NUMERICAL SIMULATION OF THREE DIMENSIONAL BOUNDARY LAYER FLOWS

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ABSTRACT

A three-dimensional boundary layer computational code is tested on a typical 3D configuration (ellipsoid at incidence).

Output of the program is compared to experimental results and to results obtained by a less sophisticated code.

A significant improvement is observed with respect to the former code; results show a qualitative good fit to experimental data, but quantitative values are still to be improved.

INTRODUCTION

Numerical simulation is becoming a more and more important tool in the Fluid Dynamics field as the experimental costs increase and the numerical ones decrease.

Experimental costs are increasing because:
- The growing requests the designer must face impose the study of quantities of configurations, and inside each of them a parametric study. The complete experimental study of a project would then require an incredible amount of wind tunnel runs.
- Meanwhile, the operating costs of the wind tunnel themselves are rapidly increasing because the needs, mentioned above, for always better precision, call for higher and higher flow quality in the test section, which in turn leads to higher wind tunnel costs.

The numerical costs are on the other hand decreasing as the computer design evolution allows the production of machines which have always higher performances and better cost-effectiveness.

Anyway, a full simulation of the flow is still beyond the possibilities of the existing technology (except for some particular cases, namely low Reynolds number flows); then, the flow must be modeled or simplified in some way for its numerical study.

These simplifications and modeling need to be carefully evaluated by checking the numerical results against experimental data in at least some test cases (validation of the code). The tests used for the validation should be as close as possible to the true cases for which computations will actually be made.

A variety of ways have been proposed and developed for the aforementioned simplification of the flows for the purpose of numerical simulation; one of the ways of modeling a three-dimensional boundary layer - which is the one of interest in this work - consists of exploiting the integral equations of the boundary layer (see next section).

This approximation is known to give satisfactory results in the case of two-dimensional flows.

As it will be shown in the present work, also 3D flows can be well computed by such a method. It should be mentioned however that integral boundary layer equations can give only the global quantities describing a boundary layer, but they bring no informations about the details of the flow field.

Then, depending on whether or not we are interested in knowing these details, the choice of the computational method will be made.

As a final remark, observe that the 3D boundary layers are a subject of great interest in modern aeronautics as three-dimensional effects are getting more and more important in the present research effort to reduce the aircraft drag.

DESCRIPTION OF THE NUMERICAL METHOD

The method implemented in the code validated during this work is an integral method developed by J. Cousteix.

It is based on the integral momentum equations in the longitudinal (streamwise) and transversal (crossflow) directions:

\[
\frac{C_{f_x}}{2} = \frac{1}{h_1} \frac{\partial \theta_{11}}{\partial z} + \theta_{11} \frac{H + \partial U_x}{U_x} \frac{\partial U_x}{\partial h_1 - \partial \theta_{11}} + K_1 \theta_{11} + \theta_{12} \frac{\partial \theta_{12}}{\partial z} + K_1 \theta_{22} \ (1)
\]
\[ \frac{C_{f x}}{2} \tan \beta_0 = \frac{1}{h_1} \frac{\partial \theta_{21}}{\partial x} + 2 \theta_{21} \left( \frac{1}{U_e} \frac{\partial U_e}{h_1 \partial x} - K_1 \right) + K_2 \theta_{11} (H + 1) + \frac{\partial \theta_{22}}{h_2 \partial z} + K_2 \theta_{22} \]

(2)

The system is completed by an entrainment equation:

\[ \frac{\partial \delta}{h_1 \partial x} - \frac{v_E}{U_e} = \frac{1}{\rho_e U_e} \frac{\partial \left[ \rho_e U_e (\delta - \delta^*_1) \right]}{h_1 \partial x} + -K_1 (\delta - \delta^*_1) - \frac{\partial \delta^*_2}{h_2 \partial z} \]

(3)

Which, if the entrainment coefficient is expressed this way:

\[ \frac{\delta}{h_1 \partial x} - \frac{v_E}{U_e} = P(G) \gamma \]

