Structural Optimization in Preliminary Aircraft Design: a Finite-Element Approach

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Abstract
The development of a multi-level system for preliminary design of aircraft structures is proposed as part of a joint research project between the National Aerospace Laboratory (NLR) and the Faculty of Aerospace Engineering of Delft University of Technology. Modern computing facilities allow sophisticated methods such as finite element analysis to be used much earlier in the design process. The present system is essentially an extension of the existing Aircraft Design and Analysis System (ADAS). ADAS has been modified to allow the definition of major structural components like ribs, spars, frames, bulkheads, floor structures, etc., and semi-automatically generate a finite element model suitable for input to a structural analysis program. The FE-model can be automatically modified as a function of general design parameters such as wing area, sweepback angle, aspect ratio. Further work is being carried out to incorporate structural optimization. The paper describes the general philosophy as basis for the development of the system as well as topics for further research. The modified ADAS-system is described including the mesh generation procedure. Some structural analysis results are presented for a typical transport aircraft subject to static loads. In conclusion the mathematical implementation of structural optimization in a multi-level environment is outlined.

List of symbols
AMP - Analysis Model Parameter
b - blending function
C - linking matrix
F, F, f - objective function at level 1, 2 and 3
G, G, g - constraint function at level 1, 2 and 3
M, M, m - number of constraints at level 1, 2 and 3
N, N, n - number of design variables at level 1, 2 and 3
Nl, n - number of modules at level 2 and 3
p - nodal coordinates
r - blending function shape factor
t - material thickness
w - nodal displacement
X, X, x - design variables of a module at level 1, 2 and 3
Y, y - design variables of all modules at level 2 and 3
\sigma - stress level
1, 2, 3 - subscript: level 1, 2 and 3
u, l - superscript: upper limit, lower limit
- denotes a vector
\sim - denotes approximated constraint functions

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the structural design be taken into account at an early stage of the design process.

**General philosophy**

Some guidelines for the implementation of a design system of this kind have emerged during the early stages of its development:

- A 'black-box' system is definitely not intended. An optimization procedure can indicate a direction in which the designer might wish to proceed, but the designer must then consider the wider issues that can not enter into a design system such as presented here. The designer must remain in charge of the design progress.
- Flexibility is called for in selecting a method for some particular design task, appropriate to the level of accuracy required and the amount of information available at that stage of design.
- Consideration of the structure in too much detail would lead to an unacceptable disruption of the design process at the preliminary design stage. Therefore the information offered to the designer with regard to the structural design is kept to a minimum. In principle the designer is always presented with a structural design which has been optimized to a greater or lesser degree.
- This leads rather naturally to a multi-level system, in which the different tasks in the design process can be kept apart to a large extent. The traditional preliminary design - and interaction with the designer - takes place at the highest level, and structural design at the intermediate and lowest levels.
- Information primarily required by the designer, i.e. at the highest level of the multi-level system, is the structure weight and its distribution. Also the effect of changes in geometry on structure weight is required, i.e. appropriate sensitivity data. Deformation of the structure is required in so far as this affects the air load distribution on the aircraft. Also it must be possible to specify various constraints on the structural design - not only the mandatory requirements with regard to strength and aeroelastic behaviour but also minimum stiffness requirements, dimensional restrictions, and so on.
- A flexible modelling of the structure is therefore needed, i.e. one which can deal not only with the usual 'sizing' problem involving thicknesses, cross-sectional areas, etc. but also with changes of shape like wing span, sweepback angle, fuselage diameter. This implies an adaptive finite element mesh which automatically adjusts itself to changes of geometry and structural layout.
- The drawing of the aircraft, i.e. created at a CAD-workstation, should be the source of geometric data defining the design. Changes made in the drawing by the designer are then taken up in subsequent rounds of analysis and optimization. The drawing of the aircraft includes not only the external form, placing of payload, fuel, etc., but also the positioning of main structural components. The finite element mesh is generated within the main structure defined in this drawing.

As already indicated, a multi-level approach is considered to be the way best of implementing a design system based on the philosophy outlined above.

