ICAS-96-1.6.4

CALCULATION OF VORTEX BREAKDOWN OVER DELTA WING BY A VORTEX-LATTICE METHOD

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Abstract

In this paper an extension of the basic formulation of vortex-lattice methods is proposed in order to cope with the *breakdown* of leading edge vortices of delta wings. The new formulation uses a *criterion* to predict the location of the phenomenon and a simple *model* to simulate the effects of the vortex bursting.

The criterion is based on the role of azimuthal vorticity in controlling the breakdown and combines the length and velocity scales of the axial and swirl motions that characterise the leading edge vortices. The flow field information necessary to apply the criterion is supplied by an unsteady vortex-lattice method capable of modelling the non-linear flow past delta wings.

The breakdown model is based on the experimental results available in literature and consists of a drastic reduction of the leading edge vortex circulation. This allows to compute the loss of vortex lift consequent to the bursting of leading edge vortices.

The numerical results are compared with experimental data for different delta wing planforms: the predictions of the onset of the phenomenon and the progression of its location along the wing chord are in agreement with experiments and the simple breakdown model is able to reproduce the initial lift reduction in accordance with experimental evidence.

The formulation here proposed can improve the computational capability of vortex-lattice methods for the simulation of delta wing flows.

1. Introduction

Modern combat airplanes utilise delta wings to satisfy the increased demand for high angle of attack performances and manoeuvrability. This type of lifting surface allows to combine the advantage of efficient supersonic cruise with high subsonic manoeuvre capability (Polhamus, 1984).

In the context of low speed characteristics, delta wing benefits refer primarily to the leading-edge vortex flows. With a sharp and back swept leading edge the flow separates along the entire edges forming strong shear layers. These layers roll up in a spiral fashion, resulting in two counter rotating vortices on the upper side of the wing.

The low pressure associated with these vortices produces an additional lift on the wing. This *vortex lift* is an important feature, because the size and the strength of the vortices increase strongly with the angle of attack, resulting in a substantial non-linear lift increment (Lamar 1977).

The design of swept winged aircraft requires the availability of computational tools, capable to cope with this non-linear vortex effect. In the recent years great advances have been achieved with numerical solutions of Euler and Navier-Stokes equations (Agrawal et al. 1990), but, unfortunately, storage capabilities and computing time are still unacceptable for the preliminary design phase.

If compressible effects are negligible, an alternative to Euler and Navier-Stokes codes is provided by boundary element methods. These methods are based on inviscid incompressible potential flow and, owing to their simplicity and their grid free feature, these computational techniques are very appropriate for aerodynamic design of delta wings (Hoeijmakers 1991).

Among these methods, it is well known that a very simple numerical procedure for calculating the magnitude of the vortex lift of a rather broad class of slender wings is provided by *non-linear vortex-lattice scheme*. This numerical technique was mainly developed by Rehbach (1973), Kandil et al. (1977). In addition to the bounding and trailing vorticity of the wing, the method introduces a free vortex sheet emanating from the leading edge and represents this vortex sheets by a lattice of vortex filaments. The roll up of the free vortex sheet is obtained aligning iteratively the wake filaments with the local flow velocity (wake relaxation procedure).

The computational efficiency of this technique is very high as a consequence of the use of a very simple formula (the Biot-Savart law) and, in spite of its

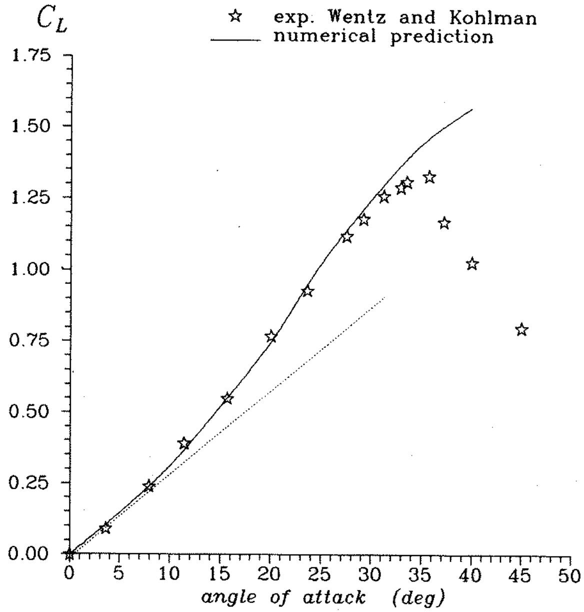

FIG. 1- Lift coefficient vs. angle of attack for a 75 deg delta wing (wing discretization of 12×24 panels)

simplicity, the method provides a satisfactory estimation of non-linear effects.

