NONLINEAR FLUTTER ANALYSIS OF WINGS AT HIGH ANGLE OF ATTACK

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Abstract

In this paper, two methods are presented to analyze nonlinear flutter of wings with separated vortex at high angle of attack. One of the methods is a Time Integration Method (TIM). Combined with the calculated unsteady aerodynamic forces for wings at high angle of attack, the structural dynamic equations of the wing are integrated by Runge-Kutta method in time domain, and the wing motion can be simulated at any flying speed. Another method is a Describing Function Method (DFM). In the DFM, the nonlinear generalized aerodynamic forces are linearized by using the concept of describing function. Then, the structural dynamic equations of the wing are solved by conventional V-g method, and the critical flutter speed can be obtained.

To verify the numerical methods, flutter tests for wings at high angle of attack are carried out in a low speed wind tunnel. The wing models are a rectangular wing and a delta wing. The wings can move in rolling and pitching. The basic angles of attack in the experiment are $14^\circ$ and $18^\circ$. It is shown that the higher the basic angle of attack, the lower the critical flutter speed. The results calculated by the above mentioned methods are in agreement with the experiment.

\begin{align*}
\mathbf{f} & = \text{generalized aerodynamic force vector} \\
H(\mathbf{\Phi},s) & = \text{indicial response of generalized aerodynamic force coefficient} \\
[I] & = \text{unit matrix} \\
[K] & = \text{generalized stiffness matrix} \\
k & = \text{reduced frequency} \\
[M] & = \text{generalized mass matrix} \\
[0] & = \text{zero matrix} \\
\phi_p(x,y) & = \text{pressure difference on the wing surface} \\
q(t) & = \text{generalized coordinate} \\
S & = \text{wing area} \\
s & = \text{laplace variable} \\
t & = \text{time} \\
U & = \text{freestream velocity} \\
V & = \text{critical flutter speed} \\
x, y & = \text{corresponding coordinates} \\
z & = \text{deformation of the wing} \\
\Phi & = \text{a kind of deformation of the wing} \\
\tau & = \text{nondimensional time} \\
\rho & = \text{density} \\
\omega & = \text{eigen frequency} \\
\omega_s & = \text{static angle of attack} \\
\psi & = \text{nondimensional rolling angle} \\
\theta & = \text{nondimensional pitching angle} \\
\text{subscript} & \quad \text{ith} \\
s & = \text{static} \\
i & = \text{ith} \\
j & = \text{jth}
\end{align*}

Nomenclature

Introduction

For nonlinear flutter analysis of wings at high angle of attack, Strganac and Mook [1] developed an integration method in time domain. With the unsteady airloads on wings obtained by an unsteady Vortex-Lattice Method, the equation of motion is integrated. In Ref. [1], only the results of a rectangular wing with big aspect ratio AR=10 were given. For another low aspect ratio rectangular wing, no detailed results were supplied. The integration method in Ref. [1] can give the vibration history of wings at any flying speeds, but the computational work is time consuming.

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By introducing a describing function for the nonlinear generalized aerodynamic force, Ueda and Dowell [2] analyzed the flutter of airfoils at transonic flow in frequency domain with better computational efficiency.

Up to now, the phenomenon of nonlinear flutter for wings with separation at high angle of attack has not been thoroughly investigated. So, wind tunnel tests for this problem would be much helpful for further research.

In this paper, both Time Integration and Describing Function Methods are developed for nonlinear flutter analysis of wings at high angle of attack. The nonlinear aerodynamic forces are provided by a subsonic unsteady numerical method—Potential Difference Method [3] developed recently by the authors. Besides, the experiments of flutter for the wings at high angle of attack were carried out, and the test results confirm the feasibility of the above mentioned methods.

Time Integration Method

The small deformation of wing structure can be expressed by

\[ z(x, y, t) = \sum_{i=1}^{N} \phi_i(x, y) \cdot q_i(t) \]  
(1)

where \( \phi_i(x, y) \) is the \( i \)th eigen mode, and \( q_i(t) \) is the corresponding generalized coordinate.

Then the equation of motion can be written as

\[ [M] [\ddot{q}] + [K] [q] = [f] \]  
(2)

where the dot means \( \frac{d}{dt} \), \([M]\) is the generalized mass matrix, \([K]\) is the generalized stiffness matrix, \([f]\) is the generalized aerodynamic force vector.

By introducing the state vector \( [e] = [q_1, q_2, q_3, \ldots, q_n, \dot{q}_1, \dot{q}_2, \dot{q}_3, \ldots, \dot{q}_n]^T \).