**Multi-level system**

A three-level system is defined for the different stages of analysis and optimization, as indicated in Figure 1. The tasks to be carried out at each level are as follows:

**Level 1**

This is the level at which the designer is active in preliminary design. This part of the system makes use of the existing ADAS preliminary design programs. These offer a wide range of options with regard to aerodynamics, performance, weights, costs, etc., largely based on semi-statistical methods. Introduction of structural design into ADAS is a logical further step in its development. ADAS enables either parameter studies to be carried out or a numerical optimization procedure to be selected. In both cases the parameters to be varied (design variables) are specified by the designer. For the structural design system currently proposed, ADAS has been extended to enable major structural components such as spars, ribs, frames and bulkheads to be defined. In addition, a mesh generation procedure has been developed that automatically produces a finite element model suitable for structural analysis and optimization at level 2. Extensions to ADAS and the mesh generation program are further described in section 2.

When optimization is performed at level 1, design variables are the preliminary design variables such as engine thrust ('rubberizing'), fuel capacity, maximum lift coefficient, etc., and other variables which also directly affect the structural design, like wing span, thickness/chord ratio, sweepback angle, etc. In addition, those variables from level 2 must be included which directly affect the constraints or objective function at level 1. In particular these are the variables defining the structure weight. Constraints at level 1 are those typically belonging to the preliminary design - range, take-off field length, etc. Representative constraints from level 2 are included to account for the necessary changes in structural design as a result of changes in design at level 1, e.g. change of sweepback angle. In this way the corresponding increase or decrease of structure weight is directly taken into account in the preliminary design. The objective function at level 1 is chosen by the designer, for example maximum fuel economy or the smallest maximum take-off weight for the aircraft.

**Level 2**

This is the level of overall design and optimization of the structure. At this level the finite element model created at level 1 is analysed, and a re-sizing procedure or numerical optimization can be carried out. The SMR B2000 program will be used for the FE-analysis with various enhancements required for optimization, including constraint gradient calculation. Modeling of the structure at this level aims to represent it only to the degree necessary for multi-disciplinary design, e.g. so that de-
cisions made in the aerodynamic design have their proper effect on structure weight. Structural modelling must be adequate to represent major redundancies, such as the load transfer at the wing/fuselage connection, input of floor loads into the fuselage shell, and redistribution of load around major cut-outs. Detail of actual stringers in the wing and fuselage and, for example, the variation of hoop stress between the frames in the pressure cabin are not called for at this level. The structural modelling, and verification that it provides an adequate stress distribution within the structure, are further described in section 3.

When used purely for the analysis of an existing design, level 2 returns stress levels and deformations of the structure to level 1. For a parametric study at level 1 it is expected that at least a systematic re-sizing procedure will have been carried out at level 2. This implies that allowable stresses have already been obtained from level 3. Loading on the structure for finite element analysis includes the aerodynamic pressure distribution, inertia loads and concentrated loads such as from engines, flaps and the undercarriage. The air load distribution can include the effect of deformations of the structure, e.g. wing bending and twist, while inertia loads include the mass of the structure itself.

For optimization at level 2, the design variables are the skin thickness and the equivalent thickness of the set of stringers forming each panel. Panel boundaries are related to physical boundaries in the structure, manufacturing joints, standard sheet sizes, and so on. A similar definition of design variables is made for spar webs, bulkheads, discrete reinforcing members, etc. For a composite structure, a larger number of equivalent thicknesses will be called for. In addition, those design variables from levels 1 and 3 which affect the constraints at level 2 are also included. Constraints are primarily stress constraints, but limitations on the deformation of the structure, aeroelastic constraints can also be chosen. The objective function is normally minimum structure weight. The optimization scheme, and the manner in which design variable linking is carried out, are described further in section 4.

Level 3

This is the level at which various detail design tasks are performed. Typical computer-tools to be used at this level are given in Refs. 1, 2 and 3. Since the major part of the development of the system at this level still has to be done, discussion here will be limited to some very general remarks. At this level the actual design of, for example, the stiffened panels forming much of the structure will be carried out. This implies that the equivalent thickness referred to above must be interpreted as an actual stringer, in a panel with skin thickness also obtained from level 2. Sensitivity data with regard to the stress levels achievable in an efficient detail design must be returned to level 2. Level 3 is not confined to panel stability problems. Suitable fatigue stress levels, hoop stress in the pressure cabin taking into account crack growth and the required inspection interval, and many other aspects of the detail design must be considered. Furthermore, in the case of a composite structure, all the information concerning the lay-up of the laminate must be generated. It would be inappropriate at level 3, seen as the lowest level in a multi-disciplinary design process, to engage in a complex analysis of all these detail design aspects. It is anticipated, therefore, that full use will be made of simplified methods, empirical data, and so on whenever this is possible. It should not be forgotten that, when the preliminary design has been completed to the satisfaction of the designer, the detail design still has to be carried out and at this stage a more accurate approach to detail design problems can be made.