For this reason codes based on a vortex-lattice scheme are computational tools suitable for the preliminary phase of aircraft design.

Figure 1 shows the resemblance between the experimental data and the numerical predictions by a vortex-lattice method, recently developed by Baron et al. (1990) and Boffadossi (1993) and based on the unsteady formulation of non-linear vortex-lattice scheme.

The method is able to capture the non-linear effects of vortex lift, but, increasing the angle of attack, the agreement with experimental results becomes less adequate.

The slope of lift curve, after the initial augmentation respect to linear trend, begins to decrease and the lift itself shows a progressive reduction, while the numerical prediction exhibits a monotonous increment. Indeed, it is well known that the maximum lift of a delta wing is limited by a phenomenon called *vortex breakdown* or *vortex bursting*.

This phenomenon is characterised by a sudden deceleration of the axial vortex flow with a dramatic transition from a tightly wound spiralling vortical flow to a larger swirling turbulent flow.

Under certain conditions related to the angle of attack and wing geometry the leading vortices can undergo to this sudden and drastic change in structure, which reduces the vortex flow benefits and can alter the aerodynamic performance. Consequently, when vortex breakdown occurs, the stability and control characteristics of the aircraft can change dramatically.

Despite the growing list of theoretical and experimental investigations of vortex breakdown — a review of the research done in this field is given in the publications of Hall (1972), Leibovitch (1978, 1984), Nelson (1991) and Delery (1994) — there is still no consensus on the mechanism governing the phenomenon, and, at present no satisfactory theory or criterion for the prediction of breakdown onset is available for delta wing design.

However the ability to predict and control this phenomenon would be very advantageous in aerodynamic design and flight dynamics and this is why it represents an active and stimulating area of research.

This paper proposes an extension of the basic capability of the non-linear vortex-lattice schemes in order to include the vortex breakdown phenomenon and, in particular, to provide the correct estimation of lift curve. To achieve this results a simple *vortex breakdown model* for the reduction of vortex circulation has been developed from the experimental data available in literature. Its formulation is based on the disruption of the coherent vortical structure and the rapid spiralling expansion of the vortex core. Both of them produce a strong reduction of vortex circulation that can be easily simulated.

This model can significantly improve the applicability of non-linear vortex-lattice schemes in the simulation of the leading edge vortex flow over delta wings, but it requires *a priori* knowledge of vortex breakdown location over the wing.

Then a criterion for the prediction of this location has been developed; it is based on the numerical solution of the flow field and combines the length and velocity scales of the axial and swirl motions that characterise the leading edge vortices. The flow field information necessary to apply the criterion is supplied by the unsteady vortex-lattice method used to compute the non-linear vortex lift of delta wings.

This paper presents a new formulation of the basic numerical technique. It combines the criterion for the prediction of vortex breakdown over the wing and the model for the simulation of the bursting effects. The paper is organised in the following way: the formulation of the vortex-lattice method here adopted will be briefly outlined in section 2. The formulation of the criterion will be introduced in section 3, while the comparison with experimental data will be presented in section 4. The simple breakdown model here adopted will be described in section 5 and, finally, some conclusions and recommendations will be formulated about the results obtained.

2. The numerical method

The computational method is based on the unsteady formulation of non-linear vortex-lattice scheme. Details of the mathematical and numerical approach are given mainly by Baron et al. (1990) and Boffadossi (1993), therefore only main features and limitations of the method are here summarised.

Wings are assumed to have negligible thickness and are simulated as rigid, plane or cambered surfaces, and wakes can be released in the flow field from any of the sharp edges. Both the lifting surface and the wake are simulated by a lattice of vortex filaments.

Wings are discretized into a finite number N of surface panels. The unknown values of the N circulations of the lifting surface panels are determined, at each time step, imposing the zero normal velocity condition on the panel control points and a linear system of N algebraic equations must then be solved at each time. Velocities are evaluated at control points through the Biot-Savart law.

Starting from rest, wakes are generated also for a steady condition in a Lagrangian process, by releasing in the field the vorticity present on the edge panels of the wings. During each time step of this process, the existing wake is moved according with the velocity field, while the vorticity is convected from the edges of the wing in the flow field in order to form a new part of the wake.

This approach is particularly suitable for multi-wing configurations and allows to compute aerodynamic loads and geometry of the wakes even with time dependent conditions. However in the context of this paper only steady conditions of isolated wing, attained after a short initial transient, are considered. but the use of an unsteady formulation of the method has the advantage to extend breakdown calculations during arbitrary manoeuvres too.

