EFFECTIVENESS OF COMBINATION OF APEX AND LEADING-EDGE VORTEX FLAP ON A
7° DEGREE DELTA-WING WITH OR WITHOUT TRAILING-EDGE FLAP

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Abstract

Experimental methods including the aerodynamic pressure and force measurements, the seven-hole probe to survey the velocity distribution in the cross flow plane as well as the new technique using the hydrogen bubble and LASEF sheet for flow visualization were conducted to study the effect of apex flap and trailing-edge flap on the aerodynamic characteristics of a 7° degree swept delta wing equipped with leading-edge vortex flap.

As usual, the leading-edge flap acts well in reducing the drag, but eliminating the lift at lower angle of attack. However, the nose-up deflected apex flap along with the tail-down deflected trailing-edge flap can increase the lift prominently, accompanying with a slight decrease of lift-drag ratio at higher angle attack and more at lower angle of attack. To deflect only the apex flap nose-up will raise the lift-drag ratio for entire region of angle of attack tested, while a mild reduction of lift appears. As we found, fitting the apex flap and the trailing-edge flap on a highly swept delta-wing equipped with leading-edge vortex flap will have advantages in increasing the lift and the lift-drag ratio.

A discrete vortex method based on the slender body theory has been applied to compute the roll-up of vortices shedding from the leading-edge, and the pressure distribution on the flap and wing. The numerical results simulate well the roll-up of vortex-layer as exemplified by a comparison of the figures with that observed in the flow visualization. The surface pressure distribution in the same cross flow planes have a similar pattern both for the computational and experimental results.

1. Introduction

The modern advanced supersonic fighters appear to have thin, highly swept, slender wings for the sake of reducing the wave drag. However, on these wings the flow separates from the leading edge at moderate angle of attack in subsonic and transonic range. The separated flow rolls up into vortices causing a drag increase and a decrease in lift-drag ratio owing to the loss of leading edge suction.

In order to minimize the excessive drag rise at higher angle of attack, the concept of employing vortex flap was suggested in late seventies(1,2). That is, by tilting the leading-edge vortex flap downwards, the pressure force acting on the flap will produce a thrustwise component effective in reducing the drag and raising the lift-drag ratio accompanying with a deficiency of slight lift reduction. For the slender wing, actually there are drawbacks such as too large drag at large angle of attack and too small lift at small angle of attack. The use of vortex flap at the leading edge can reduce the drag with reduction of lift. In order to get an improvement in the performance of take-off, landing and maneuverability, another measure must be sought. As to increasing the lift, the strake which has been utilized extensively on wings of high speed fighters seems inadequate here for the highly swept slender wing, because of its degradation of lift production. In late years, the use of apex flap to produce more lift has been proposed(3). The apex flap so called is a foremost part of the delta wing which can be deflected up and down around a spanwise axis. The addition of apex and trailing edge flaps on the delta wing has been proved effective in increasing lift(3) at the cost of drag penalty. On the other hand, as explained above, the leading edge vortex flap has the ability of reducing the drag but producing less lift. As it is, the disadvantages of leading edge vortex flap happen to be the advantages of a combination of apex flap and trailing edge flap, while the advantages of the combination of apex flap and trailing edge flap will be the disadvantages of leading edge vortex flap. It seems that to attach both the apex and leading edge flaps on a slender wing equipped with trailing edge flap, which are complementary to one another, will yield better results as we expected. For the purpose of verifying the effectiveness of the combination, we chose 7° degree swept delta planform as a basic wing furnished with detachable constant chord leading edge vortex flap which is named model A and two types apex flap to be fitted on as sketched in Fig.1. The latter two models are named model B and model C respectively. The results of low speed experiments and computation have primarily born out the validity of our concept in improving the aerodynamic characteristics of highly swept wing.

