Numerical Solution of Radiative Flowfield on the Nose Region of Blunt Bodies

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Abstract

The radiative viscous flow over hypersonic blunt bodies is calculated numerically. Chemical nonequilibrium and equilibrium are considered with viscous shock layer equations. Both line and continuum radiation models are employed. One-dimensional planar medium assumption is used to deal with radiative transport so that the spatial marching technique is available in numerical calculation. The numerical results show that the convective heating transfer rate prevails as the reentry velocity lower than 7.6 km/sec at high altitude, the radiative heating transfer rate increases with the increase of freestream velocity and density.

Nomenclature

\[ B^* \] Planck spectral radiative intensity
\[ C_i \] mass fraction of species i, \( \rho_i^* / \rho^* \)
\[ C_p^* \] specific heat at constant pressure, \( C_p^* / C_p^{\infty} \)
\[ F^* \] radiative flux
\[ h \] static enthalpy, \( h^* / U_{\infty}^{*2} \)
\[ h_i \] static enthalpy of species i, \( h_i^* / U_{\infty}^{*2} \)
\[ h_j \] shape factors for a general orthogonal coordinate system, \( h_j = 1 + \kappa y \)
\[ \kappa \] wall curvature, \( \kappa R_{\infty}^* \)
\[ I^* \] radiative intensity
\[ k \] thermal conductivity
\[ Le \] Lewis number
\[ M_i \] molecular weight of species i
\[ M \] average molecular weight of mixture
\[ n_s \] the number of species
\[ p \] pressure, \( p^* / (\rho_{\infty}^* U_{\infty}^{*2}) \)
\[ Pr \] Prandtl number
\[ q_w^* \] heating transfer rate, \( q_w^* / (\rho_{\infty}^* U_{\infty}^{*2}) \)
\[ R_n \] nose radius
\[ R \] universal gas constant
\[ T \] temperature, \( T^* / T_{ref}^* \)
\[ T_{ref}^* \] reference temperature, \( U_{\infty}^{*2} / C_p^{\infty} \)

\( U_\infty^* \) freestream velocity
\( u^* \) velocity component parallel to the body surface, \( u / U_{\infty}^* \)
\( v^* \) velocity component normal to the body surface, \( v / U_{\infty}^* \)
\( w \) local change rate of species due to chemical reactions
\( x^*, y^* \) body-oriented coordinate system, \( x^* / R_{\infty}^*, y^* / R_{\infty}^* \)
\( \alpha \) volumetric absorption coefficient
\( \beta \) mesh refinement parameter
\( \varepsilon \) Reynolds number parameter,
\( \varepsilon = [\mu_{ref}^* / (\rho_{\infty}^* U_{\infty}^{*2} R_{\infty}^*)]^{1/2} \)
\( \delta \) shock standoff distance, \( \delta^* / R_{\infty}^* \)
\( \xi, \eta \) normalized coordinates
\( \sigma \) shock angle
\( \varphi \) body angle

Subscripts

i ith species
ref reference condition
s quantities immediately behind the shock
w body surface quantities
\infty freestream condition

Superscript

* dimensional variable

I Introduction

As the space vehicles reenter the atmosphere at hypersonic speed, especially for blunt bodies with large nose radius, dissociation and ionization take place in the shock layer since the temperature is much high. Radiation transport is also an important phenomena of the flowfield, and it has the effect on the surface heating transfer rate. The radiative flowfield of reentry vehicle can be analyzed with the viscous shock-layer (VSL)
method, which is an approximation of Navier–Stokes equations for viscous flow. The VSL method maintains the advantage of Boundary Layer method and avoids the trouble caused by determining the outer edge of boundary layer. The bow shock is taken as outer edge of the flow, and a set of uniform governing equations is used in the whole region from the body to the shock. The governing equations can be solved with spatial marching technique so that less computing time is needed than that of the time–dependent method.