(where \( P(G) \) is a function obtained by the similar solutions method)

becomes:

\[ P(G) \gamma = \frac{1}{\rho_e U_e} \frac{\partial \left[ \rho_e U_e (\delta - \delta^*_1) \right]}{h_1 \partial x} + -K_1 (\delta - \delta^*_1) - \frac{\partial \delta^*_2}{h_2 \partial z} \]

(3bis)

The meaning of the symbols employed is as follows:

- \( C_{f x} \) : Skin friction coefficient.
- \( \gamma = \sqrt{\frac{C_{f x}}{2}} \)
- \( \beta_0 \) : Angle between limiting (wall) streamline and \( x \) direction (direction of the external streamline).
- \( \theta_{11}, \theta_{12}, \theta_{21}, \theta_{22} \) : Momentum thicknesses.
- \( \delta \) : Boundary layer thickness.
- \( \delta^*_1, \delta^*_2 \) : Displacement thicknesses.
- \( H \) : Shape parameter.
- \( G \) : Clauser shape parameter.
- \( K_{1,2} \) : Geodesic curvatures along the streamlines and along their normals.
- \( h_{1,2} \) : Metric coefficients of the curvilinear reference system based on the body surface.

The system is then closed by the following seven equations, again obtained by the similar solutions method:

\[ \frac{\delta^*_1}{\delta} = \gamma F_1 (G) \]

\[ \frac{1}{\gamma} = \frac{1}{k} \ln \left( H \ Re_{\theta_{11}} \right) + D^* (G) \]

\[ \theta_{11} = \frac{1}{H} = 1 - G \gamma \]

\[ \theta_{21} = \delta^*_2 + \theta_{12} \]

\[ \frac{\delta^*_2}{\theta_{12}} = \Phi_1 \frac{H}{1 - H} \]

\[ \frac{\theta_{22} \delta^*_1}{\delta^*_2} = \Phi_2 \frac{1 - H}{H} \]

\[ \tan \beta_0 = \frac{-\delta^*_1 / \delta^*_2}{\Phi_1 \frac{H}{1 - H} - \epsilon_2} \]

Here, \( F_1 \) and \( D^* \) are functions obtained by the similar solutions method. Remark also that the quantities \( \Phi_1, \Phi_2, \epsilon_1, \epsilon_2 \) are not new unknowns, as they are functions of \( G, Re_{\theta_{11}}, T = -\delta^*_1 / K \delta^*_2 \).

These functions are again obtained by the similar solutions method.

This method can then be easily extended to the case of compressible flow; indeed, it is sufficient to write the similarity solutions profile in the form

\[ F' = \frac{U_e - u}{U_e \gamma} \]

in order to have all these profiles represented by an incompressible family. If then we substitute in all the equations of the system \( \gamma \) in the place of \( \gamma \) we can solve the system in transformed, pseudo-incompressible variables which will then be decoded to the actual value of compressible variables by a suitable set of relations. (see [7] for details).

### DESCRIPTION OF THE NUMERICAL CODES EMPLOYED

- **COUSTEIX**: this was the code to be tested. It is a FORTRAN code implementing a space marching 3D boundary layer computational procedure based on the afore described Cousteix method.

The code implements also the compressibility effects and can be used to compute the features of a generic boundary layer over adiabatic wall for Mach numbers up to 4.

The detection of transition within the boundary layer is made using the empirical criterion formulated by Arnal - Habiballah - Delcourt, which allows to take into account the effect of freestream turbulence. A short explanation of this criterion is now given; consider that the beginning of the transition region depends essentially on the boundary layer stability (which can be correlated to
the momentum thickness Reynolds numbers at
the neutral stability point and at the point under
examination) and on the disturbance amplifica-
tion, which can be correlated to the flow history.
The latter quantity can be expressed by the mean
Pohlhausen parameter:
\[ \frac{1}{\theta^*} = \frac{1}{x^* - x_{cr}} \int_{x_{cr}}^{x^*} \frac{\partial^2}{\partial x^2} \frac{dU_\theta}{dx} \]
The effect of freestream turbulence, as said, can
also be put into account; an empirical correlation
for these quantities is:
\[ Re_{\theta,T} - Re_{\theta,cr} = -206 e^{25.7 \theta^*} \left[ \log(16.8 T U) - 2.77 \theta^* \right] \]
of course, it is necessary to compute the value of
Re_{\theta,cr}; this can be done by suitable empirical for-
ulas both in the case of selfsimilar and nonself-
similar flows.
The requirements of the Cousteix code are as fol-
ows: it needs the body geometry, as described by
a generic curvilinear coordinate system attached
to the body. Moreover, it needs a set of bound-
ary conditions (velocity module and direction at
the edge of the boundary layer in each point) and
initial conditions. If the calculation starts from a
stagnation point, the code is able to provide itself
a computation of initial values by using the simi-
lar solution method results; else, it needs a set of
integral thicknesses.