2. Notations

\[ x, y, z \]  wing-fixed coordinates with origin at wing apex according to Fig.2

\[ s \]  local delta wing half span

\[ c \]  center-line chord of the wing

\[ A \]  wing aspect ratio (\( A = s^2/\pi \))

\[ U_\infty \]  free stream velocity

\[ p_0 \]  total pressure

\[ p \]  static pressure

\[ q_\infty \]  free stream dynamic pressure (\( q = U_\infty^2/2 \))

\[ p_0 \]  total pressure coefficient

\[ c_p \]  static pressure coefficient

\[ (c_\rho = p/p_0) \]

\[ c_\rho \]  dynamic pressure coefficient

\[ (c_p = p/p_\infty) \]

\[ c_L \]  lift coefficient (\( c_L = L/2p_\infty s \))

\[ c_D \]  drag coefficient (\( c_D = D/2p_\infty s \))

\[ K \]  lift-drag ratio (\( K = c_L/c_D \))

\[ p_\infty \]  free stream static pressure

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pitching moment coefficient \( c_m \)

velocity potential of perturbation

local velocity

velocity of the vorticity shedding from the leading edge

component of local velocity in the wing-fixed coordinates

angle of attack

strength of vortex segment

deflection angle of leading-edge vortex flap

deflection angle of apex flap

deflection angle of trailing-edge flap

c

dimensional coordinates

2. Models and experimental set-up

3.1 Models

The model of basic delta wing used for balance measurements is made of 4mm thick duralumin plate and has a semi-span length of 165mm and total area of 9.95cm². The leading-edge vortex flaps and trailing-edge flaps were cut from 0.05mm thick duralumin plate and hinged to the basic wing. The edges are all sharpened to enhance flow separation. The apex flap of model B was obtained by cutting the foremost part of the basic delta and incorporating a transverse hinge at x/c=6.6% with its leading edge staggered from the leading-edge of vortex flaps. The leading edge of the apex flap of model C is in alignment with the leading edge of vortex flap thus keeping the whole planform a delta form, and its hinge line located at x/c=26.9% from the apex. Details of the models are given in Fig.1.

3.2 Testing facilities

Experiments were carried out in turn, in low speed wind tunnels and water channel at BUAA, Beijing, and wind tunnel of Aerodynamic Development Center of China in Sichuan Province, China. The facilities are as follows:

D-4 wind tunnel Having a test section of closed circular type with a diameter of 1.5m and a useful length of 2m. The models were mounted on the strut of a six-components mechanical balance having a variable pitch angle system with a range of approximately -4° to 30°. The maximum wind speed is 35 m/sec.

ADCC wind tunnel A closed test section is 1.4m high and 1.4m wide with a useful length of 3m. The models were mounted on a strain gage balance sting which can be rotated to vary the pitch angle from -4° to 30°. The wind speed was kept at 30m/sec for testing.

D-I wind tunnel A closed circuit wind tunnel with an elliptical test section having a span of 1.02m and jet height of 0.75m. The maximum speed is 25m/sec.

BUAA water channel It is of horizontal type having a 40cm×40cm test section. The flow speed ranges from 10cm/sec to 20cm/sec.

3.3 Description of the tests

Flow field visualization New techniques by using hydrogen bubble and LASER sheet were conducted to visualize the vortex patterns. A fine platinum wire is placed in front of the leading edge to serve as the cathode of a D.C. current. The bubbles were illuminated by a 0.5mm×1.0mm thick narrow LASER sheet directed normally to the x-axis (Fig.2) within the BUAA water channel.

Fig.2. Sketch of the test-device

Flow field measurements They were conducted by using a conical seven-hole probe with a diameter of 2.5mm (Fig.3) to measure the distribution of total pressure, static pressure as well as the flow direction in the cross flow plane. The probe is available for measuring highly deviated flow up to an angle of attack about 65°. Tests were carried out
Fig.4. Conical seven-hole probe and the coordinates of measurement in D-1 wind tunnel with Re=0.9X10^6. The flow field around model C at $\alpha=15^\circ$ and $\delta_f=10^\circ,0.1,0.1^\circ$ were successively measured in planes of $x/c=5,1,0,1.09$.

Pressure measurements
The pressure measurements were carried out in the ADCC wind tunnel. A specially designed half-model wing was fabricated for measuring the pressure distribution on the upper surface of the wing. The static pressure orifices equipped were perpendicular to the free stream, each orifice was connected to a scanning valve with transducer to record and transmit the pressure signal to a computer for data processing.

Balance measurements
A strut-type mechanical balance was adopted together with a computer used for data recording and processing. For all models, three-component measurements were carried out in the $45^\circ$ angle of attack range in the D-1 wind tunnel with Re=1.1X10^6.

