OPTIMIZATION OF HELICOPTER TAKEOFF AND LANDING*

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Abstract

The helicopter has the capability to perform takeoff and landing by different techniques. From these, the Cat. A takeoff according to FAR (Federal Aviation Regulations) Part 29 Cat. A for multiengine helicopters is of interest. Helicopters of this category are able to continue takeoff after failure of one of the engines.

The investigation deals with the optimization of the special runway Cat. A takeoff. This procedure is applied when the helicopter has to perform a takeoff similar to an aircraft. In the event of engine failure, the CDP (Critical Decision Point) is the essential criterion for the pilot's decision whether to continue takeoff or to reject takeoff. Up to now, the CDP is determined by flight tests and defined as decision point for the whole flight envelope.

The Cat. A runway takeoff with/without engine failure and the relevant performances will be discussed. A quasi-stationary simulation model on the basis of performance data fields will be presented. Parameter variations show the influence of relevant parameters on takeoff performances, e.g., the takeoff distance, by means of simulation results. The optimization task resulting in performance and trajectory optimization will be demonstrated. Finally, the optimization results will be shown and optimal takeoff techniques will be discussed.

Notations

$A_i$: polynomial coefficients
$f:F_{\text{opt}}$: function value for optimization
$g$: gravity constant
$G$: helicopter weight
$H_{\text{T.O.}}$: takeoff height
$H_{\text{CDP}}$: critical decision height
$K_r$: rotor-ground distance
$K_g$: weight coefficient $(26/\sqrt{u^2})$
$K_p,H$: power coefficient in hover $(2P/H \sqrt{u^3})$
$m$: helicopter mass
$PH$: power required for hover
$P^2\text{Engine}$: power available (takeoff power)
$P^1\text{Engine}$: power available (emergency power)
$R$: rotor radius
$S$: rotor disc area
$U_i$: optimization parameters
$U$: rotor tip speed
$U_{\text{Kg}}$: geodetical horizontal velocity
$\bar{U}_{\text{Kg}}$: non-dimensional horizontal velocity $(U_{\text{Kg}}/W_{1,h})$
$V_{\text{CDP}}$: critical decision speed
$V_{\text{Y}}$: speed for best rate of climb
$V_{\text{TOSS}}$: takeoff safety speed
$X_p$: power factor
$\rho$: air density
$W_{\text{Kg}}$: geodetical vertical velocity
$\bar{W}_{\text{Kg}}$: non-dimensional vertical velocity $(W_{\text{Kg}}/W_{1,h})$
$W_{1,h}$: induced velocity in hover
AE0: All Engines Operating
CDP: Critical Decision Point
DGLR: Deutsche Gesellschaft für Luft- und Raumfahrt
FAR: Federal Aviation Administration
IGE: In Ground Effect
OEI: One Engine Inoperative
OGSE: Out Ground Effect
PEI: Power Excess Index
PDP: Power Deficiency Parameter

I. Introduction

Takeoff and landing are critical flight regions for helicopters as well as for normal aircrafts. In contrast to aircrafts, quite different procedures are applied to helicopters. The Cat. A procedures according to FAR (Federal Aviation Regulations) Part 29 (1), (2) for multiengine helicopters with isolated engines, i.e., engines independent of one another, are of special interest. Helicopters of this category are able to continue takeoff under certain conditions after engine failure. The optimal use of the helicopter performance, taking safety requirements into account, is significant for the takeoff distance. The investigations are primarily confined to the runway takeoff including the emergency landing for rejected takeoff after engine failure.

The critical decision point, CDP, which is defined by a combination of a speed and an altitude, is essential for the pilot's decision whether to continue takeoff or to reject takeoff in the event of engine failure. If one engine fails before CDP, takeoff has to be rejected, and if the engine fails after CDP, takeoff has to be continued. With helicopters, the determination of the CDP as well as the required takeoff distances for rejected or continued takeoff are obtained by flight tests. The CDP is determined for the most critical conditions, this being maximum takeoff weight, maximum altitude, low

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temperature and minimum emergency power of the operating engine. The CDP is then defined as the decision point for the whole flight envelope.

Flight tests for the experimental determination of takeoff distance and CDP are critical tests relating to safety since the limits of helicopter performance are reached. Apart from that, these experiments take much time and involve high costs. The request for theoretical methods for the determination of characteristic performances, like takeoff distances and CDP, results from considerations relating to safety and economic efficiency. Only with the aid of theoretical methods, it is possible to undertake at low expense a systematical investigation of the essential parameters influencing takeoff performance as well as a consistent optimization of the takeoff procedure.

There have been only few theoretical studies dealing with the optimization problem of takeoff (3). Investigations regarding the optimization of takeoff with engine failure are not known.

Numerical simulation models, as developed in (4), (5) are suitable theoretical methods for this task. Apart from the accuracy with which helicopter power required is calculated, the required computation time is of decisive importance particularly for the optimization. For each step of the optimization procedure one takeoff simulation has to be carried out.