Eq.(1) is transformed to

\[ [e] = [(0) \quad (I)] [K] [q] = [f] \]  
(3)

For a given initial vector \( [e_0] \), the aerodynamic pressure difference \( \Delta P(x, y) \) on wing surface is obtained by the Potential Difference Method, and the generalized aerodynamic forces are calculated by

\[ f_i = \int_{S} \Delta P(x, y) \cdot \phi_i(x, y) \, dx \, dy \]

then, Eq.(3) can be solved by Runge-Kutta method. The state vector \( [e] \) obtained at the end of each time step provides a new boundary condition for the next time step to calculate the aerodynamic pressure. By this procedure, the wing motion can be simulated with a step-by-step discrete time history.

Describing Function Method

For a given basic angle of attack \( \alpha_0 \), the static deformation \( [q]_s \) must be calculated at first by an iteration process from the governing equation

\[ [K] [q]_s = [f_s] \]

where \([f_s]\) is the static aerelastic airload. Then, the initial state vector \([e_0]\) for the above Runge-Kutta procedure is formed by \([q]_s \) superposed by a certain disturbance of generalized displacements and velocities.

The corresponding relation in the Laplace domain is

\[ \hat{C}(\mathfrak{f}, s) = D(\mathfrak{f}, s) \cdot \mathfrak{f}/(s - ik) \]  
(6)

where the superscript \( \hat{\cdot} \) represents the Laplace transformation of a function, \( s \) is the Laplace variable and \( i = \sqrt{-1} \).

If \( k = 0 \), the Eq.(6) represents an indicial response relationship

\[ H(\mathfrak{f}, s) = D(\mathfrak{f}, s) / \mathfrak{f} \]  
(7)

In practical application, the indicial response time history of aerodynamic force is first calculated by certain nonlinear aerodynamic code with a step input \( \mathfrak{f} \). Then, by aid of curve fitting, this indicial response is approximated by a polynomial of exponential functions \( \Sigma a_t \exp(b_t \tau) \), from which it is straightforward to get an closed form expression of \( H(\mathfrak{f}, s) \) in Laplace domain. According to Eq.(7), the describing function is obtained

\[ D(\mathfrak{f}, ik) = ik \cdot H(\mathfrak{f}, ik) / \mathfrak{f} \]  
(8)

By introducing the describing functions, the generalized aerodynamic force for wings at high angle of attack...
formally has a linear relation to the
generalized displacements

\[ f = \frac{1}{2} \rho U^2 S \cdot [D] \cdot q \]

(9)

where the generalized aerodynamic force
coefficient matrix [D], different from
the linear problem, depends on not only
the reduced frequency but also the
vibration amplitudes.

So, a linear flutter equation is
gotten

\[ [M][\ddot{q}] + [K][q] = \frac{1}{2} \rho U^2 [D] \cdot q \]

(10)

This equation must be solved by an
iteration process. At first, a set of
vibration amplitudes \( \{q_k\} \) are chosen.
After the steady flow for the wing at a
basic angle of attack is computed, a
step deformation \( \delta \chi(x,y) \cdot \overline{q} \) is superposed
upon the wing impulsively, and a set of
indicial responses of generalized
aerodynamic force coefficients are
obtained in the time domain. Then, the
\( D_{ij} \) \( (i=1,N) \) are calculated by the
procedure mentioned above.

Having got the aerodynamic matrix
[D], the conventional V-g method
is adopted to solve Eq. (10), and the result
provides a critical flutter speed \( V_f \) and
flutter mode \( \{q_k\} \), which usually is not
consistent with the preassigned \( \{q_k\} \).
Then, the resultant mode \( \{q_k\} \) is used as
the initial \( \{q_k\} \) for the second V-g
flutter calculation. Such an iteration
procedure will lead to the final results,
i.e. the limit cycle flutter mode and the
flutter speed.

Experiments

The models for wind tunnel test are
a solid wooden rectangular wing and a
delta wing. A supporting system provides
rolling (\( \phi \)) and pitching (\( \theta \)) degrees
of freedom for the rigid models. The
rectangular wing has a sharp side edge
with aspect ratio AR=2, and the chord
length is 300 mm. The pitching axis is
68 mm behind the leading edge. The delta
wing has a sharp leading edge with aspect
ratio AR=2.61, and the root chord length
is 450 mm. The pitching axis is 285 mm
before the trailing edge. The basic angles
of attack in the experiments are 14° and
18° for both wings. At these attitudes,
vortex separation occurs from the side
dge for the rectangular wing and from
the leading edge for the delta wing.

A low speed wind tunnel with the
diameter 1.2 m of the test section is
used, its maximum airspeed is 60 m/s.
Fig. 1 is the photograph of the test
model.

