INVESTIGATION ON AIR REFUELING SCHEDULING

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Abstract
Air refueling is an effective measure to extend endurance and enhance fighting efficiency for aircraft. As designing the air refueling scheduling, this paper uses the knowledge of navigation, operations research and optimization theory. Refueling positions and scheduling plan are optimized comprehensively. The author obtains a truly optimal solution beyond mathematical programing techniques, and design optimal tracks of assigning tankers to receivers with fixed velocity and height, and computes optimal refueling times and fuel weights with minimum total fuel consumption. This paper takes the maximum number of tankers required, the maximum number of tankers available take-off bases, destination bases and post refueling bases etc. into consideration. Fuel consumption of a single pair of tanker and receiver is minimized with optimization of a constrained enroute method. Total fuel consumption of receivers and tankers is minimized with optimization of Conig efficacy matrix. This paper gives an example. The method of this paper suits not only for single air refueling, but also for military large-scale refueling. It serves the basis for refueling fight, training command or imitation.

I. Introduction

Transport aircraft carry various tasks from these origin bases to those other destination bases. When the distance between these bases is greater than the range of aircraft with its available fuel, it is necessary to air refuel enroute to enlarge their ranges. In the complex multi-aircraft refueling problem, solving refueling scheduling is a more complex problem that deals with the knowledge of navigation, operations research and optimization theory. This problem can be divided into two sections. First, refueling position, refueling fuel and refueling time must be determined. Second, refueling scheduling plan must be selected(1).

The first task of refueling scheduling is to locate the optimal refueling positions. Refueling positions are mainly determined by onload capabilities of receivers and offload capabilities of tankers. The onload or offload capabilities are functions of take-off gross weights, take-off fuel loads, take-off bases, destination bases and post refueling bases etc. If all other factors are held constant, each receiver's onload capability increases with flight time, and each tanker's offload capability decreases with flight time(2,3).

In order to study easily, some appropriate assumptions are required with the different refueling demands. Ref. 2 and Ref. 3 use the method that the receiver's onload capability is equal to the tanker's offload capability to determine refueling locations. This method suits for buddy-buddy approaching or flying to the same destination of tanker and receiver. For some other refueling cases, it is difficult to obtain an optimal solution. Optimization of a constrained spherical location problem is studied in Ref.4. The objective function is non-linear, and some of the constraints are a function of the decision variables. This method is complex in mathematics. Ref.5 researches the multi-aircraft refuelings, using the method of Network and Greedy.

In this paper, the following assumptions and notations are used:

1. The receivers' take-off original states (take-off weights, fuels, take-off times, take-off bases and destination bases) are fixed.

2. The tankers' take-off original states are determined by the refueling scheduling plan.
3. The earth is a perfect sphere, and receivers and tankers follow the great circle route in flighting from one point to another.

4. High weather (such as temperature) is neglected. Only constant wind is considered.

5. Refueling is a period process, not a point.

6. Flight weights and their fuel consumption rates of Tankers and receivers are allowed to vary.

7. Refueling times of one receiver in flight course is no more than twice.

8. After once refueling, the tanker lands at the appropriate post refueling base.

9. The numbers of receivers and tankers are not fixed, but their respective bases positions are fixed.

This paper's reference parameter is time or distance, according to the original states. Using optimization of a constrained enroute determines the optimal refueling positions and refueling times. Conjugacy matrix is used to solve the tanker and receiver scheduling.

II. Theory and Method

Optimal Refueling Position

Determination Receiver's Required Fuel. The difference between the fuel of receiver flighting in its whole enroute range and that of its take-off available fuel is its required fuel. The fuel is determined by iteration. The reason is that aircraft's weight will be changed while refueling occurred, and its fuel consumption will be changed.