First of all the Cat. A runway takeoff without and with engine failure will be explained. The relevant performances of the helicopter are briefly discussed. After that, there will be presented particular features of the quasi-stationary helicopter simulation of the longitudinal motion on the basis of a so-called data field simulation. For the optimization, a numerical optimization procedure from (6) is applied. The integration of the simulation in the optimization process will be explained as well as the proceeding used for optimization, e.g. the choice of optimizable parameters. The investigations are primarily confined to the optimization target: minimization of the takeoff distances for rejected and continued takeoff after engine failure.

At first, there will be assumed one CDP which is known from flight tests. This CDP, which is also given in the flight manual, is used for the whole flight envelope. In a second step, the CDP will be available as a parameter for the optimization algorithm. Because of that, the optimization results in variable CDP's which depend on gross weight, atmospheric conditions, available takeoff power and emergency power in the event of engine failure. With this variable CDP, there can be attained considerably shorter takeoff distances than in the case of a determined CDP. The optimization results are discussed in detail for one helicopter.

II. The Cat.A takeoff procedure

Takeoff procedures of multiengine helicopters according to FAR Part 29 Cat.A are characterized in that way that continued takeoff ability has to be ensured in the event of an engine failure after CDP. Figure 1 shows typical takeoff profiles without and with engine failure in CDP. (AOE: All Engines Operating, OEI: One Engine Inoperative). The takeoff starts with hover in ground effect at a minimum height Hg. At a takeoff with sufficient excess power, it is possible to perform a horizontal acceleration maneuver without height loss (Figure 1a), whereas a considerable loss of height is unavoidable if there is no excess power. The height loss essentially depends on the horizontal acceleration and the ground effect, especially on the ground vortex that is developing at low speed in front of the rotor (7).

![Normal Takeoff](image-a)

![Rejected Takeoff](image-b)

![Continued Takeoff VCDP < VToss](image-c)

![Continued Takeoff VCDP > VToss](image-d)

At transition into forward flight, the helicopter accelerates to decision speed VCDP and climbs to decision height HCDP. Without engine failure, the takeoff will be continued normally, and the climb is performed with VY, the speed for maximum rate of climb. The takeoff distance is the distance from hover until the 50ft-height is cleared.

If one engine fails before CDP (Figure 1b), the takeoff has to be rejected and is finished safely with an emergency landing. Normally, the emergency power of one engine is smaller than the power.
required for hover, so that the helicopter is touching down with horizontal speed. A maximum rate of descent during touch down can be permitted. The takeoff distance is the distance from hover up to the touch down point, the standstill of the helicopter on the ground, respectively. For the investigations carried out in this paper, the rejected takeoff distance is calculated up to the touch down point.

If one engine fails after CDP (Figure 1c,d), the takeoff has to be continued by means of the available emergency power and a minimum rate of climb of 100ft/min ≈ 0.5m/s. The speed which permits the minimum rate of climb is Vtoss (Take Off Safety Speed). At continued takeoff, there has to be distinguished between two cases. If Vcpd > Vtoss, there does not occur a height loss after CDP. If Vcpd < Vtoss, the helicopter has to be accelerated to Vtoss and generally, there occurs a considerable loss of height. The allowable minimum height reached during this maneuver is given by the FAR (1) as half the CDP height, the minimum required flare height, respectively.

These statements are valid for the assumption that Vcpd and Vtoss are below Vp,min, the speed for minimum power required in forward flight which is nearly Vy. In the following, this will be a general assumption. In this velocity range, higher velocity leads to smaller power required (see also Figure 5). The takeoff distance for continued takeoff is finished by clearing Vtoss and the 35ft-height. The longer distance of rejected takeoff and continued takeoff has to be indicated as required takeoff distance.

Figure 2 shows in a principle outline the influence of Hcpd on the flight path during rejected takeoff and continued takeoff. The influence of Vcpd is suggested in this figure.

If the choice of the decision height Hcpp is disadvantageous, either the rejected takeoff distance or the continued takeoff distance is the longer one. If the decision height is chosen "optimally," the required distances are identical and result in the shortest takeoff distance.

In this figure, it is assumed that all the represented flight paths are flyable by means of available emergency power of one engine. This is not guaranteed in principle. For both cases, Vcpd > Vtoss and Vcpd < Vtoss, basically the same relation is obtained. Reduction of the decision speed Vcpd leads to a shorter acceleration segment and thus to a reduced total takeoff distance. The reduction of Vcpd is not arbitrarily possible. The precise conditions are discussed in detail under section V.