2. FINITE ELEMENT MODEL GENERATION

Aircraft Design and Analysis System (ADAS)

The proposed system is an extension to the Aircraft Design and Analysis System (ADAS) which is a computer-based system intended for conceptual and preliminary aircraft design. The pilot version was completed in 1988 and since then many extensions and enhancements have been incorporated. The ADAS system provides a computer environment wherein design data and analysis methods are easily manipulated. Automated functions for routine tasks speed up or simplify the design process, while sufficient flexibility is retained to make ADAS applicable to a wide range of design studies.

ADAS system architecture

Figure 2 is a schematic representation of the generic ADAS system architecture. It identifies the major ADAS modules and information flows:

* DSProg (design program) is the name of a user-supplied Fortran subroutine which contains the algorithm to solve a particular design problem. DSProg may call subprograms available from the program library. The program library represents a method base with standard analysis methods and utility routines which can be used as building blocks to develop more complex design programs. The contents of the program library is regularly upgraded to incorporate new requirements and to accommodate advances in design technology.

* Geometric information is represented by a 3-view configuration drawing created with the CAD-program AutoCAD. Figure 3 gives an example of a typical ADAS/AutoCAD drawing. The aircraft must be defined according to a specific protocol which stipulates drawing conventions for each individual element, e.g. layer names, point sequence in a line, maximum number of points, etc. An interface program transfers significant geometry information from the AutoCAD drawing into the ADAS internal geometry representation via a Data Exchange Format (DXF) file. Non-geometry data are stored in regular text files.

* The ADAS Executive Program controls the processing of the
- In optimization mode control is passed to a Sequential Quadratic Programming (SQP) algorithm for optimizing a (nonlinear) objective function subject to (nonlinear) constraints by variation of selected design parameters (free variables).

Parametric survey mode and optimization mode may be combined. Which control mode to select depends largely on the kind of design problem, e.g. the number of design parameters, whether sensitivitiy information is required, the complexity of the analysis methods, the demand on computing resources, etc.

• AutoPLOT is a self-contained program that can generate several types of XYZ-plots from general parametric data. The graphs are reproduced in AutoCAD.

Definition of skin panels with AutoCAD

As stated in the introduction, in preliminary design it is not appropriate to generate a highly detailed structural model. An acceptable compromise is to use a simplified model for compound structures, such as skin + stringers, fuselage frames, etc. and to treat those as elements with specific mechanical properties. The advantage of this approach is that it simplifies not only the FE-model but also the mesh generation program.

The first task in the development of a mesh generator was to extend the ADAS/AutoCAD geometry interface. For example, the designer can use AutoCAD to define quadrangular areas representing the skin of the lifting surfaces and the fuselage, as shown in Figure 5. These mesh areas indicate regions on the outer skin where a regular mesh of elements is to be generated using the computer-tools described in the following section.

Mesh generation

The second task involved the development of a general mesh generation procedure. Because of the many alternative in wing locations, tailplane configurations, engine mounting, as well as different possible solutions for a particular structural design problem, etc., it is impractical to develop a single, stand-
alone program that can generate a satisfactory mesh distribution for any aircraft configuration. Instead, the concept of a 'toolbox' of subprograms was adopted that can be used by the designer to assemble a specific mesh generation program. This approach not only provides a high degree of flexibility but is also relatively easy to implement.

A consistent procedure is used to create a regular mesh of 4-node elements for a 3D surface enclosed by 4 given spatial boundary curves, as shown in Figure 6. The internal nodes drawing can be limited. Other examples are subprograms to construct floor structures and flat or spherical bulkheads. Two-noded bar elements can be defined for local reinforcement. At present bar elements have to be connectected to existing nodes.