In level cruise, the aircraft weight W equals left Y, and thrust T equals drag Q, so

\[
\frac{dW}{dt} = \frac{dW}{dx} \cdot \frac{dx}{dt} = - c_v T = - \frac{c_v W}{(Y / Q)}
\]

where, \( c_v \) is aircraft's fuel consumption rate. \( x \) is distance between take-off base and flight position. The combined parameter is assumed as:

\[
K = \frac{V(Y / Q)}{c_v}
\]

we have

\[
\frac{dW}{dx} = - \frac{W}{K}
\]

For the receiver, the general relationship between weight and distance is

\[
W_R(x) = W_{R0} e^{-\frac{x}{K_R}}
\]

or

\[
W_{R}(y) = W_{R0} e^{\frac{y}{K_R}}
\]

where, the receiver's take-off gross weight \( W_{R0} \) is known. \( W_{R0} \) is the basic weight that receiver reaches its destination base. It includes aircraft's empty weight, cargo weight and prepared fuels. It is also known.

If the optimal refueling point is at the distance \( r \), the required refueling fuels of receiver is obtained as follows,

\[
\Delta F_R = W_{R0}(L - r) - W_R(r)
\]

\[
= W_{R0} e^{\frac{L - r}{K_R}} - W_{R0} e^{-\frac{r}{K_R}}
\]

There, \( S \) is the distance of the receiver's whole range.

Determination the Possible Refueling Area. The possible refueling area is determined by the aircraft's properties and its surroundings. For example, as shown in Fig.1, the possible refueling point is between the earliest possible refueling point and the latest possi-

![Diagram](https://via.placeholder.com/150)

Fig.1 Determination the possible refueling area
ble refueling point on receiver’s flight route. The earliest possible refueling point is the point that receiver with full fuels can reach the destination base. The latest possible refueling point is the point that the receiver with its take-off available fuels can flight the farthest point. Before this area, receiver with refueling can’t carry its task. After this area, receiver can’t be refueled. In general, the earliest possible refueling point will not fall into aircraft’s climb part, and the latest possible refueling point will not fall into aircraft’s down part.

**Determination the Optimal Refueling Position.** Optimization refueling position is to minimize the whole fuel consumption according to the optimal function. It relates to tankers and receivers original states. \( W_g(t) \) is the function of the optimal refueling position \( r \). So it is optimized by Eq. (4). Different tankers serve the same receiver will have the different results, optimal refueling positions and required fuels. It is important for the whole refueling scheduling.

For once refueling, the general relationship between weight and flight distance of tanker is as Eq. (2) and Eq. (3)

\[
W_r(x) = W_{r_0} e^{-\frac{x}{R_r}}
\]  
(5)

or

\[
W_r(y) = W_{r_0} e^{\frac{y}{2R_r}}
\]  
(6)

It’s assumed that the offload fuel that the tanker can provide to the receiver is \( \Delta F_s \). \( l \) is the distance of the tanker reaching the refueling point. So

\[
\Delta F_s = W_{r_0} e^{-\frac{l}{R_r}} - W_{r_0} e^{\frac{l}{2R_r}}
\]  
(7)

where, \( W_{r_0} \) is the basic weight that the tanker reaches its post refueling base, and is known. \( S \) is the distance of the tanker’s whole range.

The original optimal refueling position is defined, according to the equivalence of onload and offload capability, i.e.

\[
\Delta F_s = \Delta F_s
\]

Then, \( W_{r_0} \) can be defined by formular from Eq. (4) and Eq. (7), then the tanker’s take-off time can be defined.

In this paper, the area of possible refueling method of iteration is used for searching the optimal result, refueling position \( r \) and refueling time \( T \). The error between the lasted two iterations is enough small, then \( \Delta F_s \) can be defined and the accurate refueling position can be defined.

**Twice Refueling**

If the required fuels of receiver is greater than the offload fuels of tanker once, twice refueling is considered. In this paper, the first refueling position should be early, and the receiver will be full with the tanker’s offload fuel. The method of determining the second refueling position is similar to the above one.