Vortex filaments are extremely efficient from a computational point of view and their use is in practice compulsory when iterative or time marching schemes have to be used. Nevertheless, they introduce in the flow field lines along which, according to the Biot-Savart law, the induced velocity tends to infinity. The singular behaviour of vortex filaments can be such to produce numerical instabilities and may cause the solution to diverge.

This problem can be eliminated by preventing the induced velocity, at points close to the vortex axes, from increasing above a certain value. In the present work Rankine vortices are used with a viscous core diffusion model physically consistent with the turbulent diffusion mechanism of continuous shear layers (Baron et al. 1990).

Rankine vortices are assumed to be "equivalent" to the elementary portions of the physically continuous shear layers they replace in the numerical scheme. Therefore, their core radii spread in such a way that their cross sectional area and circulation are equal, at each time, to spreading and circulation of an elementary portion of continuous shear layer containing the same vorticity. This implies a rate of change with time t of their radius r given by:

$$r(t) = \sqrt{\frac{K\Gamma}{\pi}(t - t_0)}$$

where Γ is the circulation of the vortex filament, t_0 the instant of its formation and K a diffusion constant, which assumes, both for forced and unforced turbulent shear layers, a universal value equal to 0.095, as demonstrated by experimental evidence (Liepmann and Laufer, 1947; Oster and Wygnanski, 1982).

While most of the common used vortex core diffusion models turn out to be strongly dependent on the number of vortex filaments used to discretize the continuous distribution of vorticity in the flow field (Rusak et al., 1985), the present approach is virtually independent from discretization: the turbulent diffusion is explicitly related to the circulation of each vortex filament and, therefore, to the number of vortices. This brings to a kind of "self adaptation" of the model and explains why, in a variety of applications, no tuning has been required for a correct simulation of the vortex core diffusion.

The comparison presented in figure 1 is an example of the results offered by this computational technique and other satisfactory applications of the method to fixed and rotary wings are also reported by Boffadossi (1993).

3. The criterion for the prediction of vortex breakdown

The breakdown of leading edge vortices has been under study since the first experimental studies of vortex breakdown over delta wing of Peckhan and Atkinson (1957) and Lambourne and Bryer (1961). Simultaneously, the early works of Squire (1960) and Benjamin (1962) have been the starting point for many subsequent theoretical studies.

Three different classes of phenomena have been suggested as the cause or explanation of breakdown:

1) hydrodynamic instability of Ludwieg (1961),
2) the concept of critical state of Squire (1960) and Benjamin (1962),
3) the analogy to boundary-layer separation of Hall (1972).

However no theory exists which can satisfactorily explain the breakdown nor universally predict its locations consistently with experimental results.

In the last thirty years a large number of experimental investigations have been carried out in an effort to understand what is occurring and, in particular, to describe the phenomenon. Therefore the experimental evidence of the phenomenon is quite clear: the flow decelerates and stagnates on the vortex axis and a region with reverse flow appears eventually.

The research activities have included variations of all manner of possible parameters. The extensive literature on the argument is summarised in the articles of Nelson e Visser (1990) and Nelson (1991).

All the experiments have shown a strong dependence on the angle of attack and sweep angle: decreasing sweep angle or increasing the angle of attack causes the location of the breakdown to move forward, and vice versa. The investigator's reports show the vortex breakdown location as a function of the angle of attack and leading-edge sweep.

Recent works by Brown and Lopez (1990), Nelson and Visser (1990) and Hoeijmakers (1990), have highlight the important role played by the vorticity in determining the breakdown. Their basic observations shown the physical mechanism involved in the breakdown process of production of negative azimuthal vorticity. Since the distribution of the circumferential velocity component is directly associated with the axial component of the vorticity (while the circumferential component of the vorticity is connected to the axial component of the velocity), the onset of negative azimuthal vorticity is a necessary condition for the onset of vortex breakdown. In fact, the attainment of zero or negative velocity on the axis of the vortex is possible only if azimuthal vorticity becomes negative.

In particular Brown and Lopez (1990) emphasize the crucial role of both the velocity and vorticity angles in controlling the process by which the axial component of vorticity vector is converted into negative azimuthal component. This is followed by a reduction in the initial positive azimuthal component of vorticity. The origin of spiral structure, by stretching and tilting of vortex lines, could provide the mechanism that transforms axial component into circumferential one.

