A NEW METHOD FOR CALCULATION OF HELICOPTER MANEUVERING FLIGHT

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Abstract

In this paper a set of nonlinear equations governing the helicopter in maneuvering flight is developed and solved based on finite difference methods for evaluating the extreme value of function. The inverse solution technique is improved and from which a good result of the sample calculation of level turn maneuver is obtained. Beginning with a set of prescribed flight conditions, this method calculates the flight path, velocity and other kinematic parameters first, and then the control displacement, flight attitude and the load factor for a helicopter during the maneuvering are calculated. Key words: helicopter, maneuvering, calculation.

I. Introduction

With the continuous development of helicopter applications, the maneuverability targets have been placed on the design requirements for helicopters, especially, for armed helicopters. The main rotor is the key component of a helicopter. It decides the maximum lift capability which is the fundamental factor of maneuvering capability for a helicopter. Ref. 1 ~ 4 presented the theoretical or experimental results referred to the maneuverability of a helicopter.

Synthesizing the power available, the limits of control and attitude angle and so on, a set of equations governing the helicopter during maneuvering flight is established in this paper. Also a nonlinear iteration method is developed for the solution. Based on these theoretical formulae and calculation methods, almost any kind of maneuvering flights of helicopters can be calculated.

The conventional method is in this way to find the vehicle' s response to the given control inputs and then the flight path can be found. On the contrary in this paper, an inverse solution is adopted. The solution process begins with a defined flight path, then the control deflections, the attitudes and the load factor required to fly the helicopter with this maneuvering will be obtained.

The first attempts of the inverse solution method are referred to fixed wing aircrafts5,6,7. Recently, T. Cerbe and G. Reichert of Braunschweig University in W. Germany used a similar method to study optimization of helicopter take off and landing performances8, but they did not deal with the dynamics of the whole helicopter and cut out the rotor flap motion. At the same time, D. G. Thomson and R. Bradley investigated the inverse solution for a prescribed flight path based on a set of linearised equations of motions9. Their methods are valid only for the calculation of longitudinal maneuvers since it is confined to small-angle assumption in the linearized analysis.

Because of difficulties in mathematical representation and complexity of helicopter maneuvering flight, the inverse solution technique of Euler equations is still in the continuously developing stage. Flight dynamicists are making efforts to study the mathematical simulation and solution technique for helicopter maneuvering flight10,11. A new attempt is made in this paper. Firstly, the time derivatives of flap motion coefficients and induced velocity model derived from vortex theory are adopted, so that the governing equation of helicopter maneuvering flight is derived more properly1. Second, based on the mathematical principle of finite-difference methods for evaluating the extreme value of function, the nonlinear inverse solution of general governing equations is obtained simultaneously1; Finally, a sample calculation is given with level turn maneuver.

II. Governing Equations

Dynamic equations

The conventional approach for flight dynamics calculation involves the solving process of Euler equation (1) for the six unknown state variables, \( V_x, \dot{V}_x, V_y, \dot{V}_y, V_z, \dot{V}_z \) the translational velocities, and \( \omega_x, \omega_y, \omega_z \) the angular velocities, in a body fixed frame (see Fig. 1),

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\[
\begin{align*}
F_x &= mg \sin \theta_y - m (V_x' - \omega_x V_y + \omega_y V_z) = 0 \\
F_y &= mg \cos \theta_y \cos \psi - m (V_y' - \omega_x V_y + \omega_z V_z) = 0 \\
F_z &= mg \cos \theta_y \sin \psi - m (V_z' - \omega_y V_z + \omega_x V_y) = 0 \\
M_x &= -I_x \frac{d\omega_x}{dt} - (I_x - I_z) \omega_x \omega_y - I_x (\omega_y^2 - \omega_z^2) \\
&\quad - I_x \omega_x \omega_z + I_x \omega_x \omega_y + I_x \frac{d\omega_y}{dt} + I_x \frac{d\omega_z}{dt} = 0 \\
M_y &= -I_y \frac{d\omega_y}{dt} - (I_y - I_x) \omega_x \omega_y - I_y (\omega_x^2 - \omega_z^2) \\
&\quad - I_y \omega_x \omega_z + I_y \omega_x \omega_y + I_y \frac{d\omega_x}{dt} + I_y \frac{d\omega_z}{dt} = 0 \\
M_z &= -I_z \frac{d\omega_z}{dt} - (I_z - I_x) \omega_x \omega_y - I_z (\omega_x^2 - \omega_y^2) \\
&\quad - I_z \omega_x \omega_z + I_z \omega_x \omega_y + I_z \frac{d\omega_x}{dt} + I_z \frac{d\omega_y}{dt} = 0
\end{align*}
\]

Where the helicopter attitude angles \((\theta_y, \psi, \psi_y)\) are the Euler angles relating the body fixed frame to the earth fixed frame of reference.

