VIBRATION ANALYSIS OF STIFFENED CIRCULAR CYLINDRICAL THIN SHELLS

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Abstract

This paper is addressed to the free vibrations of stiffened circular cylindrical thin shells within the frame of Love's first approximation theory of elastic shells. The circular cylindrical thin shells with shear diaphragm ends is reinforced at uniform intervals by elastic stringers and rings. The effects of stiffeners are taken into account by the orthotropic material approach of stiffened shells. The governing equations are deduced from the three-dimensional equations of elastodynamics by means of Hamilton's principle together with the usual kinematic hypothesis of circular cylindrical thin shells. The governing equations are simplified for various special cases involving the material and geometrical properties of stiffened cylindrical thin shells. The uniqueness is examined in solutions of the dynamic governing equations of stiffened shells. Moreover, the stiffened shell is discretized by Semiloo shell elements and a matrix equation is obtained by means of the variational equation to determine the vibration characteristics of the stiffened shells. The influence of geometrical parameters and material properties of stiffened circular cylindrical shells is investigated on the vibration characteristics. Numerical results are plotted with respect to the parameters of stiffened cylindrical shells, and they are compared with certain experimental results.

1. Introduction

It is required that an aerospace vehicle must be light weight. In order to overcome this problem, the designer usually resorts to the use of thin shells reinforced by the stiffeners, because of their high structural efficiency. Particularly, circular cylindrical shells reinforced by stringers and rings are used in the structure of various aerospace vehicles.

The stiffener members, representing a relatively small part of the total weight of a structure, substantially influence its dynamics behavior, stability, stiffness, and strength. Hence, the prediction of dynamic characteristics of stiffened circular cylindrical shells is very important for determination of inflight behavior, fatigue life, noise generation of the aerospace vehicle and vibration isolation of the sensitive electronic instrumentation and on-board computers.

Numerous studies examining free vibration characteristics have been conducted on elastic stiffened circular cylindrical thin shells. Eggle and Sewall studied the vibration of orthogonally stiffened cylindrical shells with discrete axial stiffeners by the Ritz method. Bushnell evaluated various analytical models for vibrations of stiffened shells. The free vibrations of a thin cylindrical shell have been investigated for discrete axial and circumferential stiffeners by Mead and Bardell. Mustafa and Ali determined the frequencies of ring stiffened, stringer stiffened and orthogonally stiffened shell using super shell finite elements and a formulation of energy functional. The free vibration characteristics of stiffened circular cylindrical shells have been studied numerically using the finite element method by Mecitoğlu. Although the uniqueness in solutions of the elastodynamic problems is very important, only a few work is found in literature examining with this subject.

The purpose of this paper is (i) to derive all the governing equations for vibrations of cylindrical thin shells reinforced by stringers and rings, (ii) to examine the uniqueness in solutions of the governing shell equations, and (iii) to study numerically the vibration characteristics of some special cases.

The dynamic governing equations of the stiffened circular cylindrical shells are derived within the frame of Love’s first approximation theory of elastic thin shells. By means of Hamilton’s principle together with the usual kinematic hypothesis of cylindrical thin shells, all the governing equations are deduced in a systematic manner from the three-dimensional equations of elastodynamics under the well-known assumptions of regularity and smoothness of field variables. The stiffeners are taken to be along the usual cylindrical coordinates and their dimensions to be small compared to the radius of cylindrical shell. The effects of stiffeners are taken into account by the orthotropic material approach of stiffened shells. The governing equations are simplified for various special cases involving material and geometrical properties of stiffened circular cylindrical thin shells. The uniqueness is examined in solutions of the dynamic governing equations of stiffened shells, and a theorem of uniqueness is given which enumerates the initial and boundary conditions sufficient for the uniqueness.

The free vibrations of a stiffened circular cylindrical thin shell with shear diaphragm ends are numerically studied by the finite element method. The stiffened shell is discretized by Semiloo shell elements. Then, by means of the variational equation, the matrix equation is obtained to determine the vibration characteristics of the stiffened shell. The natural frequencies and mode shapes of the stiffened circular cylindrical shell are determined, and their accuracy is tested with earlier experimental results. The influence of geometrical parameters and material properties of the stiffened cylindrical shell is investigated on the vibration characteristics. In addition, the effect of the spacings and dimensions of stiffeners is studied. The numerical results are plotted with respect to the parameters of the stiffened cylindrical shell.

