MODELLING FOR AILERON INDUCED UNSTEADY AERODYNAMIC EFFECTS FOR PARAMETER ESTIMATION

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Abstract
Aileron inputs give rise to a trailing vortex pattern which is a function of time. To account for the unsteady aerodynamic effects due to such a vortex pattern, a model based on a simple vortex system is proposed. Expressions for the induced sidewash and downwash angles are derived and recast into a form which can be conveniently used in the equations of motion for parameter estimation. Maximum-likelihood method in frequency-domain is utilized to analyse the frequency response curves of an example airplane. Parameter estimation is carried out with and without the inclusion of unsteady aerodynamic modelling in the estimation model. For the latter case, an explanation is provided for the significant differences observed in the aileron control derivatives when unsteady aerodynamic modelling was omitted fully or partially.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>lift curve slope of the vertical and horizontal tail</td>
</tr>
<tr>
<td>$a_Y$</td>
<td>linear acceleration along y-axis, g units</td>
</tr>
<tr>
<td>$b$</td>
<td>wing span, m</td>
</tr>
<tr>
<td>$c$</td>
<td>chord, m</td>
</tr>
<tr>
<td>$C_l$, $\Delta C_l$</td>
<td>lift and indicial lift coefficient of horizontal tail</td>
</tr>
<tr>
<td>$C_{\delta}$</td>
<td>aileron control derivative, $3C_l/3\delta_a$</td>
</tr>
<tr>
<td>$C_Y$</td>
<td>sideforce coefficient of vertical tail</td>
</tr>
<tr>
<td>$E(x)$</td>
<td>exponential integral of variable $\int_0^x dt$ (ref. 9)</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity, 9.8 m/sec$^2$</td>
</tr>
<tr>
<td>$l_x$, $l_z$</td>
<td>moment of inertia about X and Z axis</td>
</tr>
<tr>
<td>$l_v$</td>
<td>vertical and horizontal tail length, m</td>
</tr>
<tr>
<td>$m$</td>
<td>mass of the airplane, kg</td>
</tr>
<tr>
<td>$p$, $r$</td>
<td>roll and yaw rate, rad/sec or deg/sec</td>
</tr>
<tr>
<td>$S_w$, $S_v$, $S_l$</td>
<td>wing, vertical tail and horizontal tail area, m$^2$</td>
</tr>
<tr>
<td>$t$, $\tau$</td>
<td>time, sec</td>
</tr>
<tr>
<td>$u$</td>
<td>free stream velocity, m/sec</td>
</tr>
<tr>
<td>$v$, $z$</td>
<td>constants in indicial sideforce Eq. (4)</td>
</tr>
<tr>
<td>$\tilde{Y}$</td>
<td>distance from c.g. to a.c. of the left and right panels of the horizontal tail, m</td>
</tr>
<tr>
<td>$\tilde{z}_v$</td>
<td>vertical height of a.c. of vertical tail from c.g. of the aircraft</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angle of attack at the wing, deg or rad</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>aileron deflection parameter, $3\alpha/3\delta_a$ (ref. 8)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>angle of sideslip, rad</td>
</tr>
<tr>
<td>$\delta_a$, $\delta_r$</td>
<td>aileron and rudder deflection, deg or rad</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>wing sweep, deg</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of air, kg/m$^3$</td>
</tr>
<tr>
<td>$\Delta \varepsilon$</td>
<td>indicial downwash angle, deg or rad</td>
</tr>
<tr>
<td>$\Delta \sigma$</td>
<td>indicial sidewash angle, deg or rad</td>
</tr>
<tr>
<td>$\phi$</td>
<td>bank angle, deg or rad</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency, rad/sec</td>
</tr>
<tr>
<td>$\infty$</td>
<td>infinity</td>
</tr>
</tbody>
</table>

Superscripts

- mean value
- derivative with respect to time
- Fourier transform
- transpose of a matrix
- total derivatives
- equivalent derivatives