From the equations that govern the steady, inviscid, rotational axis-symmetric flow, it is possible to write a simple expression for the azimuthal component of vorticity. If we consider a vortex tube, taking cylindrical co-ordinates (r, θ, x) with x along the axis of symmetry, the azimuthal component of vorticity $\omega_\vartheta|_r$ may be described, following Brown and Lopez (1990), in terms of the radius r of the stream surface by the following formula:

$$\frac{\omega_\vartheta|_r}{\omega_\vartheta|_{r_0}} = \frac{r_0}{r}\left(\frac{\alpha_0}{\beta_0}\right) - \frac{r}{r_0}\left(\frac{\alpha_0}{\beta_0} - 1\right)$$

where $\omega_\vartheta|_{r_0}$ is the azimuthal component of vorticity in the initial section of the tube and

$$\alpha_0 = \left(\frac{V_\vartheta}{V_x}\right)_{r_0} \quad , \qquad \beta_0 = \left(\frac{\omega_\vartheta}{\omega_x}\right)_{r_0}$$

are the tangents of the helix angle of velocity and vorticity respectively; r_0 is the radius of the initial section of the vortex tube.

It is evident that for $\omega_\vartheta|_{r_0}$ positive, $\omega_\vartheta|_r$ can become negative on a diverging surface only if $\alpha_0 > \beta_0$. Therefore a helix angle for the velocity exceeding the helix angle of the vorticity seems to be a necessary condition for breakdown to occur. Brown and Lopez (1990) have tested this condition against numerical Navier-Stokes calculations of swirling pipe flow and they have found that the numerical solution showed breakdown to occur only if

$$\tau_0 = \frac{\alpha_0}{\beta_0} > 1$$

The theoretical observations and the numerical experiments of Brown and Lopez can be assumed as the basis to develop a criterion for the vortex breakdown over delta wings.

To face this problem, we must examine the physics of the phenomena related to leading edge vortex flows.

The subject has been studied extensively by several experimental (Peckham and Atkinson, 1957; Earnshaw, 1961; Hummel, 1979; Nelson, 1991; Visser and Nelson, 1993) or theoretical (Hall, 1961; 1966; Verhaagen and Kruisbrink, 1987; Hoeijmakers, 1990, 1991) researches. These investigations show that vortex flow past delta wings has a double nature of *swirl* and *jet* and it can be expressly divided into three regions:

1) the *external inviscid potential* flow;

2) a *conical* rotational, but inviscid, *vortex core*, in which is collected the vorticity of the shear sheet shed by the leading edge (within this core there are appreciable axial and circumferential components of velocity);

3) a small *viscous subcore*, where diffusion and viscous phenomena are important.

Nelson and Visser (1990) observe that the *jet-like* distribution of axial velocity $V_x(r)$ concerns essentially all the conical rotational region, while the *swirl-like* distribution of circumferential component $V_\theta(r)$ of velocity basically relates to the viscous subcore, being this subcore bounded between the maximum and the minimum values of the circumferential velocity.

Therefore the *swirl* behaviour of leading edge vortices may be characterised by:

- the azimuthal velocity scale V_ϑ,
- the viscous subcore radius r_V,
- the scale of the axial component ω_x of vorticity vector,

while, the *jet* behaviour may be described in terms of

- the axial velocity scale V_x,
- the radius of the conical rotational core r,
- the scale of azimuthal component ω_ϑ of vorticity vector.

Note that there are two different transversal length scales: the viscous subcore radius and the dimension of the conical vortex core. Following Delery (1984) this dimension may be defined as the distance from the axis where the local velocity difference

$$\left[V_x - V_{x(\text{ext})}\right]$$

is equal to a given fraction of the total difference:

$$\left[V_{x(\text{axis})} - V_{x(\text{ext})}\right].$$

The swirl motion is also characterised by the circulation Γ; consequently we may assume the characteristic axial component of vorticity as the quantity:

$$\omega_x = \frac{\Gamma}{\pi r_V^2}$$

which represents an average vorticity, strictly related to the circulation of the vortex and the sectional area of its viscous core.

On the contrary, the characteristic azimuthal component of velocity concerns both the two regions: the viscous subcore and the conical vortex core. Since it must represent an average velocity in this two regions, we assume that:

$$V_\vartheta = \frac{\omega_x r}{2} = \frac{\Gamma}{2\pi r_V^2} r$$

Conversely, we may assume the characteristic azimuthal component of vorticity to have the following value:

$$\omega_\theta = \frac{V_x}{r}$$

being this quantity only related with the jet-like profile of velocity.

It is important to highlight that all the preceding quantities do not represent the actual value of velocity or vorticity of any point in the flow field, but they are only the characteristic scales associated with a vortex. They may be used to represent the global state of the vortex, and in particular to compute the parameter τ, previously introduced, by the formula:

$$\tau = \left(\frac{\alpha}{\beta}\right) = \frac{\left(\frac{V_\vartheta}{V_x}\right)}{\left(\frac{\omega_\vartheta}{\omega_x}\right)} = \left(\frac{\Gamma}{\sqrt{2}\pi V_x}\right)^2 \left(\frac{r}{r_V^2}\right)^2$$

To evaluate this parameter it is only necessary to have some information about the typical scales. This can be obtained from the numerical solution of flow field provided by the vortex-lattice method, although the crude discretisation offered by a vortex-lattice scheme do not give an accurate distribution of velocity and vorticity profiles.