In principle, the order in which mesh areas are generated is not prescribed. When a mesh area has been generated, the subprogram returns a unique index number which can be used to reference that mesh area or the nodes therein later in the program. In addition, each mesh area can be associated with a multi-digit identification number which has the following significance:

- The sub-structure label indicates to which sub-structure, e.g. 'wing', 'fuselage' or 'tailplane', the element belongs to. For example, some elements in the wing/fuselage intersection can be defined as being part of the 'wing' while others are part of the 'fuselage'. In the FE-model sub-structures are represented as 'transparent' objects which transverse nodes but are not mechanically connected. Therefore the designer must indicate how and at which nodes sub-structures are linked. Structural analysis programs can handle sub-structures separately as 'branches'.

- The element label refers to mechanical properties in the structural elements library. For elements representing a compound structure, such as stiffened panels (skin + stringers), fuselage frames (beam + castellations), etc., specific properties can be defined.

- The part label is used to indicate significant structural components, e.g. 'spar', 'rib', 'fuselage frame', 'skin', etc. This information is used by the structural analysis program to place the external loads and impose displacement restrictions.

ADAS builds the FE-model entirely in an internal data structure using vector arrays for memory efficiency. When a FE-model is completed it can be written to a file in a format compatible with the structural analysis program, as illustrated in Figure 7. The subprogram DSEXLM provides this purpose first eliminates coincident nodes within each substructure. The remaining nodes are numbered in sequence and the corresponding X,Y,Z-ordinates are written to file. Next the individual elements are labelled and the corresponding node numbers and identification labels are appended to the file.

The internal ADAS FE-model can be translated into a DXF-file with the subprogram DSEXFM to view and inspect the model within AutoCAD, as shown in Figure 8. The design...

![Figure 6: Mesh area enclosed by 4 boundary curves.](image)

\[
\begin{bmatrix}
p(u,v) = p(0,0) b_0 + p(1,0) b_1 + p(u,0) b_2 + p(u,1) b_3 - p(0,1) b_v - p(1,1) b_v + p(0,0) b_v \n
0 \leq u \leq 1
\n0 \leq v \leq 1
\nwhere:
\n\begin{align*}
b_0 &= 1 + (16 r_x - 8) u^4 + (18 - 32 r_x) u^3 + (16 r_x - 11) u^2 (0 \leq u \leq 1) \\
b_1 &= 1 - b_v \\
b_2 &= 1 + (16 r_y - 8) v^4 + (18 - 32 r_y) v^3 + (16 r_y - 11) v^2 (0 \leq v \leq 1) \\
b_3 &= 1 - b_v
\end{align*}
\]

The factors \( r_x \) and \( r_y \) control the shape of the blending curves, e.g. \( r_x = r_y = 0.5 \) give cubic polynomials. This procedure takes place in two steps. First, the coordinates of the nodes on the boundary curves are computed. For example, a subprogram is provided to compute the boundary curves for a mesh area as defined in an ADAS/AutoCAD drawing using the geometry of airfoil sections specified at several spanwise stations and assuming a ruled surface between them. Another subprogram then generates the actual grid. Other subprograms are available to set up boundary curves by 'copying' nodes of previously created mesh areas. For example, a spar or rib structure can be constructed between nodes in the upper and lower skin panels.

In this way the topology remains automatically intact even if the global geometry, i.e. aspect ratio, wing area, sweepback angle, is changed. It also saves CPU-time as nodes do not have to be recomputed. Furthermore, the number of individual structural components to be defined in the ADAS/AutoCAD...
the model is kept to a minimum. While the complexity of the model is reduced, care should be taken that essential features of the structural behaviour are not sacrificed. Deciding which features are essential and which are not is of paramount importance. This section discusses some methods for improving the structural behaviour of a simplified FE-model.

**Stressed skin elements**

The coarseness of the mesh, as evident in Figure 9, implies that details such as the cross-section of individual stringers are too fine to handle in an FE-model at level 2. Stressed skin elements are therefore assumed to act essentially as membrane elements. Stiffened skin elements of any material or construction type are modelled using several layers of material. In the case of a conventional aluminium structure, one layer of isotropic material represents the skin itself. A second homogeneous layer, with unidirectional stiffness only, represents the properties of the stiffeners. The design variables for such a compound element are the thickness of each layer. In the case of a composite structure, three or more orthotropic layers represent the orientation and thickness of the laminae, the design variables being the orientation and the thickness of these layers. These design variables have the same values for all the elements in a mesh area.