4. Discussion of results
4.1 Constant chord vortex flaps
The pattern of the flow passing around the model equipped with constant chord vortex flaps can be seen clearly from the wind tunnel. For example, we can see the motion of the flow separation from the leading edge and the roll-up of the vortex layer for $\alpha=15^\circ$ at $x/c=0.5$ in Fig.4. There are also total pressure contours and velocity distribution graphs where two minimum total pressure points corresponding to the main and the secondary vortex centers can be found. It is obvious, if the vortex attachment can be prevented from moving onto the front delta wing, the negative peak pressure will remain over the vortex flaps, producing strong drag reduction effect. Actually, it is difficult to maintain such an ideal flow pattern along the $x$-axis, as the flow moves from apex towards the trailing edge. In the past, a lot of measures have been plotted by means of modifying the shape of the vortex flap in an attempt to strengthen the drag reduction effect(9). On the other hand, one can achieve this object by utilizing the advantage of vortex interaction such as we intend to study by using apex flap.

The balance measurements have been made that on the whole the vortex flap functions better at $\delta_f=30^\circ$ than flaps at other $\delta_f$ tested. Therefore for all tests reported here, we kept $\delta_f=30^\circ$. The curves indicating the aerodynamic characteristics of model A including $\alpha$, $\omega$, $C_L$, $C_D$, and $C_L/C_D$ relations are given in Fig.5. Observing these curves with that of the basic delta wing, it is obvious that the drag decreases and the lift-drag ratio increases appreciably with a weak reduction of lift. In addition, the pitching moment curve is nearly linear and the stalling angle is postponed due to the delay of vortex breakdown.

4.2 Aerodynamic effects of the apex flap
We have two kinds of apex flap equipped on the model B and model C, the latter has been studied in more detail. For the model C, the flow visualization experiments were taken in turn for $\alpha=7^\circ,15^\circ$ and the deflection of apex $\delta_f=10^\circ,0.1,0.1^\circ$ in the water channel. Photographs are given in Fig.6 showing how the vortex originating from the apex flap are growing as it moves into the region above the basic delta wing and the vortex shedding from the leading edge of vortex flap. From the $c_L/c_D$ curve it can be recognized that $\alpha=7^\circ$ and $15^\circ$ are nearly corresponding to the state of $c_L/c_D$ maximum and $c_L/c_D=0.5$ respectively.

4. Flow pattern
From the pictures in the Fig.6 a) and b), we can see that at $\delta_f=7^\circ$, the flow is basically attached to the upper surface of the apex flap no matter how large the deflection $\delta_f$ may be, although the flow pattern is different for different $\delta_f$. When the apex flap is tilted nose-down to $\delta_f=-10^\circ$, no vortex originating from the edge of apex flap can be discerned, because the apex has a locally negative $\alpha$. Rotating the apex flap upwards to $\delta_f=0^\circ$, now the flap having a positive local $\alpha$ will render the flow to develop into vortex after separating from the leading edge. This developed vortex extending down-stream till close to the trailing edge can be discerned in the pictures for all $\alpha$ tested. Increasing $\delta_f$ to $\delta_f=10^\circ$, we can obviously see the scale of the vortex is enlarged because of the large local angle of attack now reached. In the Fig.6 c) and d), photographs show are the flow patterns visualized at $\alpha=15^\circ$. Comparing these pictures with that of $\alpha=7^\circ$, we can find the strong vortices separated from the leading edge flap and apex flap for all $\delta_f$ tested gradually merge into each other as they move downstream. For $\alpha=10^\circ$ case, the vortex of apex flap seems to spread out and the velocity induced in the cross-flow plane is so weak that it is not easily be discerned, however we still can find their vague traces there. Maybe the breakdown has happened, causing the drag to increase and the lift-drag ratio to decrease as were confirmed by the experimental results of balance measurements to be explained later.