As a matter of fact the leading edge vortex is represented in a vortex-lattice scheme by a small number n of vortex filaments. If only typical scales are needed it is sufficient to consider *an equivalent vortex core*, in which all the vortex filaments have been collapsed.

This vortex has a sectional area equal to the sum of the sectional area of all the vortex filaments, and so it is easily computed as

$$r^2 = \sum_{i=1}^{n} r_i^2$$

being r_i $(i=1,...n)$ the radii of the Rankine cores of the n filaments representing the leading edge vortex at the chordwise location we are considering.

Analogously the *total circulation* is the sum of the circulation of all the n vortex filaments:

$$\Gamma = \sum_{i=1}^{n} \Gamma_i$$

while the *average axial velocity* may be defined as:

$$V_x = \frac{\sum_{i=1}^{n} V_{xi}\, r_i^2}{r^2}$$

The number of filaments, spiralling together to form the leading edge vortex, increases progressively from the apex of the wing to the trailing edge. Consequently the value of the total circulation, the average axial velocity and the core radius change along the wing chord.

Also the parameter τ may be estimated as a function of the location along the wing chord:

$$\tau = \tau(x/c) = \left(\frac{\Gamma}{\sqrt{2}\pi V_x}\right)^2 \left(\frac{r}{r_V^2}\right)^2 \qquad (1)$$

but we must observe that the value of r_V has not been defined yet on the basis of the numerical solution.

Hall (1966) has shown that circumferential and axial velocity distribution of the leading edge vortex can be approximated using the concepts of *inner/outer solutions* with conical rotational core, but *non conical* subcore. His theory, also confirmed by Earnshaw's experimental investigations (1961), suggests:

$$\frac{r}{c} = k_1 \frac{x}{c} \qquad \frac{r_V^2}{c^2} = k_2 \frac{x}{c}$$

where k_1, k_2 are appropriate dimensionless constants.

If we introduce a new constant k, putting:

$$k = \frac{k_1}{k_2},$$

the ratio between the rotational core and the square of viscous subcore may be expressed as:

$$\frac{r}{r_V^2} = \frac{k}{c} \qquad (2)$$

The substitution of (2) in (1) yields

$$\tau = \frac{k^2}{c^2}\left(\frac{\Gamma}{\sqrt{2}\pi V_x}\right)^2$$

The value of the empirical constant k^2 can be found by a comparison with experimental data, imposing the condition:

$$\tau_{BD} = 1$$

Unfortunately breakdown data are characterised by an apparent spread of results (as shown in figures 2-6). Then a large body of experiments available in literature have been selected to balance the influence of the different experimental parameters .

As a matter of fact the experimental data are characterised by a not negligible spread of results (as can be seen in figures 2-6). The explanation of this large dispersion is not completely clear: Erickson (1982) examines a large number of experimental works concluding that the variation in the data due to changes in Reynolds number is no greater than the variation associated with different model, test facilities and visualisation techniques. As shown by Weinberg (1992) using delta wings of different sizes relative to the test section, significant effect of the test section wall on the measured location of the breakdown can exist in many experimental investigations.

By a comparison between numerical prediction and the different experimental data available in literature (next section) we found an appropriate value for the empirical constant (k^2=47≅15π). Of course different selections of experimental results can change this value, without diminishing the theoretical validity of the criterion proposed.

4. Comparison between numerical prediction and experimental data

In order to establish the location of the vortex breakdown, the criterion requires to calculate, for each station x/c along the chord of the wing, the global circulation $\Gamma=\Gamma(x/c)$ and the average axial velocity $V_x=V_x(x/c)$ of the vortex filaments representing the leading edge vortex.Then, it is possible to estimate the chordwise distribution of the parameter

$$\tau(x/c) = \frac{k^2}{c^2}\left(\frac{\Gamma(x/c)}{\sqrt{2}\pi V_x(x/c)}\right)^2$$

and seek the location along the chord where:

$$\tau(x_{BD}/c) = 1$$

The effect of the number of panels used to discretize the wing in the vortex-lattice solution has been investigated and it has been found that even a mesh of 12x24 panels could reasonably provide correct values of total circulation and average axial velocity. In this case a run on a pentium CPU workstation takes only few minutes.

Five wings have been considered with 60, 65, 70, 75 and 80 degree of leading edge sweep angle and the range of the angle of attack is set varying from 0 to 60 degrees.