Subscripts

r | root
ss | steady state value
v | horizontal tail
t | vertical tail
w | wing

Abbreviations

a.c. | aerodynamic center
c.g. | center of gravity

Introduction

Estimation algorithms based on quasisteady flow theory have been observed to cause additional uncertainty in parameter estimates\cite{1}. This has resulted in efforts being expended at modelling of unsteady aerodynamic effects into aircraft equations of motion for parameter estimation. The simplest way to account for these unsteady effects in aircraft dynamics is through an aerodynamic force model based on physical principle of Prandtl's lifting line theory and trailing vortex concept\cite{2}. Of late, this concept has been extensively used to modify longitudinal\cite{3,4} and lateral\cite{5,6} equations of motion for parameter estimation.

The purpose of the present study is to develop simple concepts which would permit modelling of unsteady aerodynamic effects arising from the vortex pattern from aileron input into parameter extraction algorithm. Such modelling is expected to lead to increased confidence and high fidelity in parameter estimates. Frequency response curves for unit aileron impulse input are computer generated for an example airplane. Maximum likelihood method in frequency-domain is utilized for parameter estimation from frequency response to show the effect of unsteady aerodynamics on estimated parameters. The aileron control derivatives are found to be the most affected, showing marked variance from their true values.
Simplified Asymmetric Vortex Pattern

A simplified arrangement of vortex pattern for a full span positive aileron deflection is shown in Fig. 1. Unlike the vortex system used heretofore for estimation purposes, the proposed vortex system assumes a vortex pattern for each panel of the wing due to effective angle of attack change caused by deflection of aileron surface on that panel. The vortexes have the direction as shown in Fig. 1 and the same circulation strength \( P \) as determined by the lift generated on each panel given by \( C_L \delta / 2 \).

\[
\Delta \sigma_{\text{ss}} = \frac{2C_L \delta_{\text{ss}}}{8\pi} \left[ f_1(\infty) + f_2(\infty) + f_3(\infty) + f_4(\infty) \right]
\]

(4)

Combining equations (2) and (4), we obtain

\[
\Delta \sigma(t) = \frac{d\sigma}{d\delta_{\text{ss}}} \left[ 1 - y \exp(-\frac{2z u L}{c} \right] \left[ f_1(t) + f_2(t) + f_3(t) + f_4(t) \right] \left[ f_1(\infty) + f_2(\infty) + f_3(\infty) + f_4(\infty) \right] \]

(5)

A similar expression for the induced indicial downwash is obtained using Biot–Savart law to the asymmetric vortex system in Fig. 1 which induces a downwash on the left panel (looking from behind) and an equal amount of upwash on the right panel of the horizontal tail. The induced indicial downwash angle at the a.c. of the left panel is given by

\[
\varepsilon(t) = \frac{C_L(t) L}{8\pi} \left[ g_1(t) + g_2(t) + g_3(t) + g_4(t) \right]
\]

(6)

where functions \( g_1, \ldots, g_4 \) are defined in Appendix B. The accuracy of the above expression is improved by ensuring its correctness at the known steady state conditions. For a unit step increase in aileron input \( \delta_{\text{ss}} \), the indicial downwash angle \( \Delta \varepsilon \) under steady state conditions is given by

\[
\Delta \varepsilon_{\text{ss}} = \frac{d\varepsilon}{d\delta_{\text{ss}}} \left[ \frac{C_L \delta_{\text{ss}}}{8\pi} \right] \left[ g_1(\infty) + g_2(\infty) + g_3(\infty) + g_4(\infty) \right]
\]

(7)

Combining equations (6) and (7), we obtain

\[
\Delta \varepsilon(t) = \frac{d\varepsilon}{d\delta_{\text{ss}}} \delta_{\text{ss}} \left[ 1 - y \exp(-\frac{2z u L}{c} \right] \left[ g_1(t) + g_2(t) + g_3(t) + g_4(t) \right]
\]