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3. Governing Equations

The membran strains in the middle surface of the thin circular cylindrical shell can be written as

\[ e_x = \frac{au_x}{ax} \quad e_y = \frac{1}{R} \frac{au_y}{a\theta} + w \quad e_{x\theta} = \frac{1}{2} \frac{1}{R} \frac{au_y}{a\theta} + \frac{a^2}{a\theta^2} \]  

\[(1a)\]

where \( u_x, u_y, \) and \( w \) denote the displacements of a point on the shell middle surface in the coordinate system \( x, \theta, \) and \( z; \) \( R \) the radius of cylindrical shell.

It is assumed by Donnell\(^1\) that the bending strains of a thin cylindrical shell are negligibly affected by the stretching displacement \( u_0. \) Then the bending strains of a thin circular cylindrical shell can be expressed

\[ k_x = -\frac{\partial^2 w}{\partial x^2} \quad k_y = -\frac{1}{R^2} \frac{\partial^2 w}{\partial \theta^2} \quad k_{x\theta} = \frac{1}{R} \frac{\partial^2 w}{\partial x \partial \theta} \]  

\[(1b)\]

\[\text{Figure 1 Circular cylindrical shell.}\]

The force and moment resultants are related to the membran and bending strains by

\[ N_x = C(e_x + v e_\theta) \quad N_\theta = C(e_\theta + v e_x) \quad N_{\theta x} = N_{0 x} = 2Gh e_{x\theta} \]

\[ M_x = D(k_x + v k_\theta) \quad M_\theta = D(k_\theta + v k_x) \quad M_{x\theta} = M_{\theta x} = \frac{Gh^3}{6} k_{x\theta} \]  

\[(2)\]

where \( C = \frac{Eh}{1 - \nu^2} \) is the stretching rigidity of shell, \( D = \frac{Eh}{12(1 - \nu^2)} \) the bending rigidity of shell and \( h \) the thickness of shell. Here \( E \) and \( \nu \) denote the modulus of elasticity and Poisson's ratio, respectively.

The specific strain energy of a thin circular cylindrical shell can be expressed in terms of the force and moment resultants and the strains as follows

\[ W_c = \frac{1}{2} (N_x e_x + N_{\theta x} e_{x\theta} + 2N_{0 x} e_{0 x} + M_x k_x + M_{0 x} k_{0 x} + 2M_{x\theta} k_{x\theta}) \]  

\[(3a)\]

The specific strain energy of the shell is related only to the strains by using the equations (2).

\[ W_c = \frac{1}{2} C [e_x^2 + 2v e_x e_\theta + e_\theta^2 + 2(1-\nu) e_{x\theta}^2] + \]

\[ + \frac{1}{2} D [k_x^2 + 2v k_x k_\theta + k_\theta^2 + 2(1-\nu)k_{x\theta}^2] \]  

\[(3b)\]

The strain energy of a single stringer per unit length of the stringer centroidal axis can be approximated by

\[ U_s = \frac{1}{2} E_k \int \frac{\left( e_x - (k_x \theta) \right)^2}{k_s} dA + \frac{1}{2} (GJ_k \theta^2) \]  

\[ (4a)\]

where \( E_s, A_s, \) and \( G_sJ_s \) are the modulus of elasticity, cross-sectional area, and torsional stiffness of a stringer, respectively; \( \theta \) is the distance measured along the normal to the shell middle surface (Fig. 2).

\[\text{Figure 2 Stringers and rings.}\]

On carrying out the integrations and dividing \( U_s \) by the stringer spacing \( s \) we obtain the specific strain energy of a stringer per unit area of the shell middle surface:

\[ W_s = \frac{1}{2} \left( \frac{E A_s}{s} \right) e_x^2 - \left( \frac{E A_c}{s} \right) c_x k_x \]

\[ + \frac{1}{2} \left( \frac{E I_s}{s} \right) k_x \theta^2 + \frac{1}{2} \left( \frac{G J_s}{s} \right) k_{x\theta}^2 \]  

\[(4b)\]

where \( I_s \) is the moment of inertia of a stringer about the \( z \) axis and \( c_x \) distance to the centroid from the shell middle surface (eccentricity) (Fig. 2).