Component of velocity vector in the cross-flow plane
A seven-hole probe has been used after the method developed by K.N.Everett and A.A. Gerner(4) to measure the component of velocity in the cross-flow plane. In Fig.7, the distributions of velocity component measured in planes at $x/c=0.5$ are presented. They are corresponding to apex flap de-

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Fig.4. Vortex formation of model A wing in the plane $x/c=0.5$ at $\alpha=10^\circ$

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**Fig. 5** Aerodynamic characteristics of delta wing and model A

**Fig. 6** Flow field visualization showing vortices shedding from the leading edge of model C at $\delta_v = 30^\circ$
flection $\delta_p = -10^\circ, 0^\circ, 10^\circ$ at $\alpha = 15^\circ$. In each of these graphs, two centers of vortex can be found clearly. The inner located over the main wing originated from the apex flap and the outer one at the position over the leading edge vortex flap is separated from the leading edge. After comparing these graphs, we can learn that to each center the nondimensional normal distance between the center and the wing surface, i.e. $y/s$, and the strength (the induced velocity components) of the vortex increases with the increase of angle $\delta_p$, while its spanwise distance remains nearly unchanged. Here, for the inner center, the value of $y/s$ increases from 0.26 through 0.37 to 0.4 as $\delta_p$ increases from $-10^\circ$ through $0^\circ$ to $10^\circ$ respectively, while $y/s = 0.46$ at 0.53. Similarly, the outer one situated over the flap remains nearly at $y/s = 0.35 \times 0.20$ while its normal distance changes from 0.1 through 0.12 to 0.13.

The graphs shown in Fig. 8 are the results measured in cross flow plane at $x/c = 0.09$, i.e. the p-lane at 98% of root chord behind the trailing edge. We can still find these two centers of vortex and their further movements away from the wing plane. The inner one raise from $y/c = 0.08$, through 0.1 to 0.14 as the apex flap deflection changes from $-10^\circ$ through $0^\circ$ to $10^\circ$ respectively, while $y/s = 0.4$. Similarly, the outer center of vortex separated from the leading edge remains spanwise nearly at $y/s = 0.97$ as its normal distance $y/c$ changes from 0.08 through 0.13 to 0.16, while the $\delta_p$ increases from $-10^\circ$ through $0^\circ$ to $10^\circ$ respectively. Comparing the graphs of plane $x/c = 0.5$ and $x/c = 0.09$, we can find the scale of the vortex increases while the strength (the induced velocity component) decreases, and may be in some cases like $\delta_p = -15^\circ, \delta_p = 10^\circ$, the vortex has already broken down as we have explained according to the flow visualization pictures. As a whole, the outer vortex remains basically in the region above leading edge vortex flap because of the existence of inner vortex which effectively produces a beneficial interaction in retarding the inward wandering of outer vortex. The same phenomena can be observed in the flow visualization and the experimental results of pressure measurements.

Results of pressure measurements. The pressure distributions on the wing surface at $x/c = 0.5$ plane for $\delta_p = 10^\circ, 15^\circ, 20^\circ$ and 30$^\circ$ and at plane $x/c = 0.5$ for $\delta_p = 10^\circ$ and 25$^\circ$ are shown in Fig. 9 and Fig. 10. From the above figures, we discover that as a whole, the peak of suction appearing on the surface of leading edge vortex flap is the highest for $\delta_p = -10^\circ$, it decreases gradually as $\delta_p$ increases through $0^\circ$ to $10^\circ$ where the suction peak reduces to a minimum. These results reflect the flow patterns we have seen in the flow visualization tests and agree with the results of flow field measurements. As we have explained above, although the outer vortex center nearly comes to a standstill spanwise, it will reach its higher normal position with respect to the wing surface, this will decrease the induced velocity near the wing surface and in turn the pressure as the deflection of the apex flap increases in the positive direction. As the pressure curve for plane $x/c = 0.5$ shown in Fig. 9, the spanwise distance $y/s$ remains nearly unchanged at 1.15 that agrees with the value obtained from velocity measurement in cross flow plane. As to the inner vortex, the situation is quite opposite. Here, the suction is high at $\delta_p = 10^\circ$, less at 0$^\circ$ and became the least at $-10^\circ$, maybe it is due to the strength of the inner vortex turns stronger as $\delta_p$ increases, although the normal distance remains accompanying with. As a whole, the more lifting force contributed by the basic delta

Fig. 7 Velocity component vector in the plane $x/c = 0.5$ at $\alpha = 15^\circ$

Fig. 8 Velocity component vector in the plane $x/c = 0.09$, behind the trailing edge, at $\alpha = 15^\circ$. 