Based on the viscous shock–layer model, nonequilibrium and equilibrium chemistry, as well as radiation transport are considered for the real gas effects. As the radiation is considered, the energy equation is in an integro–differential form and the radiation flux is strong nonlinear coupled with temperature, thus it is more difficult to get the temperature profiles from solving the energy equation. The radiative intensity in all directions must be taken into account as the angular integration is carried out for the radiation flux, which makes the properties of the flowfield at one station depending on the downstream region. For nonequilibrium flow, the assumption of frozen–shock condition is overly estimated the temperature immediately behind the shock, and it needs to be improved as the radiation is considered with one temperature model. In order to obtain the solution of radiative reentry flowfield with spatial marching technique, one–dimensional planar geometry assumption is employed for radiative transport. The radiative flux divergence in energy equation is treated as a source term, and it is relaxed to get temperature profiles by iteration.

II Analysis

Governing Equations

Viscous shock–layer equations coupling with radiation contain a term of radiative flux divergence in energy equation, and a radiative transport equation is added to determine the radiative intensity. For nonequilibrium flow, the species continuity equations are also needed. In the axisymmetric body oriented coordinate system (see Fig.1), the governing equations are as follows:

global continuity equation

\[
\frac{\partial}{\partial x} (h_3 \rho u) + \frac{\partial}{\partial y} (h_1 h_3 \rho v) = 0
\]

x–momentum equation

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \kappa \rho u = - \frac{\partial p}{\partial x} + \frac{\mu}{h_1 h_3} \frac{\partial}{\partial y} \left[ h_1 \frac{\partial u}{\partial y} \right] + \frac{\mu}{h_1 h_3} \left( \frac{\partial u}{\partial y} \right)^2
\]

y–momentum equation

\[
\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \kappa \rho u = - \frac{\partial p}{\partial y} + \frac{\mu}{h_1 h_3} \frac{\partial}{\partial x} \left[ h_1 \frac{\partial v}{\partial x} \right] + \frac{\mu}{h_1 h_3} \left( \frac{\partial v}{\partial x} \right)^2
\]

energy equation

\[
\rho u C_p \frac{\partial T}{\partial x} + \rho v C_p \frac{\partial T}{\partial y} = - \frac{\partial}{\partial x} \left( u \frac{\partial p}{\partial x} \right) - \frac{\partial}{\partial y} \left( v \frac{\partial p}{\partial y} \right)
\]

\[
+ \frac{\epsilon^2}{h_1 h_3} \frac{\partial}{\partial y} \left( h_1 \frac{\partial T}{\partial y} \right) + \frac{2}{\mu} \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} \right)^2
\]

\[
+ \frac{\mu \kappa}{Pr} C_p \frac{\partial T}{\partial y} - \frac{\epsilon^2}{h_1 h_3} \frac{\partial}{\partial y} \left( h_1 \frac{\partial w_i}{\partial y} + Q_R \right)
\]

for chemical nonequilibrium flow

\[
\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = - \frac{\partial}{\partial x} \left( u \frac{\partial p}{\partial x} \right) - \frac{\partial}{\partial y} \left( v \frac{\partial p}{\partial y} \right)
\]

\[
+ \frac{\epsilon^2}{h_1 h_3} \frac{\partial}{\partial y} \left( h_1 \frac{\partial T}{\partial y} \right) + \frac{2}{\mu} \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} \right)^2
\]

\[
+ \frac{\mu \kappa}{Pr} C_p \frac{\partial T}{\partial y} - \frac{\epsilon^2}{h_1 h_3} \frac{\partial}{\partial y} \left( h_1 \frac{\partial w_i}{\partial y} + Q_R \right)
\]

for chemical equilibrium flow

\[
\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = w_i
\]

\[
+ \frac{\epsilon^2}{h_1 h_3} \frac{\partial}{\partial y} \left( h_1 \frac{\partial C_i}{\partial y} \right)
\]

\[
i = 1, ..., n
\]

for chemical nonequilibrium flow

\[
C_i = C_i (p, h)
\]

for chemical equilibrium flow

\[
P = \frac{\rho RT}{M C_p}
\]

for chemical nonequilibrium flow

\[
\rho = \rho (p, h)
\]

for equilibrium flow

\[
\nabla \cdot \vec{F} = \alpha [B_r - I_r]
\]

where \( \vec{F} \) is the unit vector along optical path.