A similar expression can be written for the rings

\[ W_r = \frac{1}{2} \left( \frac{E A_r}{s} \right) e_x^2 - \left( \frac{E A_c}{s} \right) c_x k_x \]

\[ + \frac{1}{2} \left( \frac{E I_r}{s} \right) k_x \theta^2 + \frac{1}{2} \left( \frac{G J_r}{s} \right) k_{x\theta}^2 \]  

\[(5)\]

By adding the specific strain energy of the stringer and ring to that of the circular cylindrical shell, we obtain the total specific strain energy of the stiffened circular cylindrical shell.

\[ W = W_c + W_s + W_r \]

\[ = E_1 e_x^2 + E_2 e_\theta^2 + E_3 e_{x\theta}^2 + G_{1x} k_x^2 + G_{1\theta} k_\theta^2 + G_{1x\theta} k_{x\theta}^2 - H_{1x} k_x - H_{1\theta} k_\theta \]

\[(6)\]

where the denotations by

\[\{E_1, E_2, E_{12}\} = \frac{Eh}{2(1-\nu^2)} \{1, 1, 2\} + \frac{1}{2} \{E A_s, E A_c, 0\} \]

\[\{F_1, F_2, F_{12}\} = \frac{Eh^3}{24(1-\nu^2)} \{1, 2, 1\} + \frac{1}{2} \{E I_s, E I_c, 0\} \]

\[G_1 = 2Gh, \quad G_2 = \frac{Gh^3}{6} + \frac{1}{2} \{G J_s, G J_c, 0\} \]

\[H_1 = \left( \frac{E A_c}{s} \right)_c, \quad H_2 = \left( \frac{E A_c}{s} \right)_c \]
4. Uniqueness of Solutions

In the previous section, a set of two-dimensional, differential, approximate equations of Donnell's type is derived for the dynamic response of a cylindrical elastic shell with stringers and rings. The two-dimensional governing equations of stiffened cylindrical shell are constructed by use of Hamilton's principle within the limits of the well-known Kirchhoff-Love hypothesis of thin shells. Now, the boundary and initial conditions are obtained which are sufficient to ensure the uniqueness in solutions of the dynamical governing equations. Of the several arguments to be used to establish the uniqueness of solutions in elasticity, the classical energy argument is used. The energy argument relies upon the positive-definiteness of strain and kinetic energies. Kirchhoff [11] used the energy argument at establishing uniqueness in elastostatics, so did Neumann [12] in elastodynamics and Weiner [13] in thermoelasticity. A uniqueness theorem of Neumann's type is proved for solutions of the initial mixed-boundary value problems defined by the two-dimensional governing equations of stiffened cylindrical shell (cf. [14] for elastic shells).

To begin with, consider two possible sets of solutions to the governing equations of stiffened cylindrical shell, namely,

\[ \alpha = 1.2 \]  

Let the difference set of two solutions be denoted by

\[ \Lambda = \Lambda_2 - \Lambda_1. \]

The difference set of solutions apparently satisfies all the governing equations of stiffened cylindrical shell due to their linearity. It will be shown that the homogeneous linear governing equations possess only the zero solution, that is, the two sets of solutions (12) are equivalent under the pertinent boundary and initial conditions. In so doing, we introduce a relation of the form

\[ \Gamma = \int_T (\Gamma_X + \Gamma_\theta + \Gamma_w) dt = 0 \]

with

\[ \Gamma_X = \int_A \tau_{xX} dA \quad \Gamma_{\theta} = \int_A \tau_{\theta\theta} dA \quad \Gamma_w = \int_A \tau_w dA \]

where \( \tau_x, \tau_\theta, \) and \( \tau_w \) are defined by the equations (9a-c).

Now, let us calculate the rates of the kinetic and strain energies of the stiffened cylindrical shell in terms of the displacement components. The rate of the kinetic energy is expressed with respect to the difference set of solutions in the form

\[ K = \int_A \left( m (a_{xX} u_x + a_{\theta\theta} u_\theta + a_w w) \right) dA \]

In this equation, \( a_x = u_x, a_\theta, \) and \( a_w \) are the components of acceleration, and the equation (7) are used.