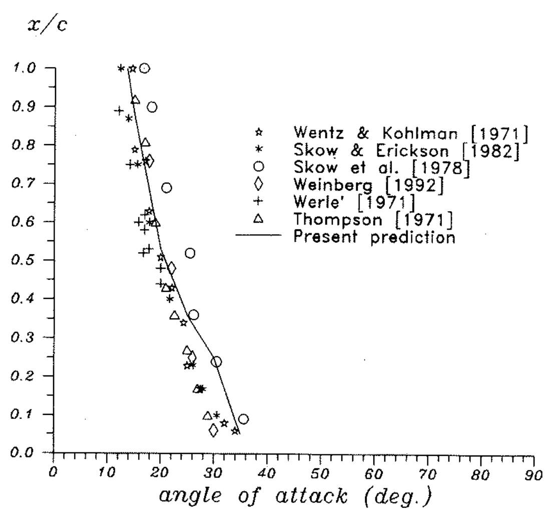

FIG 2 - Breakdown locations vs. angle of attack for delta wing with 60-deg leading edge sweep: comparison between experimental results and the present prediction.

The results of predictions are compared with experimental data in figures 2-6. The onset of breakdown and the progression of its location over the wing, increasing the angle of attack, are consistent with the experimental data and even when the position of the breakdown is near the apex, in the first quarter of the wing, the reliability of the prediction appears reasonably satisfactory.

In all cases the predicted position is located among the experimental data. Only for the 70 degrees delta wing, the angle of attack for the occurrence of the phenomenon over the trailing edge is a little under-estimated.

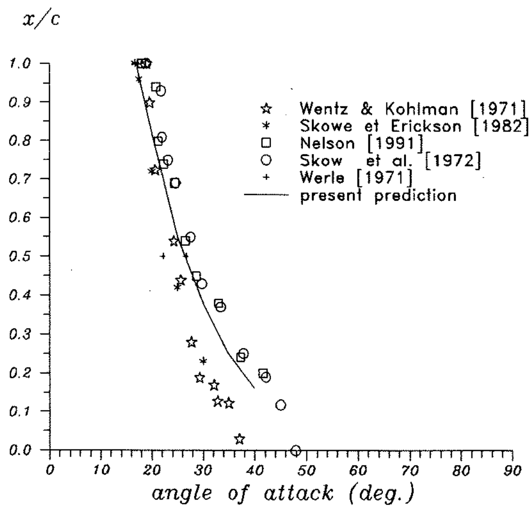

FIG 3 - Breakdown locations vs. angle of attack for delta wing with 65-deg leading edge sweep: comparison between experimental results and the present prediction.

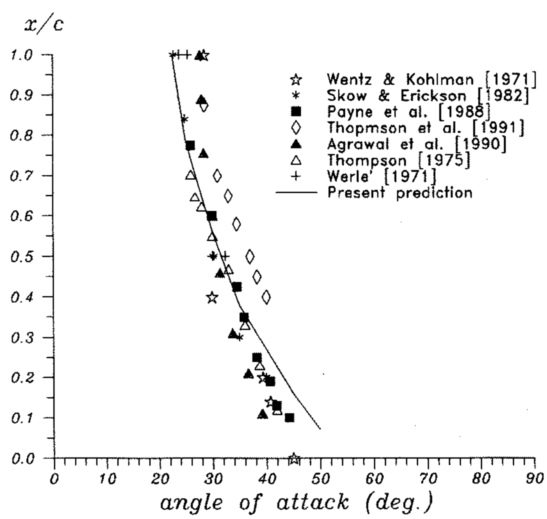

FIG 4 - Breakdown locations vs. angle of attack for delta wing with 70-deg leading edge sweep: comparison between experimental results and the present prediction.

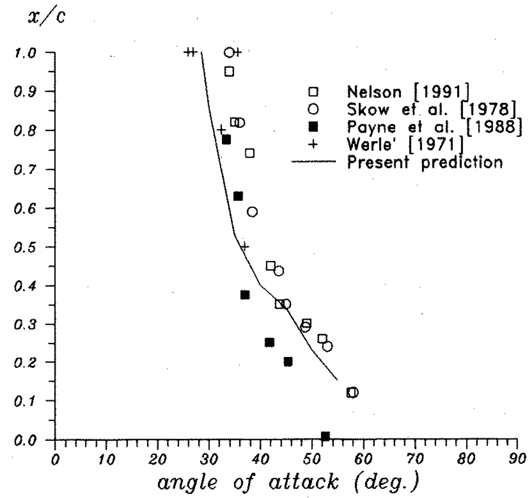

FIG 5 - Breakdown locations vs. angle of attack for delta wing with 75-deg leading edge sweep: comparison between experimental results and the present prediction.