The supporting springs are adjustable,
and hence can provide different
eigen frequencies for the model. The \( \omega^\phi \)
and \( \omega^\theta \) denote the rolling and pitching
frequency respectively. For the rectan-
gular wing, two sets of \( \omega^\phi, \omega^\theta \) are
selected. One set is \( \omega^\phi=1.875 \) Hz
and \( \omega^\theta=2.5 \) Hz. Another set is \( \omega^\phi=1.875 \) Hz
and \( \omega^\theta=2.75 \) Hz. For the delta wing, only
one set \( (\omega^\phi=2.0 \) Hz, \( \omega^\theta=2.375 \) Hz) is used.

![The test system](image)

Fig.1 The test system
(a) the supporting system
(b) the rectangular wing model
in the wind tunnel

Results and Conclusions

The test results of the critical
flutter speed and the results calculated
by both TIM and DPM are listed in
Table-1, where \( \alpha \) is the basic angle of
attack.
TABLE-1 COMPARISON OF CALCULATED
RESULTS WITH EXPERIMENT

<table>
<thead>
<tr>
<th>CASE No.</th>
<th>TEST WING</th>
<th>$\alpha^o$</th>
<th>$\omega_r$ (Hz)</th>
<th>$\omega_\theta$ (Hz)</th>
<th>TEST $V_r$ (m/s)</th>
<th>TIM $V_r$ (m/s)</th>
<th>DFM $V_r$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RECT.</td>
<td>14</td>
<td>1.875</td>
<td>2.5</td>
<td>11</td>
<td>10.25</td>
<td>10.47</td>
</tr>
<tr>
<td>2</td>
<td>RECT.</td>
<td>18</td>
<td>1.875</td>
<td>2.5</td>
<td>10</td>
<td>9.25</td>
<td>9.129</td>
</tr>
<tr>
<td>3</td>
<td>RECT.</td>
<td>14</td>
<td>1.875</td>
<td>2.75</td>
<td>13</td>
<td>11.30</td>
<td>11.9635</td>
</tr>
<tr>
<td>4</td>
<td>RECT.</td>
<td>18</td>
<td>1.875</td>
<td>2.75</td>
<td>11</td>
<td>10.20</td>
<td>10.1936</td>
</tr>
<tr>
<td>5</td>
<td>DELTA</td>
<td>14</td>
<td>2.0</td>
<td>2.375</td>
<td>21</td>
<td>19.50</td>
<td>18.6414</td>
</tr>
<tr>
<td>6</td>
<td>DELTA</td>
<td>18</td>
<td>2.0</td>
<td>2.375</td>
<td>20</td>
<td>18.50</td>
<td>17.56</td>
</tr>
</tbody>
</table>

where the RECT. means rectangular.

The vibration time history obtained by TIM for case-3 at the airspeeds 11 m/s and 11.5 m/s are shown in Fig.2 and Fig.3 respectively. The former is a subcritical state, and the latter is a supercritical state.

Fig.4 and Fig.5 depict the results of the TIM for case-5 and case-6 at the airspeeds 19.5 m/s and 18.5 m/s respectively. Both states are at the critical flutter point.

For the calculation of the generalized aerodynamic force coefficients in the DFM, an eleven terms polynomial of exponential functions is used in the curve fitting process for the indicial response.

Since there is a gap between the wing root and the shield plate, and the influence of this gap cannot be taken into account accurately in the numerical analysis. This is one of the reasons for the discrepancy between the test and calculation results. Nevertheless, the numerical results may be considered to be satisfactory.

Compared with the TIM, the advantage of the DFM lies in the much less computational time needed to get a flutter point. But the DFM can only give the critical flutter point, while the TIM can give not only the critical flutter point, but also the subcritical and supercritical responses. Besides, the linearized approximation of the generalized aerodynamic forces in DFM is not involved in the TIM. The superposition of modal generalized aerodynamic forces used in the DFM is tenable only upon the engineering consideration.

From the results of the present work, it can be seen that the higher the basic angle of attack, the lower the critical flutter speed. This means that the vortex separation for wings at high angle of attack deteriorates the flutter characteristics of the wing, which is important for combat aircraft in maneuver flight.

References

3. Ye, Z., Yang, Y. and Zhao, L., Subsonic Steady, Unsteady Aerodynamic Calculation For Wings at High Angle of Attack. 17th ICAS Congress.

Fig.2 The $\varphi$ and $\theta$ versus the nondimensional time
(U$\infty$ = 11.0 m/s)
Fig. 3 The $\psi$ and $\theta$ versus the nondimensional time
($U_\infty = 11.5$ m/s)

Fig. 4 The $\psi$ and $\theta$ versus the nondimensional time
($U_\infty = 19.5$ m/s)

Fig. 5 The $\psi$ and $\theta$ versus the nondimensional time
($U_\infty = 18.5$ m/s)