**Scheduling of Tankers and Receivers**

If the number of receivers is \( n \), the possible refueling points is \( 2n \). The number of tankers is \( m \). Mathematically, if \( K = \max(\ m, 2n \) \), the efficacy matrix is a square matrix. It’s optimal subjective function is

\[
GK = \sum_{i=1}^{k} \sum_{j=1}^{h} c_{ij}x_{ij} = Mln
\]  
(8)

Where, \( C_{ij} \) is the efficacy (fuel cost) coefficient of the tanker \( i \) assignning to the receiver \( j \). The constraint conditions is

\[
\sum_{j=1}^{h} x_{ij} = 1 \quad (i = 1,2,\ldots,k)
\]

\[
\sum_{i=1}^{k} x_{ij} = 1 \quad (j = 1,2,\ldots,k)
\]

\[
x_{ij} = 0 \quad \text{(tanker} \ i \ \text{not assigning to receiver} \ j \text{)}
\]

\[
x_{ij} = 1 \quad \text{(tanker} \ i \ \text{assigning to receiver} \ j \text{)}
\]

This conditions must be satisfied by the refueling point. We can use Conig efficacy matrix to solve the problem. Its theory basis is: the optimal assignment will not be changed while a line or a row of the efficacy matrix adds or reduces a constant. The optimal assignment is not completed until at least a zero element in one line or one row is reached.

1. \( m \geq 2n \) The available tanker number is greater than the required number of tanker. So, all refueling tasks will be finished.

2. \( m < 2n \) The available tanker number is less than the required number of tanker. Not all refueling tasks will be finished. We think that the available tankers is a kind of source. This problem is how to use the source fully to finish the task.
We have a assumption that the tanker have a waitline flight of 15 minutes when it reaches the refueling position early. The computation results of refueling positions, refueling times, refueling fuels and the scheduling assignment are shown in the following table 1 and table 2.

### Table 1  Refueling Positions and Refueling Times of Receivers

<table>
<thead>
<tr>
<th>Receiver No.</th>
<th>Required Fuel (kg)</th>
<th>First position</th>
<th>First Refueling time</th>
<th>Point No.</th>
<th>Second position</th>
<th>Second Refueling time</th>
<th>Point No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6280.12</td>
<td>(27.0,111.0)</td>
<td>09:24:14</td>
<td>2</td>
<td>(16.4,115.6)</td>
<td>11:23:13</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>5392.16</td>
<td>(16.0,111.8)</td>
<td>10:06:47</td>
<td>7</td>
<td>(4.4,109.4)</td>
<td>12:05:43</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>5444.32</td>
<td>(26.8,114.7)</td>
<td>09:38:07</td>
<td>4</td>
<td>(11.1,113.0)</td>
<td>12:06:59</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>3378.65</td>
<td>(13.5,107.5)</td>
<td>10:07:50</td>
<td>8</td>
<td>(0.0, 0.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7023.54</td>
<td>(10.0,109.5)</td>
<td>10:24:05</td>
<td>9</td>
<td>(12.7,111.2)</td>
<td>11:31:11</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>7166.86</td>
<td>(24.0,114.2)</td>
<td>09:37:49</td>
<td>3</td>
<td>(15.3,109.6)</td>
<td>12:14:06</td>
<td>17</td>
</tr>
<tr>
<td>7</td>
<td>6934.18</td>
<td>(24.7,113.4)</td>
<td>09:17:10</td>
<td>1</td>
<td>(15.4,108.2)</td>
<td>11:59:30</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>3889.86</td>
<td>(14.0,113.4)</td>
<td>09:50:29</td>
<td>5</td>
<td>(12.0,113.9)</td>
<td>11:24:56</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>* * *</td>
<td>( 0.0, 0.0)</td>
<td></td>
<td></td>
<td>( 0.0, 0.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4714.49</td>
<td>(16.1,110.3)</td>
<td>09:56:50</td>
<td>6</td>
<td>(14.3,113.5)</td>
<td>11:57:07</td>
<td>13</td>
</tr>
</tbody>
</table>