Radiative Transport

Radiative flux at one point in the shock layer

\[
\vec{F} = \int_{\Omega} \int_{\Omega} I \vec{F} \, d\Omega \, d\Omega
\]

where \( d\Omega \) is solid angle about the unit vector \( \vec{F} \), \( \nu \) is frequency.

The radiative flux divergence appeared in energy equation is

\[
Q_R = - \nabla \cdot \vec{F}
\]

Since the shock layer is thin at the hypersonic reentry velocity, it is assumed that the gradient in normal direction of body surface is much larger than that in tangent direction, thus the radiation transport equation can be expressed as follows:
\[
\begin{align*}
\cos \psi = - (B_y - I_y) \quad (13)
\end{align*}
\]

where \( \psi \) is the angle between radiative flux vector and the normal direction from the body surface.

The radiative flux divergence \( Q_R \) in energy equation can be expressed as:
\[
Q_R = - \nabla \cdot F
= \int \int \alpha_r (I - B_y) \, d\Omega \, d\nu
\quad (14)
\]

the range of integration for \( d\Omega \) in equation (14) is carried out through 4\( \pi \) solid angle, thus the radiative intensity coming from downstream region has the effect on the upstream properties. In order to make the spatial marching technique available, the angular integration in equation (14) is carried out using the forward--reverse approximation, that is, the intensity is divided into two angular groups: those rays passing through the plane of symmetry in the positive direction from wall to shock and those rays passing in the negative direction from shock to wall. The forward--reverse approximation represents all rays in the positive direction by a single ray \( I_r^+ \) with an average direction cosine \( 1/I \), and all rays in the negative direction by a single ray \( I_r^- \) with also an average direction cosine \( 1/I \), see Fig.2.

Solving equation (13) with the assumption of one-dimensional planar radiation model transport, we have:
\[
\begin{align*}
I_r^+ (y) &= \int \alpha_r (y) B_r (y) \exp \left[ \frac{1}{I} \int \alpha_r (y') dy' \right] \frac{1}{I} \, dy' \quad (15) \\
I_r^- (y) &= \int \alpha_r (y) B_r (y) \exp \left[ \frac{1}{I} \int \alpha_r (y') dy' \right] \frac{1}{I} \, dy' \quad (16)
\end{align*}
\]

The absorption coefficient \( \alpha_r \) is divided into two parts
\[
\alpha_r = \alpha_r^C + \alpha_r^L \quad (17)
\]

where \( \alpha_r^C \) and \( \alpha_r^L \) corresponds to continuum and line contribution of the radiation.

Introduce emissive function \( E(y', y) \) and equivalent width variable \( W_i (y, y') \) as follows: (15)
\[
\begin{align*}
E(y', y) &= 1 - \exp \left[ \frac{1}{I} \int \alpha_r (y') dy' \right] \quad (18) \\
W_i (y, y') &= \exp \left[ \frac{1}{I} \int \alpha_r (y') dy' \right] \left[ 1 - \exp \left[ \frac{1}{I} \int \alpha_r (y') dy' \right] \right] \quad (19)
\end{align*}
\]

Radiative flux \( F \) can be expressed in two parts corresponds to \( \alpha_r^C \) and \( \alpha_r^L \) as follows:
\[
F = F^C + F^L 
\quad (20)
\]

where:
\[
\begin{align*}
F^C &= \pi \int \int B_r \, dE(y', y) - \int B_r \, dE(y, y') \, dv \\
F^L &= \pi \sum \int \int B_r \, dW_i (y', y) - \int B_r \, dW_i (y, y') \, dv
\end{align*}
\]