- **VSAERO** This program consists of a 3D panel
  method for inviscid flow and of a 2D Head method
  based on the external flow streamlines for the com-
  putation of the boundary layer. A viscous/inviscid
  iteration is then performed. It was used for three
  purposes:

1. Computation of the boundary conditions for
   the Cousteix method. This way of doing it
   was preferred to the other possible choice
   (using the theoretical formula for inviscid
   pressure distribution around an ellipsoid)
   because it allows to already take into ac-
   count the presence of a boundary layer while
   computing the potential flow. This boundary
   layer won't be the "correct" one but the ap-
   proximation is largely sufficient to the pur-
   pose (see fig. 1)
2. Computation of initial conditions for the
   Cousteix method. As mentioned, this
   method needs a set of initial boundary layer
   thicknesses; these are taken from the bound-
   ary layer section of this code. Notice that,
   as this is done in the nose part of the body,
   the differences between 2D and 3D boundary
   layer are still not important so that these val-
   ues can be trusted.

![Figure 1: Comparison of \( C_p \) distribution along the el-
ipsoid at \( \chi = 37.5° \); VSAERO (solid line) vs. experi-
mental values (crosses).](image)

3. Comparison case as less advanced boundary
   layer solution method.

### TEST CASE

The flow over ellipsoids at incidence was chosen as
a testcase because of the many advantages it offers:
- It is a highly 3D flow, and includes all the pheno-
  mena typical of this kind of flows.
- It is easy to build a mathematical model for it.
- It exists a wide body of experimental results about
  it.

Two experimental data groups were employed, from
DFVLR and from Politecnico di Torino/Chinese Aero-
dynamical Institute. Both these data blocks were re-
ferred to 6:1 ellipsoids (see fig. 2 ), but absolute di-
mensions were different; so were the test velocities,
resulting in different test Reynolds numbers.
The experimental conditions are resumed in the fol-
lowing table:

<table>
<thead>
<tr>
<th>Testcase</th>
<th>( V ) (m/s)</th>
<th>( a ) (m)</th>
<th>( \alpha ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Göttingen</td>
<td>55</td>
<td>1.2</td>
<td>10</td>
</tr>
<tr>
<td>Politecnico</td>
<td>30</td>
<td>0.75</td>
<td>14</td>
</tr>
</tbody>
</table>

In these cases, the transition was imposed by a strip
at 20% of the ellipsoid chord.
The transition studies were conducted on the
Göttingen ellipsoid. The experimental results were ob-
tained in the various DFVLR wind tunnels, allowing
different levels of freestream turbulence.

### RESULTS

At first, some different systems of reference were
tested over the ellipsoid; this allowed to observe that
the Cousteix code is quite sensitive to the reference system employed; after these tries, the system represented in fig. 3 was chosen.

Consider at first the 3D $C_f$ mapping of figures 4,5.

Observe that the wind side mapping is not very different in the 2 cases, while in the lee side case important differences arise. The most important of these is the presence of the two lighter strakes ($\Leftrightarrow$ lower $C_f$) predicted by the Cousteix method on the ellipsoid back. They indicate the presence of the two vortices that are known to develop on the back of a 3D body at incidence. The VSAERO code does not detect this.

Now consider the 2D quantitative comparisons of figs. 6 to 11.

It is evident from figs. 6,7 that Cousteix's method is sensitive to a physical phenomenon (increase of $C_f$ after a minimum) that VSAERO do not perceive.

Figure 2: Geometrical configuration

Figure 3: Reference system employed for the Cousteix computations.