(8)

Other than the unsteady effects in sideways and downwash given by equations (5) and (8), lift build up at the horizontal and vertical tails also contribute to the unsteady aerodynamic effects. As suggested in refs. 4 and 5, expressions similar to Eq. (3) for the indicial lift build up can be used to model such effects, e.g., the sideforce build up at the vertical tail can be modelled by the indicial sideforce function

\[
\Delta C_{\text{y}} = -a \left( 1 - y \exp(-\frac{2z u L}{c} \right]
\]

(9)

However, a brief study suggested that the effects of the unsteady aerodynamics on aircraft motion could be modelled reasonably accurately even if the load build up in lift and sideforce were omitted while the contribution due to \( \Delta C_{\text{y}} \) in downwash and sideways were retained (Fig. 4 discussed later). In keeping with this simplification, the load build up effects have therefore been omitted from further analysis (\( y-z=0 \) in Eqs. 5 and 8).

The expressions for the induced indicial sideways and downwash angles in Eqs. (5) and (8) are very long and cumbersome and as such not in a suitable form for use in aircraft equations of motion.
for parameter estimation. To that purpose, $\Delta \sigma$ and $\Delta \epsilon$ were calculated for a few specific cases and after some trials, the following expressions for the induced angles were observed to approximate equations (5) and (8) reasonably accurately

$$\Delta \sigma(t) = -\frac{dE}{dt}\frac{ut^2 + 2F_1}{c_r} \left[ 1 - D\exp\left(-\frac{2Eut^2 + 2F_1}{c_r}\right) \right]$$

$$\Delta \epsilon(t) = \frac{dE}{dt}\frac{\sigma}{c_r}$$

$$x \left[ 1 - \frac{Lc_r}{(1 - \gamma \tan \Lambda) - (ut/2) - c_r} - M \exp\left(-\frac{2Eut^2}{c_r}\right) \right]$$

Constants D, E and F in Eq.(10) and L, M and N in Eq.(11) are function of wing and tail geometry and location. These are obtained by curve fitting Eq.(10) to Eq.(5), and Eq.(11) to Eq.(8). Figure 2 and Fig. 3, respectively, show the exact and approximate time histories of the indicial sidewash and downwash angles for $\Lambda = 0$. 

In Fig. 2, except for the region very near to origin, the matching is reasonably good. Parameters $\frac{dE}{dt}$ and $\sigma$ were calculated from refs. 7 and 8. Since accurate methods for computing $\frac{d\sigma}{dt}$ were not readily available, its value was obtained from Eq.(4). In comparison to Eqs. (5) and (8), Eqs. (10) and (11) now give much simplified expressions for $\Delta \sigma$ and $\Delta \epsilon$ which are amenable to Laplace transform and, therefore, can be conveniently used in equations of motion for parameter estimation.

The induced forces for arbitrary angle of attack at the tail surfaces are obtained using Duhamel's integral

$$C_y(t) = \int_0^t \Delta C_y(t - \tau) [\beta(\tau) + \Delta \dot{\beta}(\tau)] d\tau$$

$$C_\epsilon(t) = \int_0^t \Delta C_\epsilon(t - \tau) [\alpha(\tau) + \Delta \dot{\alpha}(\tau)] d\tau$$

Fig. 2: Exact and Approximate Time Histories for Indicial Sidewash

Fig. 3: Exact and Approximate Time Histories for Indicial Downwash

The coupled perturbed lateral equations of motion in the frequency-domain can be written as

$$\dot{\mathbf{X}} = \mathbf{B} \mathbf{X}$$

$$\dot{\mathbf{Z}} = \mathbf{C} \mathbf{X}$$

where $\mathbf{X}$ and $\mathbf{Z}$ are respectively the state and output vectors, $\mathbf{A}$ and $\mathbf{B}$ are the system matrices and $\mathbf{C}$ is the observation matrix.