![Fig.1 Body fixed axes system](image)

For the flapping motion of blades, only the zero and first harmonics are concerned. The elastic deflection of the blade can be written as

\[
\dot{Y}_t = \dot{\eta}_t(\tau) \sin(\psi) = \eta_t(\tau) \left[ a_t(t) - a_1(t) \cos \psi - b_1(t) \sin \psi \right]
\]

Starting from the force on the blade section, taking the method of separation of variables and the property of orthogonality of the mode shape, the partial differential equation of the blade elastic axis can be changed into the following set of tip-path-plane equations,

\[
\begin{bmatrix}
\ddot{a}_t \\
\ddot{b}_t \\
\ddot{\dot{Y}}_t
\end{bmatrix} + \begin{bmatrix}
A_3 \\
B_3 \\
C_3
\end{bmatrix} \begin{bmatrix}
a_t \\
b_t \\
\dot{Y}_t
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

**Kinematic equations**

The relations of velocity and acceleration are presented in detail in Ref. 12. Seeing Fig. 2, the velocity components in the earth fixed frame are

\[
\begin{align*}
X_e &= V \cos \theta_y \cos \psi \\
Y_e &= V \sin \theta_y \\
Z_e &= V \cos \theta_y \sin (-\psi_y)
\end{align*}
\]

The accelerations can be found by differentiating equations (3).

![Fig.2 Description of the maneuvering flight path](image)

According to the projection relations in the body fixed frame, the following relations of angular velocities can be derived

\[
\begin{align*}
\dot{\theta}_y &= a_0 \sin \psi + \alpha \cos \psi \\
\dot{\psi}_y &= (1/cos \theta_y) \alpha \cos \theta_y - \omega \sin \theta_y
\end{align*}
\]

**Angles equations**

The preceding equations include eight angles \(\theta_y, \psi, \psi_y, \alpha, \beta, \phi, \phi_y, \psi_y\) but only five of them are independent variables. They are related with each other by the following relations which can be derived through mutual transformation among the earth fixed frame, body fixed frame and wind reference axes

\[
\begin{align*}
\sin \beta &= \sin \psi \sin \theta_y \cos (\psi_y - \phi) + \cos \psi \sin \theta_y \\
\sin \alpha &= \cos \psi \cos \theta_y \cos \psi_y + \sin \psi \cos \theta_y \\
\sin \gamma &= \sin \theta_y \cos \alpha \sin \beta - \cos \alpha \cos \psi \sin \theta_y \\
\sin \gamma &= \sin \psi \cos \psi_y \cos \phi + \cos \psi \sin \psi_y / \cos \phi
\end{align*}
\]

**Complementary equations**

Using the transformation relations between body axes and wind axes, the following equations for the angular velocities can be derived.

\[
\begin{align*}
\dot{\psi}_y &= \dot{\psi}_y + \psi_y \\
\dot{\phi}_y &= \dot{\phi}_y + \phi_y + \dot{\psi}_y
\end{align*}
\]

The equations (1) through (6) are called the equations governing the helicopter during any maneuvering flight.

The Euler equations are the dominative ones. The gov-
ering equations can be simplified according to the defined conditions for a specific maneuver. For example, the calculation of level turn maneuver in this paper is carried out by setting $\delta_0=\beta=0$.

**II. The Solving Process**

Taking the level coordinated turn as a sample calculation, the inverse solution technique can be described with the following two stages.

The first stage, calculation of kinematic parameters

The calculation of kinematic parameters is to make provision for solving the history of the control deflections and flight attitudes. According to the predefined parameters, such as equivalent radius $R_e$, transient proportional factor $K_e$, velocity $V$ and total turning angle $\psi_a$ (see Fig. 3) and using the methods presented in Ref. 12, the variation of flight path, turn rate and load factor with time during this maneuver can be calculated.

\[ \dot{\psi}_a = \psi_a + \frac{1}{t} \left( -\frac{t}{t+t} \right) \left( \sin \psi_a \right) \sin \theta_a \]

\[ \dot{\psi}_a = \psi_a + \frac{1}{t} \left( -\frac{t}{t+t} \right) \left( \sin \psi_a \right) \sin \theta_a \]

\[ \dot{\psi}_a = \psi_a + \frac{1}{t} \left( -\frac{t}{t+t} \right) \left( \sin \psi_a \right) \sin \theta_a \]

where $\theta_a$, $\psi_a$ and $\gamma_a$ at time $t$ can be calculated by using the backward difference method, i.e.

\[ \dot{\theta}_a = \frac{\theta_a - \theta_a-1}{t} \]

\[ \dot{\gamma}_a = \frac{\gamma_a - \gamma_a-1}{t} \]

Also, $\dot{\theta}_a$ and $\dot{\gamma}_a$ can be treated similarly.