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### Table 2  Refueling Scheduling of Tankers

<table>
<thead>
<tr>
<th>Tanker No.</th>
<th>Take-off weight(kg)</th>
<th>Take-off time</th>
<th>Refueling point No.</th>
<th>Refueling time</th>
<th>Post Refueling-point fuel(kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>66730.9</td>
<td>08:38:41</td>
<td>8</td>
<td>3378.65</td>
<td>(30.1,110.4)</td>
</tr>
<tr>
<td>2</td>
<td>57571.6</td>
<td>11:18:11</td>
<td>14</td>
<td>3467.09</td>
<td>(23.0,113.0)</td>
</tr>
<tr>
<td>3</td>
<td>54221.7</td>
<td>09:08:05</td>
<td>3</td>
<td>3583.43</td>
<td>(25.0,121.1)</td>
</tr>
<tr>
<td>4</td>
<td>64368.8</td>
<td>07:16:31</td>
<td>7</td>
<td>2696.08</td>
<td>(20.0,110.1)</td>
</tr>
<tr>
<td>5</td>
<td>63459.9</td>
<td>09:51:08</td>
<td>11</td>
<td>1944.93</td>
<td>(20.0,120.0)</td>
</tr>
<tr>
<td>6</td>
<td>67544.6</td>
<td>10:06:59</td>
<td>16</td>
<td>2233.03</td>
<td>(25.0,105.0)</td>
</tr>
<tr>
<td>7</td>
<td>63054.7</td>
<td>10:37:22</td>
<td>13</td>
<td>2357.25</td>
<td>(19.0,115.0)</td>
</tr>
<tr>
<td>8</td>
<td>66914.1</td>
<td>09:10:27</td>
<td>6</td>
<td>2357.25</td>
<td>(24.0,120.0)</td>
</tr>
<tr>
<td>9</td>
<td>61135.7</td>
<td>08:15:10</td>
<td>5</td>
<td>1944.93</td>
<td>(20.0,110.0)</td>
</tr>
<tr>
<td>10</td>
<td>54425.1</td>
<td>08:40:41</td>
<td>2</td>
<td>3140.06</td>
<td>(33.0,115.0)</td>
</tr>
<tr>
<td>11</td>
<td>70078.1</td>
<td>10:11:13</td>
<td>15</td>
<td>2696.08</td>
<td>(20.0,120.0)</td>
</tr>
<tr>
<td>12</td>
<td>55300.8</td>
<td>08:45:48</td>
<td>1</td>
<td>3467.09</td>
<td>(25.0,105.0)</td>
</tr>
<tr>
<td>13</td>
<td>58439.7</td>
<td>08:13:39</td>
<td>4</td>
<td>3163.10</td>
<td>(19.0,115.0)</td>
</tr>
<tr>
<td>14</td>
<td>57063.0</td>
<td>10:20:58</td>
<td>10</td>
<td>3140.06</td>
<td>(24.0,120.0)</td>
</tr>
<tr>
<td>15</td>
<td>62162.1</td>
<td>10:26:13</td>
<td>17</td>
<td>3583.43</td>
<td>(20.0,110.0)</td>
</tr>
</tbody>
</table>
IV. Conclusions

This paper studies the determination of the optimal refueling position and the assignment on air refueling scheduling. Optimal refueling position is determined by optimization of a constrained enroute method. This method suits for all kinds of respective positions of tankers and receivers. Optimal refueling scheduling is determined by optimization of Conig efficiency matrix. This method is suitable for arbitrary number of tankers and receivers. The results are reasonable and practical, and can serve for air force directly.

Reference


(2). Coffman C. R., Finding Optimal Fuel and Mid-Air Refueling Location Requirements for C-5A Aircraft (R). Master’s Thesis. AD A1518307, 1984