$$\mathbf{X} = \begin{bmatrix} \beta \dot{\delta}_a, \beta \dot{\delta}_a, \beta \ddot{\delta}_a, \beta \dddot{\delta}_a, \beta \dddot{\delta}_a \end{bmatrix}$$

$$\mathbf{Z} = \begin{bmatrix} \beta \dot{\delta}_a, \beta \dot{\delta}_a, \beta \ddot{\delta}_a, \beta \dddot{\delta}_a, \beta \dddot{\delta}_a \end{bmatrix}$$

The system matrices in Eq. 18–19 are formulated as
The parameters were estimated from the simulated flight data (for \(A=0\)) while retaining the unsteady aerodynamic modelling in the estimation algorithm. As expected for simulated case, the estimated parameters compared exactly with their true values (see column 3 of Table I). The measured and estimated responses also matched well for all the observation variables.

More interesting case was to omit the unsteady aerodynamic modelling from the estimation algorithm and thereby observe how the unsteady effects present in the flight data get absorbed in estimated equivalent parameters. Parameter extraction was performed in two different modes

Mode I where only time-lag effects were omitted from the estimation algorithm \(\bar{D}_1=\bar{D}_2=0\)

Mode II where unsteady aerodynamic modelling was altogether omitted from the estimation algorithm \(\bar{P}_1=\bar{P}_2=0\)

It may be noted that the simulated measured responses used for mode I and II always contained complete unsteady effects.

The estimated parameters for mode I (column 4) and mode II (column 5) are given in Table I. It may be seen that only the aileron control derivatives \(C_{\delta \beta}^\alpha, C_{\delta \beta}^\alpha, C_{\delta \beta}^\alpha\) and \(C_{\delta \beta}^\alpha\) show significant difference between mode I and mode II; specifically, \(C_{\delta \beta}^\alpha\) even showing a change of sign. However, when a comparison was made of the estimated responses for mode I and mode II, the variations between the two were insignificant as may be seen in Fig. 5. This intriguing contradiction at first instance could be explained by defining the equivalent estimated derivatives as follows.

From Matrix B of Eq.(18), the frequency dependent control derivatives are given by

\[
\begin{align*}
C_{\delta \beta}^\alpha &= (C_{\delta \beta}^\alpha)_{ss} - K_3 P_2 \\
C_{\delta \beta}^\alpha &= (C_{\delta \beta}^\alpha)_{ss} - K_3 P_2 \\
C_{\delta \beta}^\alpha &= (C_{\delta \beta}^\alpha)_{ss} + K_3 P_2
\end{align*}
\]

The equivalent estimated derivatives for mode I are defined with \(\bar{D}=0\) in Eq.(20), and are written as
FIG. 5 COMPARISON OF ESTIMATED RESPONSE FOR MODE I AND MODE II WITH MEASURED RESPONSE
\[ C_{y\delta \alpha}'' = (C_{y\delta \alpha})_{ss} - K_{y \alpha} \left( \frac{dc}{d\delta}\right)_{ss} \] (21a)

\[ C_{l\delta \alpha}'' = (C_{l\delta \alpha})_{ss} - K_{l \alpha} \left( \frac{dc}{d\alpha}\right)_{ss} - K_{l \alpha} \left( \frac{dc}{d\alpha}\right)_{ss} \alpha \delta \] (21b)

\[ C_{n\delta \alpha}'' = (C_{n\delta \alpha})_{ss} + K_{n \alpha} \left( \frac{dc}{d\delta}\right)_{ss} \] (21c)

Similarly, the equivalent estimated derivatives for mode II are defined with $\bar{P} = 0$ in Eq. (20), and these are the same as the steady state derivatives appearing as the first term on the right hand side of Eq. (20).

The equivalent estimated control derivatives for mode I are shown in column 6 of Table 1 while the equivalent derivatives for mode II are the same as listed in column 5 of Table 1. A comparison of these equivalent derivatives shows that not only the sign discrepancy of $C_{y\delta \alpha}$ has disappeared but also the numerical values are close to each other. This explains the apparent anomaly observed between the estimated values and corresponding responses for mode I and mode II.