Likewise, the rate of the total strain energy is obtained by use of the equation (6a) as follows
\[ U = \int \left\{ \left[ \frac{2E_1}{2} \frac{\partial u_x}{\partial x} + \frac{E_{12}}{R} \left( \frac{\partial u_y}{\partial y} + w \right) + H_1 \frac{\partial^2 u_x}{\partial y^2} \right] \frac{\partial u_x}{\partial x} + \frac{1}{R} \left( E_{12} \frac{\partial u_y}{\partial x} + 2E_2 \frac{\partial u_y}{\partial y} + w \right) \right\} dA \]

in terms of the difference set of solutions. Integrating this equation by parts, one arrives at the rate of the form

\[ \dot{U} = - \int \left\{ \left[ \frac{2E_1}{2} \frac{\partial^2 u_x}{\partial x^2} + \frac{E_{12}}{R} \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial u_y}{\partial y} \right] \frac{\partial u_x}{\partial x} + \frac{1}{R} \left( E_{12} \frac{\partial^2 u_y}{\partial x \partial y} + 2E_2 \frac{\partial^2 u_y}{\partial y^2} + w \right) \right\} dA \]

are introduced.

In view of the energy rates (14) and (16), the relation (13) is expressed as

\[ \gamma = - \int \left( \dot{K} + \dot{U} \right) dt + \int_{t}^{T} \left[ \psi \dot{\psi} R \phi_\alpha - \psi_{\psi} \phi_\alpha \right] d\theta = 0 \]

An integration of this equation with respect to time yields

\[ K(t_t) + U(t_t) = K(t_0) + U(t_0) + \int_{t}^{T} \dot{\Psi} dt \]

with

\[ \Psi = \int_{C} \psi \dot{\psi} R \phi_\alpha + \int_{L} \psi_{\psi} \phi_\alpha d\xi \]

The kinetic and strain energy densities are positive-definite, by definition, and initially zero, so that the total kinetic energy and strain energy, \( K \) and \( U \), calculated by integration from the difference set of solutions for the stiffened cylindrical elastic shell have the same properties. Thus, it follows from the equation (19) that

\[ K(t_t) = U(t_t) = K(t_0) = U(t_0) = 0 \]

This implies a trivial solution for the difference set of solutions, \( \lambda \), since the remaining term \( \psi \) of the equation (19) vanishes in view of the equation (11). Hence, the uniqueness is ensured in solutions of the governing equations of stiffened cylindrical elastic shell. The following theorem of uniqueness is then concluded.

**Theorem** - Given the regular region of a stiffened cylindrical elastic shell in the Euclidean three-dimensional space, then there exists at most one set of single-valued solutions, \( \lambda \), namely,

\[ \lambda = [u_x, u_y, w] \in C_1; \quad \xi, \eta, \phi \in C_2; \quad \psi \in C_0; \quad \nabla \psi \nabla \phi \in C_0; \quad M_{\phi \eta}, M_{\xi \eta}, M_{\xi \phi} \in C_0 \]

which satisfies all the governing equations of the stiffened shell, provided that the kinetic and strain energies are positive-definite, and the boundary (10) and initial conditions (11) are prescribed. \( C_m \) refers to the function with derivatives of order up to and including (m) and (n) with respect to space coordinates (x, y) and time t.

### 5. Method of Solution

In this section some properties of the Semiloof element developed by Irons\(^3\) are briefly reviewed. Figure 3 shows the Semiloof cylindrical shell element where the global coordinate system (x, y, z), isoparametric curvilinear
$$a_{x} = \frac{dN}{d\alpha} Q^e, \quad a_{\theta} = \frac{dN}{d\theta} Q^e$$  \hspace{1cm} (24a)$$

It is used to the following equation to get the \(x, \theta\) derivatives of the shape functions

$$\begin{bmatrix}
\frac{\partial N_1}{\partial \alpha} \\
\frac{\partial N_1}{\partial \theta}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial}{\partial \alpha} \\
\frac{\partial}{\partial \theta}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial N_1}{\partial \alpha} \\
\frac{\partial N_1}{\partial \theta}
\end{bmatrix}
$$  \hspace{1cm} (25)$$