It has already been pointed out that the CDP is experimentally determined by flight tests. In these flight tests, the CDP is established for the most unfavourable conditions, these being maximum weight, maximum range of height and temperature. In determining the conditions for flight tests and their analysis, the characteristical variables P.E.I. (Power Excess Index) and P.D.P. (Power Deficiency Parameter) which are defined in (2), are applied:

\[
P.E.I. = \frac{P_{2\text{Engine}} - P_{H,\text{IGE}} (H_{T.O.}=1\text{m})}{G}
\]

\[
P.D.P. = \frac{P_{H,\text{IGE}} - P_{1\text{Engine}}}{G}
\]

How these flight tests for the Cat.A takeoff have to be carried out is explained more detailed in (2). This results in the diagram, Figure 3, out of which the height Hcpp is established for a critical P.D.P.. The decisive factor is that the represented boundaries are valid for one already determined speed Vcpd. The speed Vtoss is as well presupposed as known, resulting from special flight tests independent of the takeoff flight tests.

The limit for continued takeoff results for Vcpd < Vtoss from the condition that a possible takeoff is being restricted by the minimum height permitted over ground and the height loss due to required acceleration up to Vtoss (Figure 1d).
For \( V_{cpp} > V_{toss} \) (continued takeoff without height loss), there could be achieved a similar limit by establishing a maximum takeoff distance. For this, the choice of \( V_{cpp} \) relative to \( V_{toss} \) is of importance.

The limit for rejected takeoff results in general from a rejected takeoff not being performable within a predetermined distance. As a maximum distance for rejected takeoff e.g. the required continued takeoff distance for a current P.D.P. can be predetermined. After the intersection point of the boundaries, rejected takeoff would require a lower \( H_{cpp} \) than continued takeoff. The point of intersection yields \( H_{cpp} \) and the critical P.D.P.

If a rejected takeoff distance longer than the required continued takeoff distance would be accepted, a higher \( H_{cpp} \) and a higher critical P.D.P. would be taken from the diagram. Additionally, there also exists a maximum P.D.P., where a safe climb with a minimum rate of climb of 100ft/min, and thus a continued takeoff capability, is not assured.

For a P.D.P. < P.D.P.crit, there exists a region in which the decision height could be varied. The established \( H_{cpp} \) does not necessarily give the shortest takeoff distance for every P.D.P.

The assumption of a rejected takeoff which can be performed savelly, presupposes a takeoff outside the boundaries represented in the Height-Velocity-Diagram. Figure 4 shows the H-V-diagram with the dangerous areas for engine failure (One Engine Inoperative). For the certification, the boundaries have to be determined by flight-tests.

![Figure 4 Typical Height-Velocity-Diagram](image)

The H-V-diagram is established for engine failure during hover (high hover and low hover point) and horizontal flight (knee point). The FAR require a minimal distance of 5kts from the H-V-limits for a Cat.A takeoff. The boundary for low height and high speed does not apply to most helicopters and does not exist for the helicopter B0 105, which is examined here. The boundaries, taken strictly, are valid for horizontal flight. An additional rate of climb can be considered as advantageous in view of a safe emergency landing after engine failure. An increasing rate of climb leads primarily to a longer distance for an emergency landing, to a longer rejected takeoff distance, respectively. Dealing with the theoretical and experimental determination of the H-V-limits, there exist numerous investigations (8), (9).

The characteristic variables P.E.I. and P.D.P. have an essential influence on the takeoff procedure. While P.E.I. is influencing the distance up to the CDP, P.D.P. has an effect mainly on the distance after the CDP. Figure 5 shows the power required of a helicopter for hover and forward flight, out of and in ground effect (OGE: Out Ground Effect, IGE: In Ground Effect), for a ground effect height of \( H_{T,0} = 1m \). Starting from hover with increasing speed, power required decreases rapidly up to a minimum power until it increases due to fuselage drag. For takeoff, the region up to a speed near \( V_{y} \) is interesting.

![Figure 5 Power Required at Forward Flight with/without Ground Effect](image)

Additionally, the critical decision speed \( V_{cpp} \) and the speed \( V_{toss} \) are marked, both of which are in general smaller than \( V_{p,\min} \). The ground effect has an essential influence on the takeoff procedure. Ground effect reduces helicopter power required depending on the height of the helicopter rotor above ground as well as on the airspeed. The decrease of ground effect with increasing speed is visible in the figure. In a range of \( V=15\text{m/s} \), the influence of ground effect may be neglected. Furthermore, a minimum and a maximum available takeoff power for two engines as well as the emergency power of one engine are shown. According to FAR, a takeoff from a minimum height of \( H_{T,0} = 1m \) without collective increase (that is, nearly constant power) has to be assured. This results in the fact that minimum takeoff power has to be the power required for hover in ground effect \( P_{H,IGE} \) for \( H_{T,0} = 1m \) (takeoff without power excess, P.E.I. = 0m/s). For a given takeoff power, this may limit the maximum takeoff weight. The maximum power available for two engines and the emergency power for one engine depend on the limits of engine and gear. For a Cat.A takeoff, the emergency power has to be higher than the minimum power required for horizontal flight in order to guarantee the capability for continued takeoff.
III. Data field simulation

For the simulation and optimization of the Cat.A takeoff, primarily the general performances of the helicopter are of interest. The safe execution of the takeoff and the required takeoff distance depend directly on power required, power available and maneuver strategy. Important about that is the calculation of power required and power available as accurate as possible.