As seen from Table 1, for the aileron control derivatives, a comparison of the true values (column 2) and equivalent estimated values (columns 5 and 6) shows that the change in $C_{l\delta \alpha}$ values is quite insignificant as compared to changes in $C_{y\delta \alpha}$ and $C_{n\delta \alpha}$ values. To explain this, a comparison of unsteady contributions due to downwash at horizontal tail and sidewash at vertical tail was carried out. Figure 6 illustrates the relative contributions to roll rate. As seen from Fig. 6, the response with downwash effects only is close to the true response, and thereby suggests that contributions to $C_{l\delta \alpha}$ due to downwash effects are small for the airplane considered. A large horizontal tail may, however, lead to downwash effects being comparable to unsteady effects in sidewash and thus also affect the $C_{l\delta \alpha}$ values significantly.

![Measured Response with unsteady effects included](image)

**FIG. 6 COMPARISON OF MEASURED RESPONSES WITH UNSTEADY EFFECTS IN DOWNWASH AND/OR SIDEWASH**

**Conclusions**

A simplified vortex model has been developed to account for unsteady aerodynamic effects due to aileron deflections. Parameters have been extracted in two different modes, omitting, respectively, the time

<table>
<thead>
<tr>
<th>Parameters</th>
<th>True Value</th>
<th>Estimated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-C_{y\beta}$</td>
<td>0.3671</td>
<td>0.4337 (0.0166)</td>
</tr>
<tr>
<td>$-C_{y\alpha}$</td>
<td>0.0400</td>
<td>0.3973 (0.0142)</td>
</tr>
<tr>
<td>$C_{y\delta \alpha}$</td>
<td>0.0000</td>
<td>0.1223 (0.0177)</td>
</tr>
<tr>
<td>$-C_{l\beta}$</td>
<td>0.1231</td>
<td>0.1790 (0.0078)</td>
</tr>
<tr>
<td>$-C_{l\alpha}$</td>
<td>0.5320</td>
<td>0.5949 (0.0017)</td>
</tr>
<tr>
<td>$C_{l\delta \alpha}$</td>
<td>0.1706</td>
<td>0.2357 (0.0133)</td>
</tr>
<tr>
<td>$C_{n\delta \alpha}$</td>
<td>0.2590</td>
<td>0.2761 (0.0005)</td>
</tr>
<tr>
<td>$-C_{n\beta}$</td>
<td>0.1474</td>
<td>0.1909 (0.0025)</td>
</tr>
<tr>
<td>$-C_{n\alpha}$</td>
<td>0.0420</td>
<td>-0.0852 (0.0015)</td>
</tr>
<tr>
<td>$-C_{n\delta \alpha}$</td>
<td>0.2404</td>
<td>0.2803 (0.0075)</td>
</tr>
<tr>
<td>$-C_{\delta \alpha}$</td>
<td>0.0250</td>
<td>0.0715 (0.0007)</td>
</tr>
</tbody>
</table>

\( a \) & Cramer-Rao bounds

**TABLE 1 COMPARISON OF PARAMETER ESTIMATES FOR DIFFERENT ESTIMATION MODELS**

1360
lag effects and the steady state vortex system in estimation model for mode I and mode II. For the airplane considered, the estimated derivatives show a marked variation from their true values. Significant difference between the values of the aileron control derivatives extracted in mode I and mode II was also observed and an explanation is offered by invoking the concept of equivalent derivatives. A brief study suggests that the induced sideward effects are more pronounced than the downwash effects.