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Fig. 9 Pressure distribution curves for plane $x/c=0.5$ a) $\alpha=10^\circ$ b) $\alpha=15^\circ$ c) $\alpha=20^\circ$
d) $\alpha=30^\circ$, $x-$ $\delta_p=10^\circ$, $\epsilon-$ $\delta_p=0^\circ$, $\omega-$ $\delta_p=10^\circ$

Fig. 10 Pressure distribution curves for wing for positive $\delta_p$ may be compensated by the lifting capacity lost at the leading edge vortex flap, and vice versa when $\delta_p$ becomes negative. This may be the reason that the lift is nearly unchanged no matter the $\delta_p$ deflects in either direction as will be seen from the results of balance measurements later. Looking over the pressure distribution curves in Fig. 9 for the plane $x/c=0.5$ as example, it appears that the position of suction peak on the vortex flap moves inwards to the main wing as $\alpha$ increases from $10^\circ$ to $30^\circ$. On the other hand, the suction peak induced by the inner vortex moves in opposite direction from inner towards outside. Gradually, the two peaks draw in closer and closer and finally merge together at $\alpha=25^\circ-30^\circ$. In the mean time, these three pressure distribution curves corresponding to $\delta_p=10^\circ, 0^\circ, 10^\circ$ tend to coincide as $\delta_p$ increases over $25^\circ$.

Comparing again the pressure distribution over vortex flap at plane $x/c=0.5$, the suction peaks appear on all three pressure curves corresponding to $\delta_p=10^\circ, 0^\circ, 10^\circ$ at lower angle of attack as $\alpha=10^\circ$. When $\alpha$ increases, the pressure distribution curve of $\delta_p=10^\circ$ becomes flatter and the peak disappears. Then, when $\alpha=20^\circ$, the flatness of $\delta_p=0^\circ$ curve appears too. As $\alpha$ reaches to $25^\circ$, pressure peaks could hardly be found in all three pressure curves. That means the outer vortex has gradually become unstable and the vortex breakdown phenomena may be realized.

The pressure distribution curves of plane $x/c=0.9$ for $\alpha=10^\circ$ and $25^\circ$ are given in Fig. 10, where we can still find the suction peak on the flap region is higher at $\delta_p=10^\circ$ than the other two curves and the situation is opposite for the suction peak on the main wing region, similar to that appearing in plane $x/c=0.5$. These three curves are much closer to each other with lower value of $\alpha$ in this cross flow plane as the trailing edge approached.

4.3 Computation of discrete vortex method based on slender body theory

The roll-up of vortices shedding from the leading edge and the pressure distribution on the flap and wing have been calculated for plane $x/c=0.5$ at $\alpha=15^\circ$ as shown in Fig. 11, 12, based on the method of distributing discrete vortices in the cross flow plane. The equations developed on the basis of slender body theory, the fundamentals of this method is described in the appendix.

From Fig. 11 we can see the computational shapes of the separated vortices and their similarity to that got from visualization as indicated in Fig. 6 where the inner and outer vortices can be seen clearly. A family of pressure coefficient increment $\Delta c_p$ curves for different $\delta_p$ ranging from $-\delta_p$ to $\delta_p$ has been calculated and drawn in Fig. 12, where the change of inner and outer suction peak as $\delta_p$ varies from $-10^\circ$ to $10^\circ$ presents the same tendency, that is the outer $\Delta c_p$ peak becomes larger than that of inner peak for $\delta_p=0$ and vice versa for $\delta_p < 0$, as we have discussed before about the results of pressure measurements. The $\Delta c_p$ comprises the contribution presented by the vorticity $y$ in x direction of the cross flow plane considered, which corresponds to major term $(-y\delta_p)$ in c expression, accounting mainly the contribution of two separated vortices over the half-span to the pressure.