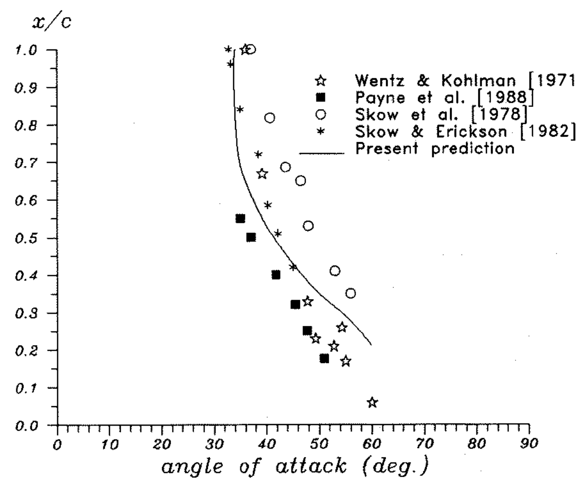

FIG 6 - Breakdown locations vs. angle of attack for delta wing with 80-deg leading edge sweep: comparison between experimental results and the present prediction.

6. A simple model for the simulation of breakdown effects in a vortex-lattice method

The development of a model for the simulation of vortex breakdown must relate to the physical mechanism of vortex bursting.

This phenomenon is characterised by a sudden deceleration of the axial flow, a rapid expansion of the vortex core and a substantial change in the coherent structure of the vortex. In the core the motion of fluid changes from a *jet like* to a *wake-like* flow and downstream of stagnation point the flow usually becomes strongly unsteady and turbulent.

Two types of vortex breakdown have been identified: *bubble type* and *spiral type*. However for the Reynolds number range of interest here usually the spiral type of breakdown occurs. In this case three successive stages can be observed:

1) a sudden deceleration of the fluid moving along the axis of the vortex;
2) an abrupt kink where the axial filament is deflected into spiral configuration which performing a whirling motion about the central axis, persist for a few turns;
3) a breakdown to large scale turbulence.

The scenario is very complex, but the main result of vortex bursting is a strong reduction of vortex circulation, consequent to the disruption of the coherent vortical structure and the rapid expansion of the core. Of course the main effect of this decreasing of the circulation is the disappearance of the vortex lift contribution.

The reduction of vortex circulation is apparent in the LDA velocity data acquired by Iwanski over a 70° delta wing at α=30°, as reported in figure 16 of (Nelson and Visser 1990).

The chordwise distribution of integrated axial vorticity estimated by Iwanski shows a reduction of about 60% in the vortex circulation. This reduction starts 7% of the wing chord before the location of breakdown and terminates 13% after. This corresponds to assume a length of the transition zone equal to about 20% of the chord.

The Iwanski's experimental observation suggests a "first order" model for breakdown, only based on the decrease of the vortex circulation. Interpolating the Iwanski's data with an arc of ellipse we found the following model for the reduction of vortex circulation:

$$\frac{\Gamma_{BD}(x)}{\Gamma(x)} = 1 - 0.6\sqrt{1 - \frac{(x - x_{BD} - 0.2c)^2}{(0.2c)^2}}$$

where x_{BD} denotes the location of breakdown, and $x > x_{BD}$ the chordwise position we are considering.

The figures 7 and 8 present a comparison between the numerical predictions, obtained by combining the criterion and this simple model of vortex breakdown (*BD model*), with two experiments available in literature.

It is apparent that the BD model can reproduce correctly the initial loss of vortex lift related to the vortex bursting, in particular for the case presented in figure 7 of a 65 deg. delta wing. Note that the difference between the numerical results and the experimental data shown in figure 8 may be partially related to different onset of the bursting over the trailing edge, as can be seen in figure 4.

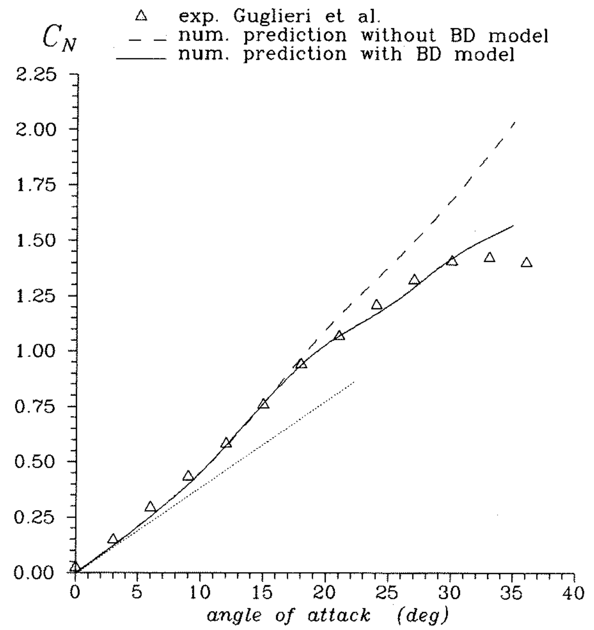

FIG. 7- Comparison between experimental data and numerical prediction by vortex-lattice method, (with and without the simulation of vortex breakdown) for a 65 deg delta wing.