**References**


**APPENDIX A**

The function \( f_1,\ldots, f_6 \) represent the contribution of bound, shed and trailing vortices of the left panel of the wing to the sideward at the a.c. of the vertical tail.

\[
f_1(t) = \frac{\left[ m_1 + (m_2 - l) / m_2 \right] / h_2}{\sin \theta} \]

\[
f_2(t) = \frac{\left[ (l_1 - m_2) \sin \theta / m_2 + m_2 \sin \theta + \left( \frac{b}{2 m_2} \cos \theta \right) / h_2 \right]}{\sin \theta} \]

\[
f_3(t) = \frac{\left[ m_1 + m_2 \right] / m_2 - m_1 / m_2 \right]}{h_2} \]

\[
f_6(t) = -\left[ m_2 + (m_2 - l) / m_2 \right] / h_2 \]

where

\[
m_1 = \frac{b}{2} \tan \theta - 1 \quad m_2 = \frac{c_t + \frac{u}{2}}{2} \]

\[
m_3 = \frac{1}{\sqrt{l_1 + z_2^2}} \quad m_4 = \sqrt{m_1^2 + (b/2)^2 + z_2^2} \]

\[
m_5 = \sqrt{(l_1 - m_2)^2 + z_2^2} \quad m_6 = \sqrt{(m_1 + m_2)^2 + (b/2)^2 + z_2^2} \]

\[
h_1 = \sqrt{(l_1 \cos \theta)^2 + z_2^2} \quad h_2 = \sqrt{(l_1 - m_2) \cos \theta)^2 + z_2^2} \]

For the vortex pattern in Fig. 1, the upwash at the a.c. of the right panel of the horizontal tail will be equal to the downwash on the left panel of the horizontal tail.

**APPENDIX B**

In the expressions that follow, the contribution to the downwash at the a.c. of the left horizontal tail by the bound, shed and trailing vortices is given by \( g \), for the vortex pattern of the left panel of the wing and \( g_r \), for that of the right panel \( (i=1,2,3) \). Function \( g \), represents the contribution of the center trailing vortex to the downwash.

\[
g_i(\bar{y}, t) = (-1)^{i-1} \left[ \cos A + \cos B \right] / h, \quad i=1,2,3 \]

\[
g_r(\bar{y}, t) = -g_i(-\bar{y}, t), \quad i=1,2,3 \]

where

\[
\cos A_i = \frac{m_i \sin \theta + m_c \cos \theta}{m_c} \quad \cos B_i = \frac{(m_1 + m_2) \sin \theta + m_c \cos \theta}{m_c} \quad \cos A_j = \frac{(m_1 + m_2) \sin \theta + m_c \cos \theta}{m_c} \quad \cos B_j = \frac{(1 - m_i) \sin \theta + \bar{y} \cos \theta}{m_c} \]

and

\[
h_1 = \left| \frac{\cos A - \bar{y} \sin A}{h_1} \right| \quad h_2 = \left| \frac{(1 - m_i) \cos A - \bar{y} \sin A}{h_2} \right| \]

The function \( g_i(\bar{y}, t) \) is given by

\[
g_i(\bar{y}, t) = \frac{1}{\bar{y}} \left[ \frac{1}{h_1} - \frac{1 - m_i}{h_2} \right] \]

For convenience, constants \( m_1 \) to \( m_3 \) are defined as

\[
m_1 = \frac{b}{2} \tan \theta - 1 \quad m_2 = \frac{c_t + \frac{u}{2}}{2} \]

\[
m_3 = \frac{1}{\sqrt{l_1 + z_2^2}} \quad m_4 = \sqrt{m_1^2 + (b/2)^2 + z_2^2} \]

\[
m_5 = \sqrt{(l_1 - m_2)^2 + z_2^2} \quad m_6 = \sqrt{(m_1 + m_2)^2 + (b/2)^2 + z_2^2} \]

\[
h_1 = \sqrt{(l_1 \cos \theta)^2 + z_2^2} \quad h_2 = \sqrt{(l_1 - m_2) \cos \theta)^2 + z_2^2} \]

For the vortex pattern in Fig. 1, the induced upwash at the a.c. of the right panel is equal to the downwash on the left panel of the horizontal tail.