Helicopter power required is, apart from horizontal velocity and ground effect height, dependent on gross weight, atmospheric conditions, vertical velocity and translational accelerations. These effects have to be considered in principle for the simulation of maneuvers and especially for the simulation of takeoff procedures.

In (5), there have been presented different simulation models for the simulation of takeoff and landing procedures. Apart from a complex three-dimensional simulation model, a quasi-stationary two-dimensional model and a quasi-stationary data field model have been developed. Whereas the three-dimensional model contains the six degrees of freedom of the rigid body, the two-dimensional model contains only the longitudinal motion, regarding the quasi-stationary degree in pitch. Angular velocities and accelerations have a neglectable influence on power required.

Both models are not suitable for optimization because of computation time. Therefore, basing on data reduction methods (10), a quasi-stationary simulation on the basis of data fields has been developed. The data fields contain power required for stationary flight states with and without ground effect in non-dimensional form. By means of reduction of power required, as presented more detailed in (10), a compressed representation of power required can be attained. It is defined a power coefficient \( K_P \), which depends on the four parameters weight coefficient, \( K_g \), non-dimensional horizontal and vertical velocity, \( \bar{U}_{Kg} \) and \( \bar{w}_{Kg} \), and the relative ground distance \( H_g/R \). These dependencies are combined in two diagrams.

Figure 6 shows the power coefficient \( K_P \) depending on the weight coefficient \( K_g \) for hover without ground effect. The relation results in a curve that can be described by a polynomial:

\[
K_{P,H} = K_{P,H}(K_g=0) + K_{P,H}(K_g)
= A_0 + A_1 K_g + A_2 K_g^2 + ...
\]

Figure 7 demonstrates the power factor \( X_P \) as a function of the non-dimensional velocities \( \bar{U}_{Kg} \) and \( \bar{w}_{Kg} \) for one ground effect height. In dependence on \( H_g/R \), there result different levels in the diagram. The power coefficient for any flight state is:

\[
K_p = K_{P,H}(K_g=0) + X_P(\bar{U}_{Kg}, \bar{w}_{Kg}, H_g/R) \cdot K_{P,H}(K_g)
\]

Figure 7  Power Factor at Forward Flight with Climb/Descent

Different possibilities exist for the representation of the power factor \( X_P \). It has turned out that three-dimensional polynomials, for computation time reasons, are less suitable than the data field form. Moreover, the data fields cannot be approximated by polynomials in all velocity regions with sufficient accuracy. For the data field, the choice of data point distance and interpolation procedure is of importance. With a linear interpolation and an accordingly small data point distance, there can be attained, compared with a spline-interpolation, a distinctive advantage in computation time with the same accuracy. Computation time for one simulation is only a few percent of the simulated real time. Only with that, the numerical optimization of a complete takeoff procedure is made possible because in every optimization step, one takeoff simulation is carried out.

The data fields represented here have been attained from stationary performance calculations with the quasi-stationary two-dimensional model (5).

The influence of ground effect is described in this model by a modified source model (11), (12). The data field can be just as well directly established from flight tests. It is obvious that the flight tests have to cover the whole data field. Up to now, this possibility has not been realized. The comparison of power required after calculations and flight tests is included in (5),(10). There has been shown a good accordance for hover, forward flight up to the minimum power
(this is the velocity range relevant for takeoff) and for climb. For descent, the calculation of power required shows still inaccuracies which are caused by the empirical curves employed for calculation of the induced velocity at the rotor disc.

During takeoff, there occur translational accelerated flight states. Because the data fields contain only stationary states, the accelerated states have to be transformed into an equivalent stationary flight state e.g. a horizontally accelerated forward flight becomes an equivalent stationary climb with an equivalent gross weight. Both flight states are similar concerning inflow angle, control input and power required.

During simulation, from the three variables (horizontal acceleration, vertical acceleration and power required) two variables are always given for every simulation step whereas the third variable is determined from the data field. In the most simple case, the accelerations are given and power required has to be determined. The translational velocities and the flight path result from integration which is in this case performed analytically. Thus, for a given flight path, the power required is calculated. On the other hand, the computation of the flight path is possible when power required is given as a function of time. In this case additionally one of the translational accelerations has to be given. The other acceleration is iteratively determined by means of the data field at any timestep. The acceleration is then numerically integrated to velocity and distance. During takeoff simulation, different cases are applicable to the takeoff segments.

Even though data field calculation has theoretically not necessarily to result in definite solutions, this problem does in praxis not occur because the flight state is only slightly varied from one time step to the next (at=0.1s). In the direct vicinity of a flight state, a definite solution is attained from the data field. It has to be assured that the data field limits are not exceeded. In principle, only an interpolation within the data field is permitted, extrapolation is not allowed.

Figure 8 presents the result of a rejected takeoff simulation. The power required and the height are shown as a function of takeoff distance.