Then the following parameters should be calculated progressively

\[ \omega_a = \psi_a + \sin \theta_a \dot{\psi}_a \]

\[ \omega_a = \cos \theta_a \cos \psi_a \dot{\psi}_a + \sin \theta_a \dot{\gamma}_a \]

\[ a = \omega_a + \psi_a \sin \theta_a \]

\[ \dot{a} = \frac{\omega_a - \psi_a \cos \theta_a \dot{\gamma}_a}{\cos \theta_a} \]

\[ \dot{\omega}_a = \frac{1}{\sin \theta_a} \left[ \ddot{\psi}_a \cos \theta_a - \ddot{\gamma}_a \sin \theta_a \right] \]

\[ \dot{a} = \frac{\omega_a - \psi_a \cos \theta_a \dot{\gamma}_a}{\cos \theta_a} \]

\[ \dot{\omega}_a = \frac{1}{\sin \theta_a} \left[ \ddot{\psi}_a \cos \theta_a - \ddot{\gamma}_a \sin \theta_a \right] \]

Finally, the calculation of rotor flap motion and componential forces and moments of the rotor and other parts could be done. The solution of equation (1) begins with substitution of all componential forces and moments as well as all kinematic variables into (1). Generally speaking, the numerical solution methods for such a set of nonlinear equations have not been ripe. The Levenberg-Marquardt finite-difference method for solving problem of the least squares is adopted in this paper. The left side of Euler equations is defined as function $f_i(x)$, the summation of square of function $f_i$, $x$ is expressed as SSQ$=\sum f_i(x)^2$. The simulation of valid solution of Euler equations requires the minimization of SSQ.

The calculation proceeds according to recurrence steps. The calculation begins with level flight ($t_0=0$). Then the every next transient state is calculated at each step of time $\Delta t$. For the transient state at $t_1$, the initial value takes the final value of $t_{1-1}$ transient state and the iteration calculation proceeds repeatedly until the result satisfies convergence criteria of the governing equations.

**IV. Analysis of a Sample Calculation**

The prescribed conditions

Helicopter Bo-105 performs a 90° left turn maneuver at constant velocity $V=157$ km/h ($u=0.2$), with the effective
radius $R_e = 250m$ and the transient proportional factor $K_n = 0.1$

The kinematic parameters

From the defined conditions, $\phi_n = \frac{\pi}{2}$, the track angle in the entry part $\phi_3 = K_n \phi_n$, and the track angle in the withdrawing part $\phi_3 = K_n \phi_n$ (see Fig. 3), then the following results can be obtained as

$t_1 = 1.5s$, $t_2 = 7.7s$, $t_3 = 9.2s$

i.e. the transient turn in either entry part or withdrawing part takes 1.5 seconds and the steady turn part takes 6.2 seconds.

The turn radius in steady turn section is $R_e = 214.67m$ and the turn rate($r/s$) in three sections are respectively section 0–1, $\phi_3 = -0.109588t^2 + 0.25436n^2$

$0 \leq t < 1.5$

section 1–2, $\phi_3 = \frac{V}{R_e} = 0.203023\quad 1.5 \leq t < 7.7$

section 2–3, $\phi_3 = 0.109588t^2 + 2.798027t^2$

$+ 23.61649t - 65.779358\quad 7.7 \leq t < 9.2$

The load factor during this maneuver is

$$n_f(t) = \sqrt{1 + (\frac{\phi_3 V}{g})^2}$$

The variation of the load factor with time is shown in Fig. 4.

![Fig. 4. The variation of load factor](image)

The maximum value is $n_f = 1.348$, occurring in the steady turn section.

The control deflections and attitude angles

The time history of control deflections is calculated, among which the lateral cyclic pitch and tail rotor collective pitch are shown in Fig. 5 and Fig. 6. The variation of pitch angle, bank angle and yaw angle of the helicopter with time are shown in Fig. 7 through Fig. 9. The thrust vector of the main rotor has to tilt for making the helicopter into turn maneuver and so the main rotor collective must increase to keep the helicopter flying at a constant altitude in the entry phase. In steady turning the main rotor collective keeps constant. Then the collective decreases in withdrawing phase and recovers gradually to the value of level flight at last. The rolling motion during turning is controlled by the lateral cyclic control (see Fig. 8). In the entry phase, the pilot deflects the stick leftward to accelerate the helicopter rolling and then pulls the stick back somewhat until the proper bank angle of the helicopter is established. In the withdrawing phase, the contrary deflection in lateral control is performed. The corresponding bank angle of the helicopter is shown in Fig. 8. When the helicopter enters left turning, the gyroscopic moment of the main rotor due to helicopter rolling leftward causes the helicopter nose up (see Fig. 7). To keep the helicopter flying at a constant speed, the stick must be pushed forward to eliminate the nose-up motion. In the steady turning phase, no rolling exists and the longitudinal cyclic pitch keeps constant. In withdrawing phase, the conditions are inverse.

![Fig. 5. The time history of lateral cyclic pitch](image)

![Fig. 6. The time history of collective control of tail rotor](image)

![Fig. 7. The variation of pitch angle of the helicopter](image)
The thrust of the tail rotor must be varied correspondently to keep the helicopter in coordinated turn (zero sideslip). The time history of the tail rotor collective is shown in Fig. 6. According to such a control process, the 90° left turn is completed as shown in Fig. 9.

The variation of the angle of attack is similar to the pitch angle of the helicopter since the helicopter turns keeping a constant altitude. The time histories of angular velocities and accelerations are also calculated. As an example, the yawing angular acceleration is shown in Fig. 11.

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