The derivative \(\frac{\partial u_0}{\partial z}\) are given by

$$\frac{\partial u_0}{\partial z} = \left( \frac{\partial u_0}{\partial z} \right) N + \left( \frac{\partial u_0}{\partial z} \right) L = \frac{1}{h} \left( H_0 + H_0 \frac{\partial N}{\partial \alpha} \right) Q^e + L \frac{\partial (\phi_{xz})^e}{\partial z}$$  \hspace{1cm} (24b)$$

where \(h\) represents the shell thickness at the point \(P\) and \(H_0\) and \(H_0\) are the components of a thickness vector \(H\) defined by the following formula

$$H = \sum_{j=1}^{9} \eta_j Z_j$$  \hspace{1cm} (26)$$

Here, \(Z_j\) is the unit normal vector of the middle surface at the Loof nodes, and the central node. It can be computed from the vector product at any point, say \(P\), as

$$Z = \alpha P \times \alpha P$$  \hspace{1cm} (27)$$

Similar expression can be obtained for \(\frac{\partial u_0}{\partial z}\). Since it is assumed that the shear \(\gamma_{xz}\) and \(\gamma_{xy}\) are zero, it may be written

$$\frac{\partial w}{\partial x} = \frac{\partial u_0}{\partial z}, \quad \frac{\partial w}{\partial \theta} = R \frac{\partial u_0}{\partial z}$$  \hspace{1cm} (28)$$

The derivatives governing the bending behavior of the shell are given by

$$\frac{\partial^2 u_0}{\partial z^2} = \frac{1}{h} \left( \frac{\partial H_0}{\partial \alpha} \frac{\partial N}{\partial \alpha} + \frac{\partial H_0}{\partial \theta} \frac{\partial N}{\partial \theta} + \frac{\partial H_0}{\partial \theta} \frac{\partial N}{\partial \alpha} \right) Q^e + \frac{\partial L}{\partial \theta} (\phi_{xz})^e$$  \hspace{1cm} (29)$$

Similarly, the derivatives \(\frac{\partial^2 u_0}{\partial x \partial z}\), \(\frac{\partial^2 u_0}{\partial y \partial z}\), and \(\frac{\partial^2 u_0}{\partial y \partial z}\) can be obtained.

The local displacement components and their derivatives are used to determine the stiffness and mass matrices after an application of the discrete Kirchhoff-Love hypothesis\(^{15}\).

Discretizing the stiffened cylindrical shell with Semiloof elements, we obtain the matrix equation

$$\ddot{M} + KQ = 0$$  \hspace{1cm} (30)$$

by means of Hamilton's principle (8) for the dynamic behavior of the stiffened shell. Here, \(M\) and \(K\) are the mass and stiffness matrices of the stiffened cylindrical shell, respectively. \(Q\) is the vector of nodal displacements and rotations.

Assuming harmonic motion, \(Q = Q^{\text{real}}\), the equation (30) reduces a linear eigenvalue problem as

$$[K - \alpha^2 M] \{Q\} = 0$$  \hspace{1cm} (31)$$

This equation is solved by using the EISPACK routines.
6. Numerical Results

The vibration characteristics are determined for stiffened circular cylindrical shells having shear diaphragm ends, which have integral stringers and/or rings. Two types of stiffened circular shell model are used. C model is reinforced by the stringers with rectangular cross-sections and F model is reinforced by the stringers and/or rings with profile shape cross-section. Geometrical and material properties of the models are listed in Table 1.

Table 1 Geometrical and material properties.

<table>
<thead>
<tr>
<th>Model</th>
<th>C</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>L(m)</td>
<td>0.39446</td>
<td>18.00</td>
</tr>
<tr>
<td>R(m)</td>
<td>0.04976</td>
<td>1.95</td>
</tr>
<tr>
<td>h(m)</td>
<td>0.00165</td>
<td>0.0012</td>
</tr>
<tr>
<td>E(N/m²)</td>
<td>68.95 x 10⁶</td>
<td>68.95 x 10⁹</td>
</tr>
<tr>
<td>μ(kg/m²)</td>
<td>2760</td>
<td>2760</td>
</tr>
<tr>
<td>ν</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The natural frequencies and the mode shapes are listed in Table 2 for a stringer stiffened shell, (model C). Natural frequencies obtained by using the super shell finite element and measured values obtained by Hopmann are also included for the purpose of comparison. Dimensions of a stringer are s₀ = 0.00535 m, s₀ = 0.003175 m and the number of external stringers is N₂ = 16. The present results are in good agreement with earlier experimental and numerical results.