Takeoff with excess power starts with hover at a height of H_{h_0}=1m. In the beginning, power is increased to takeoff power by a maximum gradient of 80kw/s. Until CDP, a constant takeoff power is assumed. The horizontal acceleration is determined iteratively within the acceleration segment, the vertical acceleration within the climb segment, respectively.

In the event of engine failure in CDP, engine power decreases from the takeoff power of two engines to the emergency power of one engine. For this decrease an e-function is assumed. The transition into descent as well as stationary descent require an amount of power which is sufficiently below emergency power. Therefore the power required for a predetermined flight path can be calculated. The flight path is predetermined in a way that in the following flare segment just the emergency power is attained.

$$m = 2000 \text{ kg}$$
$$H = 0 \text{ m}, H_{\text{NA}}$$

![Diagram](image)

Figure 8 Rejected Takeoff from Data Field Simulation

The second flight path in Figure 8 shows takeoff without excess power. Takeoff power equals power required for hover in ground effect at H_{h_0}=1m and is kept constant until CDP. Within the acceleration and climb segment, vertical acceleration is determined for a given horizontal acceleration. In the beginning, takeoff involves height loss. In contrast to takeoff with excess power, CDP is attained about 140m later. The segment for engine failure corresponds to the failure segment of takeoff with power excess. This segment is primarily influenced by emergency power of one engine, i.e. by the P.D.P., which is equal in both cases. The complete rejected takeoff distance for takeoff without excess power is considerably longer because of the longer acceleration segment. The P.E.I. has significant influence on the takeoff distance up to the CDP.

IV. The Optimization

For the optimization problem numerical procedures after (6) are applied which are available as FORTRAN-subroutines. The program EXTREM is used for the determination of a local extreme value (optimum) of a multi-variable function without knowledge of its derivations. The optimal parameters, leading to an optimal function value (minimum or maximum), are calculated by means of systematical variation of the parameters and of the search direction. Limits of all sorts can be taken into account. Figure 9 shows the proceeding of EXTREM exemplary for a function F which is only dependent on two parameters, U_1 and U_2.

Basing on the initial values for U_1 and U_2, the function value F is calculated in points 1 (estimated value), 2 and 3 (each being a search step). A parabolic extrapolation is providing for point 4. This point is the optimum of a parabola.
through the points 1 to 3. By means of a Gramm-
Schmidt-Orthogonalization, the first secondary
search direction and the extreme value 7 are
determined. The second main search direction
always results of last and penultimate secondary
search direction. Accordingly, the third main
search direction would result from points 7 and
13. For a problem with two parameters, the
optimization strategy is still clear and represen-
table. With an increasing number of optimiza-
tion parameters, computation requirements and the
difficulty to interpret the results is increas-
ing. Basically, the number of combinations should
only be as high as the optimization problem
requires. A number too low may reduce the pos-
sible solutions of a problem too much. A suitable
compromise has to be found.

![Figure 9 Principle Optimization Method of EXTREM (6)](image)

Normally, complex functions have several local
extreme values. Since the optimization algorithm
EXTREM is mainly suitable for the search of local
extreme values, there is the possibility of not
finding the global optimum. For the optimization
with EXTREM, the search for the global optimum
can be improved by predetermining different
combinations of initial estimated values. This
process is taken on by the program GLOBEX (6) which
determines estimated values for optimization
parameters by means of normally distributed
random numbers. Figure 10 shows the simplified
optimization structure.

The precondition for the optimization start with
EXTREM is the knowledge of estimated values for the
optimization parameters which are not
violating the given limits. The search for
permuted initial values can be taken on by GLOBEX as well before three overriding optimization
segments are initiated. Each optimization step is subdivided into several optimization
degrees which in turn consist of several optimi-
ization steps. Each optimization step comprises
the calculation of the function value i.e. a
complete takeoff simulation by means of the data
field model. Rejection of the optimization in the
individual steps, degrees and segments is possi-
ble on the basis of different criteria e.g. a
minimum change of the optimal function value.
Further details are included in (6).

According to the optimization criteria, the
required takeoff distance, time needed for
takeoff or fuel consumption are handed over
to EXTREM as function value. Primarily, the
takeoff optimal in distance, with regard to
safety requirements, is investigated here. This

is the more interesting case in praxis.

![Figure 10 Integration of the Data Field Simulation into the Optimization Structure](image)

It has already been mentioned that the choice
of the optimizable parameters has a decisive
influence on the optimization expenditure and
on the interpretability of the results. If a
result is not physically explainable, it has
only numerical significance. Every result should
be examined by plausibility reflections. This
task cannot be taken on by any optimization
algorithm. For the determination of the optimum
takeoff procedure, there is a choice of accelera-
tions, velocities, distances, power and time as
parameters. By means of a suitable subdivision of
the Cat.A takeoff into individual segments which
take the physical characteristics of the Cat.A
takeoff into account, the optimization variables
can be limited to a maximum of eleven, respecti-
vely nine parameters. The parameters include
characteristical accelerations, velocities and
heights. Figure 11 shows for a sample rejected
takeoff the individual segments and optimization
variables.