**Stressed skin panels**

A mesh area representing, for example, a skin panel may have several unsupported ‘free’ nodes as shown in Figure 10. As the membrane elements of the skin do not have in-plane stiffness, these nodes must be supported. This problem is solved in a way that further reduces the number of degrees of freedom. The normal displacements of free node \(i\) is constrained by linear interpolation of the displacements of the nearest structurally supported nodes at \(i = 1\) and \(i = n\), according to:

\[
    w_i = w_1 + \frac{n-i}{n-1} (w_n - w_1)
\]

This approach yields two degrees of freedom for a free node, and three degrees of freedom for a structurally supported node. The total number of degrees of freedom of the model has thus been reduced to less than 50% of the alternative, i.e. using elements with out-of-plane bending stiffness with six degrees of freedom per node.
Lifting surfaces

Skin panels, spar webs and rib webs of lifting surfaces are modelled using the compound membrane elements described above. Bar elements are added where skins and webs are attached, so that loads may effectively be introduced. The normal displacements of the free nodes are interpolated in chordwise direction between the front and rear spar. It will be observed from Figure 10 that the spar webs are built up using single elements to cover the entire spar depth. This causes in-plane bending in the spar web elements as the wing deflects. The in-plane bending stiffness of various available 4-node shell or membrane elements differs considerably, and in general is much greater than would be expected from an Engineer’s Theory of Bending consideration. This results, for example, in considerably higher divergence and flutter speeds. Use of a finer mesh is an obvious solution, but would also result in a much larger number of degrees of freedom. In order to provide a realistic in-plane bending stiffness a ‘dummy’ material with corrected stiffness constants is used. Fortunately, the accessibility of the B2000 finite element code makes it relatively easy to implement a ‘special purpose’ element specifically for spar webs.

Figure 11 shows a contour plot of calculated Von Mises stress levels in an ADAS wing skin under an elliptical load

![Figure 11: Calculated stress contours in a wing box structure as compared to experimental results.](image)

should be noted that as the fuselage structure was fixed at the wing/fuselage intersection, down load on the horizontal tail-plane causes a small positive tilt. Finally, it should be pointed out that this type of frame modelling is not considered suitable for heavy frames such as employed in the centre-section and as reinforcements for large cut-outs. In these cases, frame elements with a realistic axial stiffness will be used, as described in the next section.

Large cut-outs

As pointed out earlier, there is a limit to the amount of detail that should be included in the FE-model. A typical feature of fuselage structures is the presence of numerous cut-outs of varying sizes, such as windows, passenger and cargo doors, wheel bays, etc. It is apparent from Figure 9 that the presence of the window cut-outs is impossible to take into account, as one skin element is roughly the size of a window. With the coarseness of the mesh, it is doubtful whether even larger cut-outs, such as doors and wheel bays, can be modelled very well. In addition, if only one half of the fuselage is to be designed, for reasons of economy or speed, this will imply that only a symmetric configuration with similar doors on each side can be modelled. However, even if the cut-outs themselves cannot be modelled in detail, their influence on the surrounding structure can still be taken into account. For this reason, cut-outs are included in the structure model as follows.

In the structural elements library, a material with very low stiffness constants is defined. The ‘removed’ skin representing the cut-out is modelled using this material, so that the normal and shear stress carrying capability of the skin is neutralized. The reason for using this rubber-like material instead of simply removing the skin elements is that, with the door still physically present, pressure loads may be applied to the skin in a regular way. To transfer these loads to the edges of the cut-out,
longitudinal beams with zero axial stiffness are placed in the cut-out, as shown in Figure 13. These beams thus perform the

\[ \text{Figure 13: Structural modelling of large cut-outs.} \]

structural task of a physical door. A keel beam is provided under the floor, serving as a crash beam and also provides an alternative load path near the cut-outs. A cut-out is reinforced by heavy frames directly fore and aft, and by beam elements on the top and bottom edges of the cut-out. These beams are extended beyond the frames in the form of bar elements without bending stiffness.

Centre-sections

A considerable variety is found among actual centre-section designs, making it difficult to define a standard centre-section for the purpose of preliminary design. However, the following features are generally found in the centre-section of transport aircraft: Structural irregularities, such as cut-outs and reinforcements, are concentrated in the fuselage, so that the wing box structure passes through the fuselage relatively undisturbed. Heavy root ribs are mounted in the wing, usually just outside the fuselage. At these points the wing is attached to heavy fuselage frames, located at the same stations as the wing spars in the centre-section.

