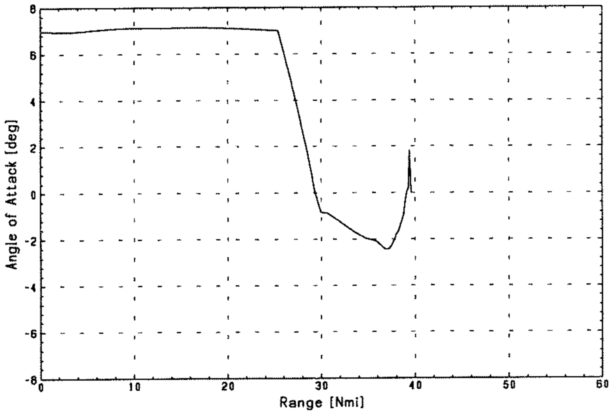

Figure 6. Optimum angle of attack versus range for mission 1

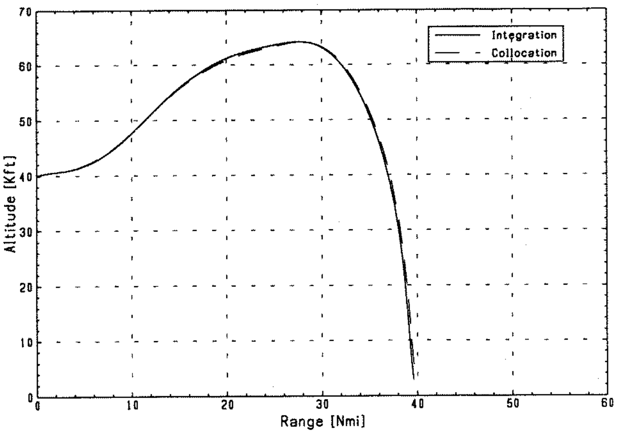

Figure 7. Comparison of range-altitude results from optimization (collocation), and actual integration of the equations

Further investigation of Figure 6 shows that in the terminal segments of the trajectory there is a rather sharp fluctuation in the angle of attack input. This fluctuation is due to the coarse mesh (30 segments) used after burnout. This sharp fluctuation may be eliminated by decreasing time interval at the final segments. To see the effect of this fluctuation on actual missile trajectory this optimized input is filtered by a low pass filter

$$\frac{a}{s+a} \tag{10}$$

For mission 1, after few trials a is chosen as 0.15. The filtered angle of attack versus range is given in Figure 8. Simulation conducted with this filtered angle of attack showed that the new range is within 0.5 Nmi of the unfiltered result.

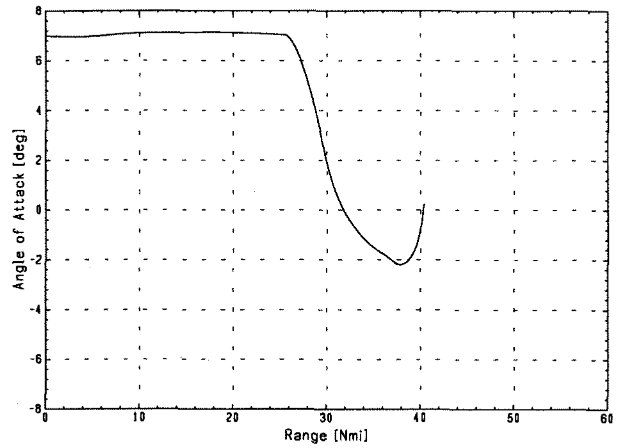

Figure 8. Filtered angle of attack history for mission1

The question arises if the missile is physically capable to fly this trajectory, since no limitations on the maneuverability of the missile is incorporated in the optimization. It is decided that such missiles can handle load factors in the range of -3g to +3g. The load factor history using the filtered angle of attack input of Figure 8 is plotted in Figure 9.