Figure 4: Mapping of the $C_f$ on the wind side of the Politecnico ellipsoid; a) VSAERO, b) Cousteix results.

Figure 5: Mapping of the $C_f$ on the lee side of the Politecnico ellipsoid; a) VSAERO, b) Cousteix results.
Figure 6: $C_f$ mapping along the ellipsoid, $\chi = 7.5^0$; Cousteix vs. experimental (Göttingen ellipsoid).

Figure 7: $C_f$ mapping along the ellipsoid, $\chi = 7.5^0$; VSAERO vs. experimental (Göttingen ellipsoid).

Figure 8: $H$ mapping along the ellipsoid, $\chi = 7.5^0$; Cousteix vs. experimental (Göttingen ellipsoid).

Figure 9: $\theta_1$ mapping along the ellipsoid, $\chi = 7.5^0$; Cousteix vs. experimental (Göttingen ellipsoid).

Figure 10: $C_f$ mapping along the ellipsoid, $\chi = 22.5^0$; Cousteix vs. experimental (Göttingen ellipsoid).

Figure 11: $C_f$ mapping along the ellipsoid, $\chi = 52.5^0$; Cousteix vs. experimental (Göttingen ellipsoid).
Figure 12: $C_f$ mapping along the ellipsoid, $\chi = 112.5^0$; Cousteix vs. experimental (Göttingen ellipsoid).

Figure 13: Influence of $T_u$, freestream turbulence, on the transition point.

Nonetheless, the quantitative analysis of this phenomenon is clearly perfectible. Actually, the program decidedly overestimates the extent of the $C_f$ distribution peak.

Distribution of $H$ and $\theta$ are well computed.

Going around the ellipsoid, we can observe that Cousteix method indicates the presence of the peak just at the same location as in experimental results. On the lower part, to conclude (fig. 12), we remark that, though the trend is correctly indicated (very flat $C_f$ curve) the program underestimates the values of the skin friction coefficient.

Moving to the analysis of the transition studies (performed only with Cousteix method) it can be observed in fig. 13 that the program gives a good trend for the variation of transition position with variation of $T_u$, but again the values of the point of transition are badly computed. The program is able to capture the plateau of transition positions at low turbulence, and it computes quite well its position. The fact that the experimental curve presents a peak can be attributed to the fact that also the turbulence spectrum has an influence on the transition.

Fig. 14 shows the influence of the Reynolds number on transition as predicted by Cousteix method and measured. The two numerical curves were obtained by keeping the $T_u$ constant or linearly varying with airspeed. It can be seen that the difference is not important. Going to the comparison with experiments, again we can observe that the trend is correctly predicted but the values are not very good.

The last parameter considered is the influence of the incidence on the transition; to show this effect, observe fig. 15 (experimental plots) and figs. 16 to 18: again, the Cousteix method captures the physical phenomena but quantifies them incorrectly (there is an overestimate of the incidence effect).

**CONCLUSIONS**

The program tested during this work showed to be satisfactorily reliable.

As described above, it gives good qualitative informations about the flowfield, and quantitative errors are acceptable. Comparison with the VSAERO program allows to state that the new computational procedure is a clear advance in the 3D flowfield computation. Actually, the latter gets some 3D aspects of the flowfield which are out of the VSAERO capabilities, although some imperfections appear in some points (e.g. the underestimate of the $C_f$ value on the lee side of the ellipsoid).
Figure 15: Influence of the incidence on the transition: experimental results.

Figure 16: Influence of the incidence on the transition: Cousteix, $\alpha = 5^0$.

Figure 17: Influence of the incidence on the transition: Cousteix, $\alpha = 10^0$.

Figure 18: Influence of the incidence on the transition: Cousteix, $\alpha = 30^0$.

Transition is computed with enough precision, and the code showed to be sensitive to many of the parameters that theory and wind tunnel work demonstrated to be important.

Nevertheless, this sensitivity, which under the qualitative viewpoint is good and even very good, is not always supported by a similarly good quantitative agreement. Indeed, the effect of the incidence is often overestimated, while the ones of Reynolds number and external turbulence are underestimated.

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References