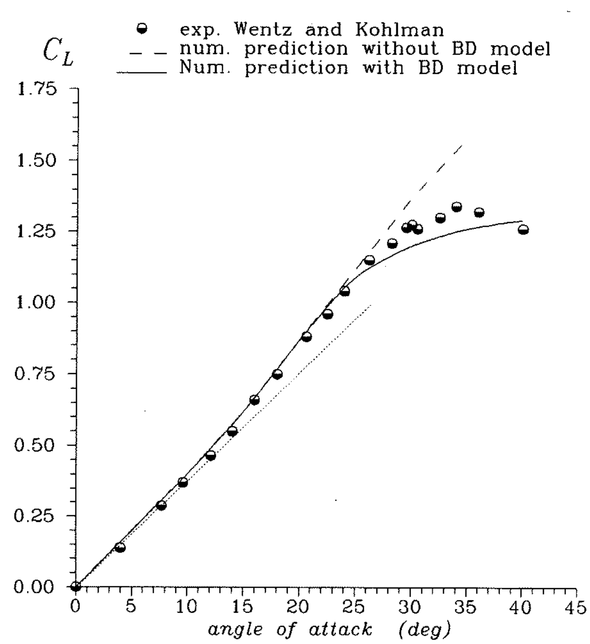

FIG. 8- - Comparison between experimental data and numerical prediction by vortex-lattice method, (with and without the simulation of vortex breakdown) for a 70 deg delta wing.

Conversely, the simulation of the entire stall phenomenon is less satisfactory, because the numerical prediction seems to increase monotonously. The model is not able to predict the maximum lift coefficient. This fact may be attributed essentially to the simplicity of the model proposed, which is based only to the reduction of vortex circulation, without reproducing the actual stretching and tilting process of vortical lines, namely the conversion of axial vorticity into circumferential one.

Nevertheless it is important to note that Iwanski's data refers only to a specific test case and no flow field investigations are available for breakdown positions near the wing apex, as characteristic when stall condition are established over the entire wing.

However the introduction of a simple *BD* model, in the existing codes, can improve the computational capability of the basic formulation of vortex-lattice methods, as shown in figure 7 and 8.

7. Conclusions

Non-linear vortex techniques represent an efficient method to compute the vortex lift of delta wings, until the occurrence of vortex breakdown phenomenon.

Starting from the basic technique of an unsteady non-linear vortex-lattice method a new formulation has been developed. It combines a criterion for the prediction of the phenomenon over the wing and a simple model for the simulation of breakdown effects; it can improve the basic computational capability for delta wing flows by vortex-lattice methods.

The criterion relies on the role of vorticity field to control the phenomenon. A physical analysis, which has recognised the two main features of leading vortex flows, the *swirl* and the *jet* behaviours, has been performed and it has been possible to identify in the vortex lattice solution the physical quantities that represent the characteristic scale of the different phenomena.

This has led to a criterion for the breakdown based on a simple parameter $\tau(x/c)$, which combines the total circulation and the average axial velocity of the vortex. The empirical constant that appears in the criterion does not depend on wing geometry, angle of attack or panel discretization.

Therefore the criterion proposed to predict the breakdown of leading-edge vortices *is not a best-fitting of experimental data*, it has a real physical significance because it is based on local flow properties. The application of the criterion requires the knowledge of flow field information, and an unsteady non-linear vortex-lattice capable of modelling flow past delta wing must be used to supply the necessary information.

Unfortunately breakdown data are characterised by an apparent spread of results that requires more accurate experimental investigation.

The predictions of the onset of breakdown and the progression of its location over the wing, by increasing the angle of attack, are shown to be consistent with experimental data. The reliability of the prediction appears to be reasonably satisfactory even when the position of the breakdown is near the apex in the first quarter of the wing.

The knowledge of the breakdown position allows to use a model to simulate the bursting. Even a simple model, based on the reduction of vortex circulation, is able to reproduce the loss of vortex lift consequent to the breakdown.

More sophisticated models are necessary to improve the prediction of maximum lift of delta wings and to correctly describe the entire stall phenomenon of this kind of wings, but this goal requires a more complete experimental investigation of vortex the breakdown phenomenon.

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