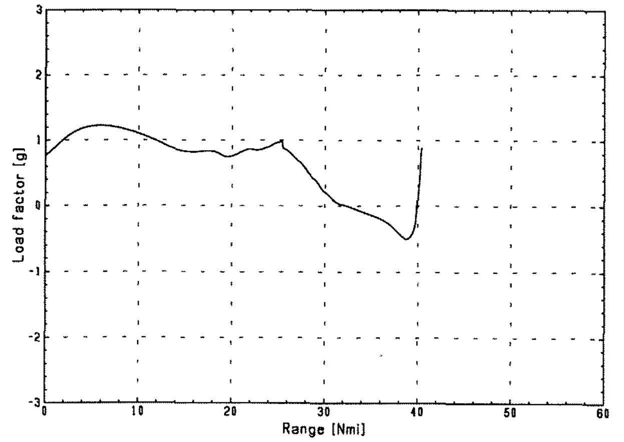

Figure 9. Load factor versus range for mission 1

The results indicate that load factors attained during this maximum range flight is well within the indicated range. It should be noted, however, that load factor limitations may also be included in the optimization routine just like constraint on angle of attack, which was programmed to be $-8^\circ \leq \alpha \leq 8^\circ$.

Required impact conditions for mission 2 with HEW configuration are listed in Table 5. For this mission, smaller impact angles and impact velocities can be tolerated. Consequently we may deliver the missile for a longer range while satisfying the impact constraints. Convergence of the flight trajectory from the initial trajectory as given in Figure 4 to the final maximum range trajectory is given in Figure 10. The figure legend indicates the major iteration number of the optimization routine as before. It can be observed from this graph that the maximum range obtained is larger than the mission 1 as expected. Final impact conditions obtained for HEW configuration are listed in Table 6. The new range is 52.2 Nmi, while flight time after burnout is 126.45 s. To attain this range the missile benefits from gliding and diving maneuvers. This motion may easily be observed from the angle of attack versus range plot given in Figure11.

V_f	0.9 Mach
γ_f	-50°
α_f	0°
h_f	3200 ft

Table 5. Required impact conditions for mission 2

r_{N2}	52.2 Nmi
V_{N2}	0.9 Mach
γ_{N2}	-50°
α_{N2}	0°
h_{N2}	3200 ft
t_2	126.45 sec

Table 6. Obtained impact conditions for mission 2

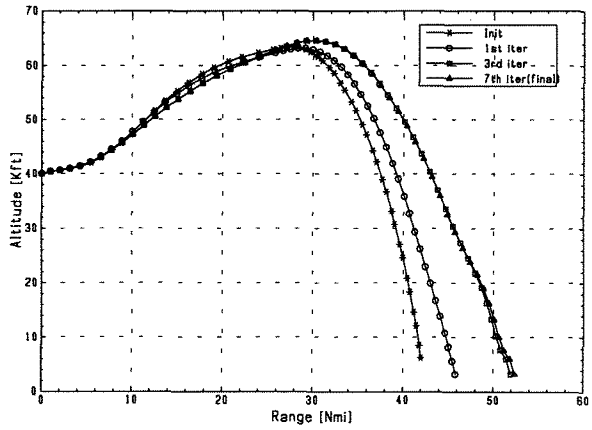

Figure 10. Convergence of the trajectories to maximum range in mission 2

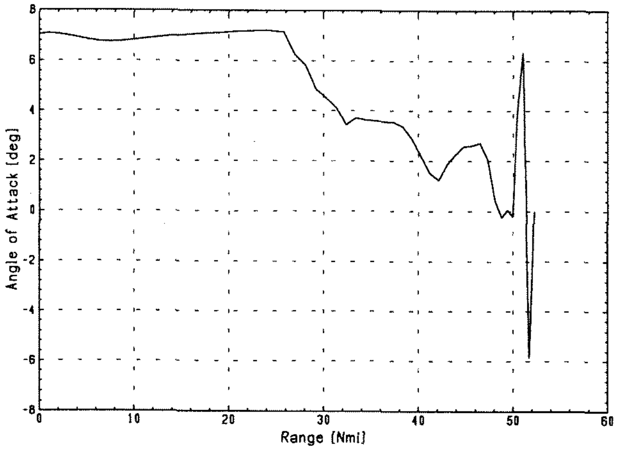

Figure 11.Angle of attack versus range for mission 2

As in mission 1, sharp fluctuation in the angle of attack at the terminal phase is observable. As discussed above terminal section requires finer mesh. Similarly, the angle of attack obtained in the optimization is again filtered using low pass filter (Eq. 10). Here a is chosen as 0.3. The filtered angle of attack for this mission is plotted in Figure 12. Simulation results showed that the maximum range with filtered angle of attack is again within 0.5 Nmi.