Fig. 11 Roll-up separated vortices over model C computed for cross flow plane $x=0.5$ at $\alpha=15^\circ$

Fig. 12 Pressure coefficient increment of model C computed for cross flow plane $x=0.5$ at $\alpha=15^\circ$ with $\delta_p=10^\circ, 0^\circ, 10^\circ$
vortex flap has two effects: one is the effect of a weak decrease of lift at $\alpha=0^\circ$ to $25^\circ$ range, because of smaller local angle of attack at flap due to the deflection of $\delta_v$, and the forward inclination of the lift vector there, the other is to increase the $c_{l_{\text{max}}}$ and the corresponding critical angle of attack because of capability to delay the vortex breakdown just like we have seen in the flow visualization that the breakdown point has already reached the trailing edge at $\alpha=25^\circ$.

From the $K-c_l$ curves, we can see that the lift-drag ratio $K$ has been improved quite a lot after attaching the vortex flap on the basic delta wing with $\delta_v=30^\circ$ due to the powerful effect of vortex flap in decreasing the drag, although the $c_{l_{\text{max}}}$ slightly decreases. For example, we have 15% increase of $K$ as well as $20-25\%$ decrease of induced drag and about 20% increase of $K$ at $c_{l_{\text{max}}}=0.4$ for $\alpha=0^\circ$ range.

As to the drag coefficient, the $c_D$ value is nearly equal to that with full suction at $K<10^{-3}$ as the separated vortex being situated completely above the flap. After that, the vortex will move inward to the region of main delta wing and thus it renders the drag reduction to diminish gradually, yet it will keep the $c_D$ always smaller than that without the suction forces.

The $c_{l_{\text{max}}}$ curve of model A becomes more linear due to the existence of vortex flap and the delay of vortex breakdown.

Model C and its combination with trailing edge flap Model C, one of the model with apex flap (Fig.1), has been taken as a main case in studying the effect of apex flap to improve the aerodynamic characteristics. We still set the deflection of the leading edge flap $\delta_v=30^\circ$ and three deflections of apex flap, i.e. $\delta_{a}=10^\circ,0^\circ,10^\circ$ to carry out the balance measurements. The results gained are shown in Fig.13 with corresponding curves for the basic wing. In comparison with the basic wing, it reveals that the model C still possesses the property of lower lift at moderate and low $\alpha$ range (we have mentioned similar property of leading edge vortex flap). One important result is the deflection of apex flap gives no apparent influence on lift whether $\delta_{a}$ is positive or negative. This result is also true for model B according to the experiments and agrees with that data in reference (3). However, judging from the $c_{l_{\text{max}}}$ curves which are different for different deflection of apex flap, we can find that the pitching up moment is larger for model C with apex flap nose-up than that with nose-down. This means the lift exerting on apex flap has to be larger for apex flap nose up than for nose down. This deduction together with the inference that the total lift is nearly independent on the deflection of $\delta_v$ makes us now to have the conclusion that the lift contributed by afterpart of wing behind the apex flap should be larger for $\delta_{a}<0$ case than that of $\delta_{a}>0$. Looking over the graphs shown in Fig.9 to Fig.12, where the curves of pressure measurement and computation have been given, the loading on wing section is always larger for $\delta_{a}<0$ than $\delta_{a}>0$, and this further substantiates the above conclusion.

If we attach the trailing edge flap on model C like the form shown in Fig.1(d), the combination will produce considerable increment of lift with flap deflected downward (the aerodynamic characteristic curves are given in Fig.14). At the same
time the drag will increase which renders the $K_{\text{max}}$ decrease, but the percentage of decrease becomes smaller at the angle of attack increases, and in any case the $K$ value will exceed that of model A.

![Diagram](image)

Fig.14 Aerodynamic characteristics of model C with trailing-edge flap

On the whole, deflecting the apex flap, $K_{\text{max}}$ increase from about 7 in our experiments for the case of model A (delta wing plus leading edge flap) to 7.5 or 8 for model C (complete configuration with trailing edge flap detached), i.e., we have 7-10% increment of $K$, while the lift coefficient is only slightly reduced. As to the combination of apex flap and leading edge vortex flap and trailing edge flap on a 74 degree swept delta wing, it is found that downward deflection of trailing edge flap will cause the lift coefficient to increase while $K_{\text{max}}$ decreases. However for larger angle of attack (9 degrees for example) $K$ becomes larger than that of model A.

The experimental results of model B show they are similar to that of model C. The aerodynamic characteristic curves of model C are given in Fig.15, to which no further discussion will be made.