- **HT.0.** - hover height at beginning of
takeoff
- **Ukg,A** - horizontal acceleration (for
takeoff without excess power)
in acceleration segment
- **Ukg,D** - horizontal deceleration/accelera-
tion after engine failure
- **Ukg,opt** - horizontal velocity in descent
- **Ukg,max** - vertical acceleration for transi-
tion from climb into descent
- **Ukg,FL** - horizontal deceleration in flare
- **Wkg,FL** - vertical acceleration for flare
- **Wkg,FH** - vertical velocity for descent/flare
- **HFL** - flare height
- **VCOP** - critical decision speed
- **HCOP** - critical decision height

For optimization of the takeoff distance, it
has turned out to be favourable if the vertical
acceleration in flare is increased and immedi-
ately decreased by means of a permitted gradient.
This results in a minimum required flare height for a given vertical velocity that has to be reduced. If this ideal flare maneuver is a priori assumed, the three vertical flare variables can be reduced to one. Chosen as variable is the vertical velocity \( w_{kg,FL} \) at the beginning of the flare segment.

![Graph showing optimization parameters for rejected takeoff with power excess](image)

Figure 11: Optimization Parameters for Rejected Takeoff with Power Excess

V. Results

For the practical application, primarily optimization of the Cat A takeoff distance in the event of engine failure is of importance. An optimization of the time required for takeoff or of fuel consumption, which can be of interest for the individual case, has been carried out as well for normal takeoff without engine failure but is not further discussed here. It should be mentioned that the results optimal for time and fuel at normal takeoff nearly correspond to the results optimal for distance.

The takeoff distance which has to be indicated for the Cat A. A takeoff is the longest emerging distance for engine failure, therefore it is either the rejected takeoff distance or the continued takeoff distance. The optimization aim is therefore to turn the largest emerging takeoff distance into a minimum. The takeoff distance depends on P.E.I., P.D.P. and the choice of the CDP.

Figure 12 shows the flight paths and power required for three different rejected takeoff maneuvers. A medium helicopter mass of \( m=2000 \text{kg} \) and an atmospheric condition \( H=0 \text{m}, \text{INA} \) have been assumed. Takeoff is performed with the maximal available takeoff power. For a predetermined CDP (according to the flight manual of the B0 105) with \( H_{cpp}=30\text{ft} \) and \( V_{cpp}=45\text{kts} \), flight path 1 is obtained. This flight path is already a result of optimization and presupposes an optimal flight maneuver considering maximum values for accelerations and descent velocity, just as a reduction in horizontal velocity to an optimal value for descent/flare. The reduction of horizontal velocity is carried out after engine failure.

![Diagram showing optimized rejected takeoff with power excess](image)

Figure 12: Optimized Rejected Takeoff with Power Excess

If the takeoff height, which is \( H=1\text{m} \) for 1, is available for optimization, the required takeoff distance, flight path 2, is reduced. Because of the smaller height difference towards the CDP, the CDP is reached earlier. The condition that the velocity \( V_{cpp} \) has to be reached prior to the height \( H_{cpp} \), is limiting the possible takeoff height. In this case, there does not exist a limitation by the H-V-diagram. If, instead of the takeoff height, the decision height \( H_{cpp} \) is available for optimization, a nearly similar result is obtained. The height difference between takeoff height and decision height is basically influencing the takeoff distance.

For flight path 3, the takeoff height has been determined with \( H=1\text{m} \). Now the decision height \( H_{cpp} \) and the decision speed \( V_{cpp} \) is made available for optimization. The optimization results in an optimal CDP with \( H_{cpp}=12\text{ft} \) and \( V_{cpp}=30\text{kts} \). Compared with the initial flight path, this results in a considerable reduction of the takeoff distance by more than 30%. The decision speed \( V_{cpp} \) is the essential influential factor for the reduction of the takeoff distance. In reducing \( V_{cpp} \), the acceleration segment is reduced and the dynamically attained maximal height is significantly decreased. This can be explained by the lower climb speed in CDP. The influence of the decision height, the takeoff height, respectively, is, on the other hand, significantly smaller.

In the following, the influence of P.E.I. and P.D.P. is discussed. While the parameter P.E.I. determines the takeoff distance up to CDP, the parameter P.D.P. has an essential influence on
the distance after CDP and on the CDP itself. Figure 13 shows three flight paths for the flight mass \( m = 2000 \text{kg} \) and \( H = 0 \text{m}, \text{INA} \). The takeoff is performed without power excess. Thus, the assumed takeoff power equals the power required for hover in ground effect for hover 1m.