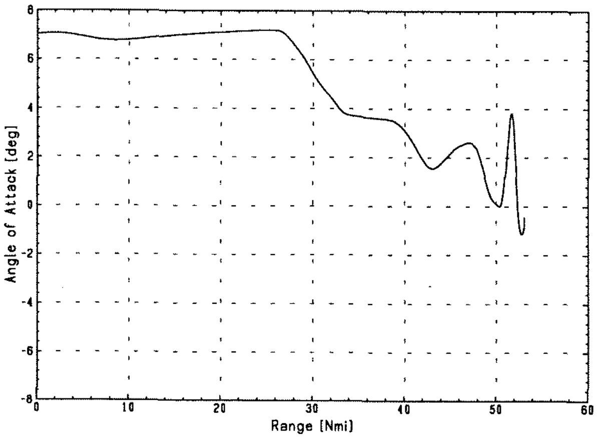

Figure 12. Filtered angle of attack versus range for mission 2

The load factor values obtained with filtered angle of attack input for mission 2 (HEW) is plotted in Figure 13. The load factor values are again within the required limits.

For comparison purposes, trajectories obtained from simulation for both missions, using filtered angle of attack inputs, are plotted together in Figure 14. The gliding motion obtained in mission 2 may be easily observed from this plot.

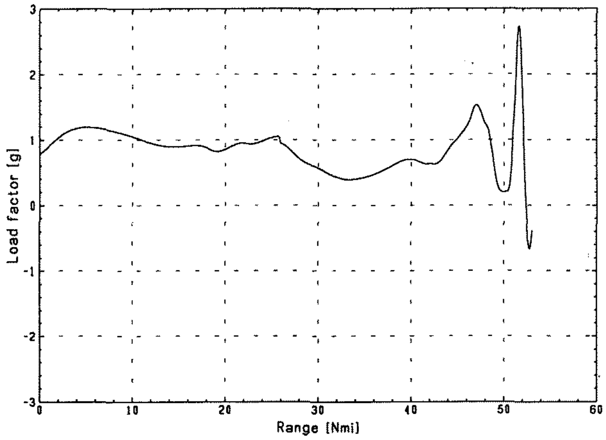

Figure 13. Load factor versus range obtained with filtered angle of attack for mission 2

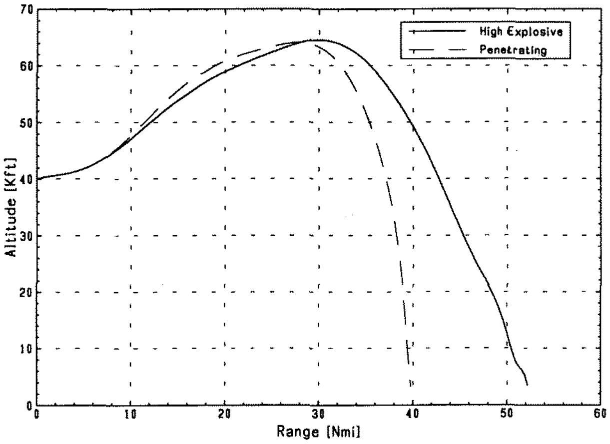

Figure 14. Trajectory simulations results using filtered angle of attack histories (Figure 8 and 12) for two missions

Conclusion

The tool developed to solve the optimum midcourse trajectories of precision guided munitions is described. It uses collocation technique with nonlinear programming. Using the tool, maximum range midcourse trajectories of a hypothetical missile are presented for two different missions. Based on the study the following are concluded:

1. Collocation technique together with nonlinear programming is quite effective to solve trajectory optimization problems. However, selection of the optimization package parameters require some experience.

2. The collocation technique requires excessive computer memory. However, it is suitable for sparse nonlinear programming and should be employed in future studies.

3. A rather coarse mesh may be chosen to speed up the optimization. However, this causes sharp fluctuations in the optimized angle of attack history and may cause excessive load factors on the missile. The angle of attack history obtained may be filtered to reduce load factor. The results indicate that ranges obtained using filtered and unfiltered angle of attack values are quite close.

4. The tool developed is also used for minimum weight missile design(18). Results of this study will be presented in a future article.

Acknowledgments

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