The radiative flux divergence \( Q_R \) in energy equation can be expressed as:
\[
Q_R = Q^C + Q^L + Q^L + Q^L
\quad (23)
\]

where:
\[
\begin{align*}
Q^C &= 2\pi \int \alpha_r (y) \int B_r (y') \, dE_i (y', y) \, d\nu \\
&+ \int B_r (y') \, dE_i (y, y') \, - 2B_r (y') \, dv \\
Q^L &= 2\pi \sum \int B_r (y') \, [S_i (y) - A_i (y, y') \, dE_i (y', y) \, d\nu \\
&+ \int B_r (y') \, dW_i (y, y') \] \\
Q^L &= 2\pi \sum \int B_r (y') \, dW_i (y, y') \\
&\exp \left[ \frac{1}{I} \int \alpha_r (y') dy' \right] \, dA_i (y', y) \\
&+ \int B_r (y') \exp \left[ \frac{1}{I} \int \alpha_r (y') dy' \right] \, dA_i (y, y') \\
&- 2B_r (y') \, S_i (y) \quad (27)
\end{align*}
\]

\[
A_i (y', y) = \int \alpha_r (y) \, d\nu \\
S_i (y) = \int \alpha_r (y) \, dv
\quad (29)
\]

The volumetric absorption coefficient \( \alpha_r^C \) and \( \alpha_r^L \) are treated the same as in reference [15].

Boundary Conditions

Boundary Conditions at the Bow Shock

The Rankine--Hugoniot equations are used to obtain the shock boundary conditions. Non--dimensional Rankine--Hugoniot equations written in body oriented coordinate (see figure 1) are as follows:
\[
\begin{align*}
u_s &= \cos \phi \cos (\sigma - \phi) + \bar{\rho} \sin \phi \sin (\sigma - \phi) \quad (30) \\
v_s &= \cos \phi \sin (\sigma - \phi) - \bar{\rho} \sin \phi \cos (\sigma - \phi) \quad (31) \\
p_s &= (1 - \bar{\rho}) \sin \phi^2 + p_\infty \quad (32) \\
h_s &= 0.5 \sin \phi + h_\infty \quad (33)
\end{align*}
\]

where \( \bar{\rho} = 1/\rho \).

Species mass fractions at the shock are:
\[
C_i (y) = C_i (p_s, h_s) 
\quad (34)
\]

The specific intensity condition is
\[ I_v = 0 \]  
which means that the specific intensity coming through the shock towards the body is specified as zero.

**Boundary Conditions at the Wall**

The wall is treated as no-slip and no injection boundary.
\[ y = 0 \quad u = 0 \quad v = 0 \quad T = T_v \]  

\[ C_i = C_i^w \quad \text{full catalytic wall for nonequilibrium flow} \]
\[ C_i = C_i^e(p_w, h_w) \quad \text{for equilibrium flow} \]

the specific intensity at the wall is 
\[ I_v^+ = 0 \]
which means that the wall is full absorption surface.

**Thermodynamic and Transport Properties**

The enthalpy and frozen specific heat of species i at constant pressure are got from the data list given by Browne (7) (8), the species viscosity and heat conductivity are given by curve fit relations as Blottner (9).

The approximate viscosity and conductivity of the gas mixture are given by:
\[ \mu = \sum_{i=1}^{n_s} X_i \mu_i \]  
\[ k = \sum_{i=1}^{n_s} X_i k_i \]

where 
\[ X_i = \frac{C_i}{M_i} \]
\[ \Phi_{ij} = \left[ 1 + \left( \frac{\mu_j}{\mu_i} \right)^{\frac{1}{2}} \left( \frac{M_i}{M_j} \right)^{\frac{1}{2}} \right]^{\frac{1}{2}} / \left[ 8 \left( 1 + \frac{M_i}{M_j} \right) \right]^{\frac{1}{2}} \]

**IV Results and Discussion**

Hypersonic flows over reentry blunt bodies are calculated using spatial marching finite-difference method. The flight altitudes are from 65km to 90km.

Fig.4 is the comparison of temperature profiles, both radiation and no radiation are presented. The grid stretch method is compared with that of Dorodnitzin transformation (11), the former is more adequate for large Reynolds number and low wall temperature, and it is more suitable for marching downstream calculation from the stagnation streamline. The results show that the temperature profiles agree quite well with each other, which checked the equilibrium code of this study and proved that a reasonable initial value can be applied for marching downstream from the stagnation streamline. It also shows that the temperature behind the shock is reduced due to the emission as the radiation is coupled.