![Diagram](image)

Fig.15 Aerodynamic characteristics of model B

Deflection of apex flap on a wing with leading edge vortex flap will not cause appreciable change in lift coefficient, however the pressure distributions are altered.

As the apex flap deflects downward ($\delta_p < 0$), the pressure distribution on the leading edge vortex flap will yield a higher and higher suction peak, producing a large thrust component, so we have a better drag reduction effect and also with an increase in K. As the apex flap deflects upward ($\delta_p$ is from negative to positive), the pressure peak lowers with a decrease in K. This effect of leading edge vortex flap may be clearly seen from the force and pressure measurements, flow visualization tests. The computation based on our modified slender body theory also verifies the above statement.

A combination use of apex and trailing edge flap on a slender delta wing equipped with leading edge vortex flap will have the following effects: In the case of apex and trailing edge flaps are both deflected downward, lift will be appreciably increased and this will compensate the well-known defect of slender wing i.e., it has a smaller lift coefficient. But in so doing, the drag increases and the values of K at smaller angles of attack decrease; we also notice that with further increase in angle of attack, the decrease in K become less evident.

Based on what stated above, with a proper arrangement of the deflections of apex and trailing edge flaps we may arrive at desired lift increase, drag reduction and increase in lift-drag ratio in different flight regimes. This demonstrates the superiority of attaching apex and trailing flaps to a slender delta wing equipped with leading edge flaps.

6. References

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5. Conclusion

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Appendix: Fundamentals of the discrete vortex method based on slender body theory

The method consists of the use of distribution of slices of vortex (or point vortex) to replace the wing and separated vortices based on the slender body theory in order to simulate the roll-up of vortices separated from the leading and trailing edges of slender wings and to calculate the load distribution on the surface (5). The method has the advantage of avoiding the difficulties encountered by using conformal transformation in the calculation for complex configurations as the conventional methods usually made.

Based on the slender body theory for ideal incompressible flow, the potential perturbation function \( \phi' = \phi(y, z; x) \) should satisfy the Laplace equation in cross flow planes normal to the x-axis:

\[
\frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial^2 \phi'}{\partial z^2} = 0
\]  

(1)

and the boundary conditions,

at far field: \( \frac{\partial \phi'}{\partial y} = 0 \), \( \frac{\partial \phi'}{\partial z} = 0 \)  

(2)

on wing surface: \( \frac{D \phi'}{D t} = 0 \)  

(3)

where \( f(y, z; x) = 0 \) and \( x = \omega \cdot t \cos \theta \).

On vortex sheet: zero force condition equivalent to the condition of coincidence of stream and vortex lines.

at leading edge: Kutta condition, zero normal velocity, \( V_n \big|_{L, E} = 0 \)  

(4)

In each cross flow plane, the wing section are divided into small segments. On each wing segment we have linearly distributed vorticity to replace the boundary and keep the vorticity strength on every neighboring segment equal at the section point. The boundary condition is to be satisfied at middle point of each wing segment. For every time step \( t \), a new vortex piece of small stretch as will be born from the leading edge in line with the wing surface with \( \Delta s = \gamma \cdot \Delta t \), where \( \gamma \) is the mean shedding velocity with which the new born vortex leaves from the leading edge. This new vortex piece will be concentrated into a point vortex which flows down with the local velocity to the next cross flow plane, where the second new born vortex will appear. In this way the computation proceeds on successively from section to section.

With proper selection of the time step \( \Delta t \) and the number of vortex piece \( N_v \), including the new born leading edge vorticity piece, we can have \( N_v \) equations to satisfy \( N_v \) boundary conditions including eq. (3) and Kutta condition eq. (4). They are:

\[
[C] [\gamma] = -[w_0], \quad i = 1, 2, \ldots, N_v \quad j = 1, 2, \ldots, N_v
\]  

(5)

where \([C]\) is the matrix of influence coefficients, \( \gamma_j \) is the strength of the \( j \)th vortex piece, \( V_n^0 \) is the normal induced velocity at the \( i \)th control point that can be determined by the known \( \gamma_i \) at the former section. Thus the \( \gamma_i \) can be determined for the considered cross flow plane by solving the eq.(2). As the computation proceeds forwards, the roll-up of the separated vortices along x-axis and the pressure loads exerting on the wing can be determined.