The model of the centre-section is made on the basis of the common features outlined above. For minimum wing-fuselage interference, the concept of ‘transparent’ sub-structures has been introduced. In a finite element model, two structures may intersect without the one affecting the other. The actual connection between the two parts is established by modelling a ‘connecting’ structure using standard shell and bar elements, as shown in Figure 14. The resulting structure is more realistic than it may at first appear. In some centre-section designs, the fuselage stringers are carried through the wing box - a physical form of ‘transparent’ structures. If the designer is not satisfied with a straightforward ‘transparent’ centre-section, the structure can be modified, e.g. by removing patches of fuselage skin and adding reinforcements around the resulting cut-out (see previous section). A significant degree of flexibility is offered to the designer, the only restriction being that fuselage frames with axial stiffness must be located at the wing spar joints.

\[ \text{Figure 14: Wing/fuselage connecting elements.} \]

Loads

The analysis results included in the next section were based on greatly simplified aircraft loads. However, a program is being implemented to automatically determine design load cases in compliance with the pertinent airworthiness requirements. Using standard ADAS aerodynamic tools, manoeuvre and gust envelopes are established for a number of critical combinations of altitude and aircraft weight. For each of these load cases, aerodynamic forces, ground reactions and engine thrust are calculated. These forces cause the aircraft to accelerate in a manner which depends on the mass distribution. A number of possible critical mass distributions are considered, and the resulting inertial forces are applied to the FE-model to balance the aerodynamic and other loads.

The present load calculation procedure assumes that the aircraft is a rigid body. Dynamic flexibility effects will be included at a later stage, under the assumption that structural deformation does not affect the aerodynamic loads.

Structural analysis for a hypothetical loading condition

The FE-model shown in Figure 9, completed with connecting elements and cut-outs as described is the previous sections, was subjected to a hypothetical static load case, shown in Figure 15. The floor and tail loads were derived by assuming vertical force and pitching moment equilibrium, respectively.

\[ \text{Figure 15: Hypothetical load case for a complete aircraft structure.} \]

Figure 16 shows a grayscale rendering of the resulting Von Mises stress levels. For the sake of clarity, the deformations have been magnified by 2.5. All elements have the same material thickness, so that areas of high stresses, i.e. at the wing/fuselage joint and at the root of the vertical tailplane, can be
4. STRUCTURAL OPTIMIZATION

Structural optimization with ADAS uses a multi-level approach. There are 3 levels, in order of refinement:

level 1: multi-disciplinary preliminary design,
level 2: mono-disciplinary preliminary design,
level 3: mono-disciplinary detail design of structural components.

Although much work has been done in the development of computer-tools for aerodynamic design and analysis\[^{[12]}\], structural design is currently the only discipline in ADAS which can be optimized at all three levels.

Multi-level optimization

As this part of the project is still in the development stage, the discussion will be confined to the mathematical implementation of structural optimization in a multi-level system.

Multi-level optimization is more than optimizing each level separately. It implies that the interaction between levels is accounted for in such a way that an optimum is found for the whole problem instead of a set of local optima for each level. This topic is still a subject of research\[^{[13],[14],[15]}\]. The optimization scheme envisioned for the current system resembles those proposed by Vanderplaats\[^{[13]}\] and is schematically illustrated in Figure 18. A design task at level 1 can be stated in mathematical terms as: find the set of level 1 variables \(\bar{X}\) and level 2 variables \(\bar{Y}\) that will minimize \(F(\bar{X}, \bar{Y})\) subject to:

\[
\begin{align*}
G_j(\bar{X}, \bar{Y}) & \leq 0 & j = 1, M \\
G_{jk}(\bar{X}, \bar{X}_k) & \leq 0 & j = 1, M_k, \quad k = 1, NL \\
X_{1j}^L & \leq X_j & j = 1, N \\
X_{ik} & \leq X_{ik}^U & i = 1, N_k, \quad k = 1, NL
\end{align*}
\]

with:

\[
\begin{align*}
\delta_{\text{actual}} & = \delta_{\text{new}} \quad \delta_{\text{allow}} \quad \text{until the local stress level } \delta_{\text{actual}} \text{ at each element is equal to a specified stress level } \delta_{\text{allow}}. \text{ For example, the FE-model shown in Figure 9 may contain a large number of redundancies, especially in the centre-section and in the wheel bay immediately behind it. A high degree of redundancy shows the convergence of a re-sizing or optimization procedure. The FE-model has been subjected to a fully stressed design procedure with the same loading case as used in the previous section. Figure 17 shows the stress distribution after a single iteration step. The continuous parts of the structure have converged well, while in the centre-section convergence is slower. Even so, all stresses have been brought within roughly 30% of the allowable stress level.}
\end{align*}
\]
\[ \tilde{G}_{jk}(\bar{X}, \bar{X}_k) = \tilde{G}_{jk}^0 + \nabla_1 \tilde{G}_{jk}(\bar{X}, \bar{X}_k) \Delta \bar{X} \]
\[ + \frac{1}{2} \Delta \bar{X}_k^T \left[ \nabla_2^2 \tilde{G}_{jk}(\bar{X}) \right] \Delta \bar{X}_k \]
\[ + \frac{1}{2} \Delta \bar{X}_k^T \left[ \nabla_2^2 \tilde{G}_{jk}(\bar{X}_k) \right] \Delta \bar{X}_k \]

Notice that \( \bar{Y} = (\bar{X}_1, \bar{X}_2, ..., \bar{X}_{n1}) \). At level 2, the problem to be solved is: find the set of level 2 variables \( \bar{X}_k \) and level 3 variables \( \bar{y}_k \) that will minimize \( F_k(\bar{X}_k, \bar{y}_k) \) subject to:

\[
\begin{align*}
G_j(\bar{X}, \bar{X}_k) &\leq 0 & j &= 1, M, \\
G_{jk}(\bar{X}, \bar{X}_k) &\leq 0 & j &= 1, M, \\
G_{jk}(\bar{X}_k, \bar{y}_k) &\leq 0 & j &= 1, M, \\
\tilde{g}_{jk}(\bar{X}_k, \bar{x}_k) &\leq 0 & j &= 1, m, & 1 &= 1, n, \\
X_{ik}^1 &\leq x_{ik}^u & i &= 1, n, \\
x_{ik}^l &\leq x_{ik}^u & i &= 1, n, & 1 &= 1, n, \\
\end{align*}
\]

with:

\[ \tilde{G}_j(\bar{X}, \bar{X}_k) = \tilde{G}_j^0 + \nabla_1 \tilde{G}_j(\bar{X}, \bar{X}_k) \Delta \bar{X}_k \]
\[ + \frac{1}{2} \Delta \bar{X}_k^T \left[ \nabla_2^2 \tilde{G}_j(\bar{X}) \right] \Delta \bar{X}_k \]

and:

\[ \tilde{g}_{jk}(\bar{X}_k, \bar{x}_k) = \tilde{g}_{jk}^0 + \nabla_2 \tilde{g}_{jk}(\bar{X}_k, \bar{x}_k) \Delta \bar{X}_k \]
\[ + \frac{1}{2} \Delta \bar{X}_k^T \left[ \nabla_2^2 \tilde{g}_{jk}(\bar{X}_k) \right] \Delta \bar{X}_k \]

Notice that \( \bar{y}_k = (\bar{x}_k, \bar{x}_{k2}, ..., \bar{x}_{kn}) \). At level 3, the problem to be solved is: find the set of level 3 variables \( \bar{x} \) that will minimize \( f_k(\bar{x}_k) \) subject to:

\[
\begin{align*}
\bar{G}_{jk}(\bar{x}_k, \bar{x}_k) &\leq 0 & j &= 1, M, \\
\bar{g}_{jk}(\bar{x}_k, \bar{x}_k) &\leq 0 & j &= 1, m, \\
\bar{x}_{ik}^l &\leq \bar{x}_{ik}^u & i &= 1, n, \\
\end{align*}
\]

with:

\[ \bar{G}_{jk}(\bar{x}_k, \bar{x}_k) = \bar{G}_{jk}^0 + \nabla_1 \bar{G}_{jk}(\bar{x}_k, \bar{x}_k) \Delta \bar{x}_k \]
\[ + \frac{1}{2} \Delta \bar{x}_k^T \left[ \nabla_2^2 \bar{G}_{jk}(\bar{x}_k) \right] \Delta \bar{x}_k \]

The sensitivity information, i.e. the approximated constraint gradients (denoted by \( \tilde{\cdot} \)) are calculated either semi-analytically or by finite difference. The quadratic terms are numerically constructed based on update formulas (BFGS).