Fig.4 is the comparison of heating transfer coefficient with Engle's (11). It shows that the radiative heating increases rapidly with the increase of freestream velocity just as that in Ref.11. The small difference between the two
results is due to the different numerical methods. Fig. 5 is the results comparison with Gupta’s (Ref.12). The freestream conditions are taken from the trajectory points of Fire II (14). It shows that the agreement of the two results is quite well.

Figs.6–7 are the results of marching downstream. Fig.6 and Fig.7 are convective and radiative heating rate distribution, respectively. The freestream velocities are from 7 km/sec to 15 km/sec, the altitude is 65 km. The results show that the radiative heating is greatly affected by freestream velocity, it may increase several orders of magnitude as the freestream velocity increases only by two times. The change of convective heating is relatively small. This is because the radiative heating depends on the temperature itself, but the convective heating depends on the gradient of temperature.

Figs.8–10 are the results of different nose radius. They show that the radiative heating increases with nose radius increasing, but, on the contrary, the convective heating is reduced.

Fig.11 is the results of radiative heating at different altitude. It shows that the radiative heating increases about 3 orders of magnitude from 83 km to 40 km. In fact, the lower the altitude, the greater the density. The species number density of the air is larger at the low altitude, thus the radiative flux increases with the decrease of the altitude.

Fig.12 is the temperature profiles at different downstream stations. It shows that the highest temperature in the nose region is at the shock, but in far downstream region, the highest temperature is near the wall, which shows the viscous effects of the hypersonic flow.

Figs.13–18 are the results of nonequilibrium flow. Seven species are included in nonequilibrium chemistry(N₂, O₂, N, O, NO, NO²*, e⁻). the chemical reactions are assumed to proceed at finite rate.

Figs.13–16 are the results of convective and radiative heating transfer, respectively. They are compared with those from Gupta (12) at the stagnation streamline, and they show that both convective and radiative heating transfer agree well with Ref.12. As the altitude is above 81 km, the convective heating is much larger than radiative heating at the velocity of 9.89 km/sec.

Figs.17–18 are the results of heating transfer at the altitude of 90 km. The comparison with Moss (13) results shows that the results of VSL method are agree well with that of DSMC method.

Concluding Remarks

It can be concluded from the study of this paper that the radiating viscous shock layer can be calculated using spatial marching finite-difference method as a planar radiative transport model is employed. The energy equation in integro-differential form can be solved as a partial differential equation with the radiative flux divergence term relaxed by iteration.

The radiative heating transfer at the body surface is negligible as the reentry velocity is lower than 7.6 km/sec, and it increases rapidly with the increase of freestream velocity, density and nose radius. As the freestream velocity is 15 km/sec, the radiative heating is larger than the convective heating.

References


[10] Shen Jianwei and Qu Zhanghua, "Numerical Calculation of Hypersonic Nonequilibrium


Fig.1 Body oriented coordinate system

Fig.2 One-dimensional planar medium for radiative transfer

Fig.3 Temperature profiles for radiation and no radiation

Fig.4 Surface heating transfer coefficient for different freestream velocity
Fig. 5 Radiative heating rate at the stagnation streamline

Fig. 6 Convective heating transfer distribution for different reentry velocity

Fig. 7 Radiative heating transfer distribution for different reentry velocity

Fig. 8 Radiative heating transfer distribution for different nose radius

Fig. 9 Convective heating transfer distribution for different nose radius

Fig. 10 Change of surface heating transfer rate with nose radius
Fig. 11 Radiative heating transfer at different altitude

Fig. 12 Temperature profiles at different station along the body

Fig. 13 Radiative heating transfer distribution at the altitude of 81.86km

Fig. 14 Convective heating transfer distribution at the altitude of 81.86km

Fig. 15 Radiative heating transfer distribution at the altitude of 84.60km

Fig. 16 Convective heating transfer distribution at the altitude of 84.60km
Fig. 17 Radiative heating transfer distribution at the altitude of 90 km

Fig. 18 Convective heating transfer distribution at the altitude of 90 km