Figure 13 OptimizedRejected Takeoff without Power Excess

Flight path 1 is valid for the predetermined, not optimized CDP (\( V_{CDP} = 45 \text{kts}, H_{CDP} = 30 \text{ft} \)) and a P.D.P. according to Figure 12. The maximum horizontal acceleration in the first takeoff segment results from the optimization. The possible acceleration is determined in a way, that the takeoff is performed without ground contact. The ground effect has a significant influence on the height loss. The optimization of the flight path, including optimization of the CDP, results in flight path 2. The optimal decision speed and the optimal decision height are close to \( V_{CDP} = 30 \text{kts} \) and \( H_{CDP} = 10 \text{ft} \). This optimal CDP equals the optimal CDP according to Figure 12. The CDP is mainly determined by the P.D.P., which is the same as in Figure 12. The optimal takeoff distance is by approximately 40% shorter than the takeoff distance for the predetermined CDP.

The influence of P.D.P. is clarified by flight path 3. Here, the emergency power and this way the P.D.P. has been changed that an optimal decision speed of \( V_{CDP} = 45 \text{kts} \) is a result of the optimization. The optimal decision height is below the decision height for flight path 1, so that, here as well, this results in a shorter takeoff distance.

Continued takeoff, which is not shown in the Figures 12 and 13, is possible in all cases. Figure 14 shows continued takeoff and rejected takeoff (from Figure 12, flight path 3 and Figure 13 flight path 2) for takeoff with and without power excess.

The optimized decision speed \( V_{CDP} \) is higher than the speed \( V_{QS} \), thus the distances for continued takeoff are below the distances for rejected takeoff. For rejected takeoff, it has been assumed that the helicopter is touching down horizontally. Thus, the optimized rejected takeoff distances are on the safe side. In praxis, a descent velocity during touch down is permitted. Because of that, the rejected takeoff distances could be further reduced.

Figure 14 Comparison of Optimized Rejected and Continued Takeoff

According to the expositions under section II, CDP and required takeoff distance depend mainly on P.E.I. and P.D.P.. By means of variation of takeoff weight, takeoff power and emergency power, there can be regarded similar P.E.I. and P.D.P.. This consideration is fictitious in so far as for a helicopter maximal takeoff power and emergency power are strictly predetermined and cannot be arbitrarily varied. Optimization calculations for helicopter masses from \( m = 1400 \text{kg} \) to \( 2400 \text{kg} \) are performed. Figure 15 shows the results which are directly comparable with Figure 12, flight path 3.

Figure 15 Optimized Rejected Takeoff Depending on Helicopter Mass, P.D.P. = const.
The attainable optimal takeoff distances correspond to 75% (a medium takeoff distance is assumed). This is a remarkable result if it is considered that very different effects influence the takeoff distance at such a complex maneuver like rejected takeoff. Apart from the takeoff distance, the optimal decision speed and the optimal decision height are in good accordance. They are in the region of $V_{CDP}=300$ kts and $H_{CDP}=12$ ft.

Optimization carried out here results in an optimal decision speed $V_{CDP} \approx V_{TOSS}$, so that in principle, a continued takeoff without height loss is possible. But this also means that with increasing P.D.P., the decision speed has to increase. Figure 16 shows the dependence of $V_{TOSS}$ from helicopter mass and from P.D.P. for constant atmospheric conditions. P.D.P. variation is attained by altering the helicopter mass as well as the emergency power. For constant emergency power, with increasing P.D.P., i.e., increasing helicopter mass, the safety speed $V_{TOSS}$ increases.

![Figure 16: Effect of Helicopter Mass and P.D.P. on Takeoff Safety Speed](image)

For a low P.D.P. up to about P.D.P.$=0.0050$m/s, the takeoff safety speed is nearly constant if constant P.D.P. is regarded. This explains the good accordance of flight paths and takeoff distances for constant P.D.P. but different helicopter mass, as shown in Figure 15, where P.D.P.$=0.0047$m/s has been predetermined. But it should be noted that for constant high P.D.P., the increase of $V_{TOSS}$ with decreasing helicopter mass is not neglectable. The optimization of takeoff with higher P.D.P., e.g., P.D.P.$=0.0060$m/s, would result in different optimal $V_{CDP}$ and thus in different optimal takeoff distances depending on the helicopter mass and the emergency power. The P.D.P. does not seem to be a sufficient parameter to characterize a takeoff in any case, an optimized takeoff, respectively. For the investigation carried out here, a constant atmospheric condition has been assumed, thus the variation of the emergency power is fictitious. But it should be kept in mind that power available depends mainly on air density and temperature.

The optimization results, Figure 12 and 13, show a significant reduction of the takeoff distance for the case that $V_{TOSS}$ is considerably smaller than the predetermined $V_{CDP}$ from the flight manual. The optimized $V_{CDP}$ is nearly $V_{TOSS}$. This result is physically obvious and could be expected. The question is now whether a similar reduction of the takeoff distance can be obtained by optimization for the case as $V_{TOSS}$ being higher than the predetermined non-optimal $V_{CDP}$.