Implementation of this scheme in ADAS offers additional features:

- The objective and constraint functions at a lower level can be used as variables at the next higher level.
- Variables at a lower level can be 'linked' to reduce the number of design variables at the higher level (see next section). However, the lower level will use the unlinked variables.
- Standard optimization algorithms can be used at each level to perform the optimization.
- It is assumed that the highest level is not directly influenced by the lowest level and vice versa. This implies that there are no constraints which depend on variables of all three levels.
- At a given level, constraints from other levels are used in an approximated form. Time consuming constraints of the given level may themselves also be used in approximated form. Each time a level is optimized, such a constraint and its gradients are calculated, after which the approximated functions are used.
- Not all the constraints of another level need to be considered. Only the subset (denoted by \( \gamma \)) of constraints from another level which are also a function of the variables of the level under consideration are needed.
- It is assumed that modules at a given level do not interact.

The key idea here is that constraints at a higher level are passed on to the lower level in approximated form, and vice versa. Because of the approximated form used, the computational cost of evaluating the constraints and gradients during optimization is dramatically reduced. Optimization at each level is an iterative process. The final optimum is achieved if this iterative process converges. The user can select the order is which the different levels are to be optimized. Although convergence of this optimization process cannot be proven mathematically, in practice the procedure tends to converge.

**Variable linking**

By selecting variables, constraints and objective, as well as the optimization strategy and other optimization directives on each level, the designer creates a so-called optimization model. Besides selection of variables, constraints etc., additional data has to be given. For constraints this is information like: allowable stresses, deformation restrictions, etc. Because this model is partially variable linking matrices. A linking matrix specifies a linear relation between variables on the model and the design variables (X). These details are called: Analysis Model Parameters (AMP). These relations can be seen as a rectangular matrix, the linking matrix C:

\[ [\text{AMP}_{\text{amp}}] = [C_{ij}] [\text{AMP}_{\text{amp}}] = [C_{ij}] [\text{AMP}_{\text{amp}}] \]

Consider, for example, the FE-model. The AMP's of this model are nodal coordinates, thicknesses etc. During optimization the thickness of the fuselage skin could change, which implies that the some elemental thicknesses have to vary. The relation between (the change of) the AMP's (element thicknesses) and the (change of) design variable 'fuselage skin thickness' is one column in the linking matrix. Because the process of defining the optimization model is very flexible, it is also possible to perform a one level optimization. For example, semi-empirical formulas can be used to predict the structural weight in order to get a good starting point for more accurate calculations with the FE-method.

5. CONCLUSIONS AND FUTURE DEVELOPMENTS

The development of a computer-based system for the preliminary design of aircraft structures has been proposed. The joint research project aims to investigate the possibility of introducing structural design early in the design process. A multi-level approach has been selected which is compatible with the existing Aircraft Design and Analysis System (ADAS) architecture. A program has been developed which rapidly generates a simplified FE-model of an ADAS/AutoCAD 3-view configuration drawing.
The problem of using a simplified FE-model with sufficiently realistic structural behaviour has been solved by using universal elements with modified stiffness coefficients to account for compound structural components, such as stiffened skin panels, spar webs, fuselage frames, etc. By placing displacement constraints on skin panels, in-plane bending can be eliminated, greatly reducing the number of degrees of freedom. An approach for modelling complicated structures, such as large cut-outs and centre-sections, has been discussed. Structural analysis was carried out for a typical aircraft structure (wing + fuselage + tail), subject to a hypothetical load case. Considering the coarseness of the FE-model, the general structural behaviour is satisfactory. The principle of a Fully Stressed Design (FSD) to obtain a first-order estimation for the element thicknesses has been demonstrated. The mathematical implementation of structural optimization in a multi-level system has been described. This approach is based on passing design variables, constraints and sensitivity information from one level to another.

Research effort in the near future will be directed to the implementation of a panel code as a more consistent method to predict the aerodynamic loads on the structure. Dynamic structural analysis, including flutter and transient loads, is foreseen as part of a follow-up project.

6. REFERENCES