Table 2 Natural frequencies of a stringer stiffened shell with shear diaphragm ends (Model C).

<table>
<thead>
<tr>
<th>mode number experiment</th>
<th>predicted natural frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>m, n</td>
<td>present element⁴ d.o.f. = 272</td>
</tr>
<tr>
<td></td>
<td>present element⁴ d.o.f. = 376</td>
</tr>
<tr>
<td></td>
<td>nine-node element⁹ d.o.f. = 592</td>
</tr>
<tr>
<td>2,1</td>
<td>4216</td>
</tr>
<tr>
<td>2,2</td>
<td>3228</td>
</tr>
<tr>
<td>2,3</td>
<td>1830</td>
</tr>
<tr>
<td>2,4</td>
<td>2600</td>
</tr>
<tr>
<td>2,5</td>
<td>4080</td>
</tr>
</tbody>
</table>

The variation of the natural frequency ratio Ω/Ω₀ of circular cylindrical shell (model C) with the depth ratio of a stringer is shown in Fig. 4 for different width ratio of stringers. Here, Ω₀ is the natural frequency of the unstiffened circular cylindrical shell. Number of stringers is N₂ = 16.

The influence of the number of the stringers and rings on the vibration characteristics is investigated for orthogonally stiffened circular cylindrical shells (model F). The cross-section of a stringer and a ring is depicted in Fig. 5.

The effects of the number of stringers on the vibration characteristics are investigated for an orthogonally stiffened circular cylindrical shell. The number of rings is taken to be 36. The variation of the lowest three natural frequencies is plotted in Fig. 6.

Fig. 4 The depth ratio effects on the natural frequency ratio of a stiffened cylindrical shell.

Fig. 5 Dimensions of profiles.

Fig. 6 Natural frequencies vs the number of stringers. N₂ = 36.
7. Discussion

The free vibrations of stiffened circular cylindrical thin shells with shear diaphragm ends are studied numerically within the frame of Love's first approximation theory of elastic shells. The governing equations are obtained from the three-dimensional equations of elastodynamics by means of a generalized variational principle together with the usual kinematic hypothesis of cylindrical thin shells. The effects of stiffeners are taken into account by the orthotropic material approach. The uniqueness is tested in solutions of the dynamic governing equations of stiffened shell. The vibration characteristics are obtained numerically by means of the finite element method. A comparison of the present results with earlier results shows good agreement.

In the uniqueness theorem, the boundary and initial conditions which render \( \psi \) to zero are shown to be sufficient for the uniqueness in solutions of the governing equations of stiffened cylindrical elastic shell. The conditions (10) and (11), and also, to specify one member of each of the products in \( \psi \) of (20) ensure a unique solution for the governing equations. Besides, the sufficient conditions can be expressed in terms of the stress resultants as well as the displacement components, and they can obtained by logarithmic convexity arguments with no restrictions on the positive-definiteness of energies.

For a circular cylindrical shell reinforced by stringers, the natural frequency slightly decreases with the depth ratio of the stringer. The influence of the width ratio intends to decrease the natural frequency. Because an increment at the depth ratio and the width ratio of a stiffener contributes to the stiffness and mass of the cylindrical shell, but the contribution to the mass of the shell is significant for the considered examples.

The number of the stringers slightly effects the dynamic behavior of the stiffened cylindrical shell. But the number of rings considerably increases the natural frequency corresponding to the mode shape \( m,n = 1,3 \), and decrease the natural frequency of mode shape \( m,n = 1,1 \). It should be noticed that the mode shape corresponding to the lowest natural frequency is not the simplest one, and this mode shape may change with the geometrical properties of the stiffened cylindrical shell and the conditions of reinforcing. Sometimes although the variation of the lowest natural frequency with a parameter is negligible, other frequencies may show considerable increasing with the parameter.

The natural frequencies corresponding some mode shapes strongly decreases with the increasing in the length of orthogonally stiffened circular cylindrical shells.

The present method can be successfully used so as to investigate the effect of rectangular cutouts on the free vibration characteristics of stiffened cylindrical shell. Also, by the method, the free vibration analysis can be carried out for the stiffened cylindrical shell with discrete stiffeners. The method can be extended to the forced-response dynamic behavior and linear buckling analysis of stiffened cylindrical shell; this is will be reported else where.

References