Figure 17 shows rejected and continued takeoff for a helicopter mass of $m=2350$ kg as a result of the optimization. Flight paths 1 are valid for a given CDP ($V_{CDP}=45$ kts, $H_{CDP}=30$ ft) and an optimized maneuver, i.e., a maximum possible acceleration at the beginning of takeoff, an optimized transition into rejected takeoff, continued takeoff, respectively, and an optimal flare segment for rejected takeoff. During the continued takeoff the allowable minimum height is reached. The critical takeoff distance is the continued takeoff distance which is considerably longer than the distance for rejected takeoff.

![Figure 17: Optimized Rejected and Continued Takeoff for High P.D.P.](image)

Flight paths 2 are valid for a takeoff with an optimized CDP. The optimization results in a CDP with $V_{CDP}=55$ kts and $H_{CDP}=12$ ft. The optimal decision speed is a little higher than the takeoff safety speed (see also Figure 16), thus a continued takeoff is possible without height loss. In this case, the rejected takeoff distance is the longer one and has to be indicated as required takeoff distance. Though the rejected takeoff distance, flight path 2, is longer than the rejected takeoff distance, flight path 1, because of the longer acceleration segment, it is significantly smaller than the continued takeoff distance, flight path 1. The remarkable reduction of the required takeoff distance is approximately 40%. This result can be physically explained.

For the ability to continue takeoff after engine failure, it is necessary to accelerate up to a velocity $V_{TOSS}$. With flight path 1, this is carried out after engine failure. With flight path 2, this is carried out before engine failure. The acceleration segment is of course significantly smaller if the takeoff power of two engines is available instead of the emergency power of one engine. Additionally, a height loss is unavoidable for flight path 1, where the conversion of potential energy into kinetic
energy is required. When $V_{TOSS}$ is reached, the height loss has to recover with a minimum rate of climb. These different effects lead to a high continued takeoff distance for the case of $V_{CDP} < V_{TOSS}$.

VI. Concluding Remarks

The optimization results can be summarized as follows. For a critical decision point, CDP, predetermined for the whole flight envelope, optimization leads to optimal takeoff techniques. For the rejected and continued takeoff, these optimal maneuvers consist of a maximum horizontal acceleration with maximum available takeoff power up to the decision speed $V_{cpp}$. Regarding the takeoff with excess power, an optimal takeoff height is attained which may be limited by the $H$-$V$-diagram, the takeoff power, respectively. Regarding the takeoff without excess power, the optimization determines an optimal acceleration in a way that takeoff is performed without ground contact.

The relation of decision speed $V_{cpp}$ and takeoff safety speed $V_{TOSS}$ is significantly influencing the optimal maneuver after engine failure in CDP. The case $V_{cpp} < V_{TOSS}$ results in rejected takeoff distances which are considerably longer than the continued takeoff distances. In case of $V_{cpp} < V_{TOSS}$ the continued takeoff requires the longer takeoff distance. The difference between rejected and continued takeoff distance is small only for the conditions for which the CDP has been determined. Differing from these conditions the difference between rejected and continued takeoff distance increases and leads to unnecessarily high takeoff distances.

In a further step the parameters $V_{cpp}$ and $H_{cpp}$ have been made available for the optimization. The optimization results in a variable CDP that depends mainly on the power deficiency parameter, P.D.P. Whereas the P.D.P. has an effect on the takeoff segment after CDP, the power excess index, P.E.I., influences primarily the takeoff segment up to the CDP. It has been shown that the optimal $V_{cpp}$ is in principle something higher than $V_{TOSS}$. The optimization carried out here leads always to continued takeoff without height loss. Thereby it is attained a remarkable reduction of about 30-40% for the optimized takeoff distance with variable CDP compared with the optimized takeoff distance with predetermined CDP. This is valid for low as well as for high helicopter mass. Because $V_{TOSS}$, and thus the optimal $V_{cpp}$, depend on P.D.P., the P.D.P. has indirectly an effect on the length of the acceleration segment. Regarding the continued takeoff distance, an acceleration up to $V_{TOSS}$ with all engines operating is significantly advantageous compared with the acceleration up to $V_{TOSS}$ with one engine inoperative.

The optimal decision height $H_{cpp}$ is in all cases less than the predetermined $H_{cpp}$. By means of $H_{cpp}$ variation, small differences between rejected and continued takeoff distance can be balanced. If a continued takeoff without height loss is assured, the decision height $H_{cpp}$ is of less importance.

The investigation will be continued to find general, easily performable, optimal takeoff techniques which will also be acceptable for the praxis. Until now, only takeoff without engine failure has been evaluated by flight tests. It would be of interest to show that a similar reduction of the takeoff distance can be attained, using the optimal takeoff procedure with variable CDP, as discussed here.

Further investigations will be carried out regarding the takeoff without power excess. Especially in this case the horizontal acceleration has a significant influence on the flight path. Acceleration too high results in ground contact during takeoff, acceleration too low results in higher takeoff distance. Whether a takeoff without ground contact is performable or not depends from the ground effect. Here, the ground vortex has an important effect on power required.

VII. References

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