Crack growth from a notch tip, under the influence of cyclic compression loading is investigated in CCT-specimens of a lower strength steel. Such cracks, caused by the residual tensile deformations at the notch at unloading, progressively decelerate until complete arrest occurs. Special consideration is taken to the build up of crack closure during crack advance. In particular, the influence of the very first unload cycle, on the elastic-plastic notch field, crack initiation, crack extension and crack arrest, is examined. Experimental measurements correlate well with numerically obtained estimates of the crack closure loads assuming plane stress conditions.

1. Introduction

Current practice in fatigue design and analysis of structural components involves several steps including stress analysis, materials testing and the use of cumulative damage rules or fracture mechanics concepts to estimate either the total life time to failure or the crack growth behaviour from an assumed initial flaw size up to critical crack length. Complex structures, e.g. a fighter aircraft, are generally subjected to a very large number of load cases and in the development of relevant load and stress spectra emphasis is normally placed on load combinations giving rise to large tensile stresses. It has, however, been recognised for some time that tensile stress is included in the resulting load spectrum never can be discarded as harmless. Examples of applications where tensile stresses may have to be considered include the lower surface of aircraft wings where it is known that compressive loads can reduce the beneficial retardation effects due to tensile overloads, e.g. [1]. In the case of weldments it is usual to use the entire stress range, irrespective of mean stress, due to the existence of residual stresses, for fatigue design and also in this case compressive stresses are found to be detrimental in the general case of spectrum loading [2]. It has also been found that fatigue cracks may grow faster at $R = -1$ than at $R = 0$, particularly in the near-threshold regime where the compressive load cycles may cause flattening of the crack surfaces and thereby reduce the closure contributions from rough crack surfaces and fretting oxidation [3, 4]. The same detrimental influence of compression cycles was found in another study showing that arrested cracks at the threshold point, $dK_{th}$, did re-initiate upon the application of compression overloads [5]. This was attributed primarily to a smaller contribution from roughness-induced closure, arising from compacting and cracking of surface asperities close behind the crack tip. Finally, it should be noticed that crack growth under fully compressive loading is frequently studied in the case of layered composite materials where delamination growth is more prone to local buckling, e.g. [6, 7].

However, in the case of metals, studies of crack growth under fully cyclic compressive loading are very scarce. This is in view of the implicit notion that fatigue cracks remain closed during compressive loading or even below a certain load known as the closure load. Nevertheless, fatigue cracks do initiate at notches under fully cyclic compression, due to residual tensile stresses upon unloading. These cracks will grow with a decelerating rate until they arrest. In this paper we continue our recent work on compression loading by examining the influence of the very first load cycle on the elastic plastic notch field, crack initiation and the subsequent growth characteristics.

2. Background

Observations of controlled crack initiation in compression date back to the 1960s when it was found [8] that fatigue cracks initiated in regions of high stress concentration in notched beryllium samples cycled under negative stresses. In fact, it was the only way to initiate fatigue precracks in this material in a controlled manner because cycling in tension resulted in catastrophic failure [8]. A fracture mechanics-based study of the growth of fatigue cracks under remote compression cycles was first reported by Hubbard [9] in center-notched specimens of 7075-T6 aluminum alloy. His results revealed that the fatigue crack grew at a progressively decreasing rate before arresting completely after a total growth of ~1-2 mm. Hubbard [9] presented simple concentrated and distributed residual stress models for the calculation of an effective crack-tip stress intensity factor with the assumption that the crack was growing under the influence of an unchanging residual tensile stress field. Although such stress intensity values roughly correlated with the effective $\Delta K$ inferred from cyclic tensile tests leading to identical growth rates, the choice of these equations and values of parameters were not given clear justification [9]. Moreover, Hubbard's 3.175-mm-thick specimens were not subjected to any (thermal) residual-stress-relieving treatment following solutionizing and crack length was monitored only on one side of the specimen. More recently Reid et al. [10] reported the initiation of fatigue cracks in 25.4-mm-thick compact (CT) specimens subjected to a single compressive overload. Their results revealed that fatigue cracks emanating from the
or notch tip grew several millimeters during subsequent constant amplitude compressive cycling before crack arrest occurred [10]. The maximum distance of growth roughly correlated with the calculated value of maximum plane-stress plastic-zone size corresponding to the compressive overload. As the state of stress changed through the thickness of the specimen, crack growth in the near-surface (plane-stress) sections of the specimen was faster than in the center-thickness (plane-strain) section [10]. Chu et al. [11] noted that the three stress ratio maximum plane-stress plastic-initiation under compression was higher than that for tension when the peak load of the compressive cycle was well below zero. In double-edge-notched specimens of ultra-high-strength steel, they observed a total crack growth of ~0.5 mm under remote compression [11].

The variation of total crack growth distance in far-field compression with the applied compressive load range was examined by Suresh [12] in compact and center-cracked geometries of a 2 1/4 Cr-1 Mo steel. In the CT specimens, increasing the compressive load range until a critical value resulted in an increase in crack growth distance. Beyond this critical value of load range, the total (saturation) crack length did not vary appreciably with far-field loads. Suresh [12] also presented experimental evidence to demonstrate that the phenomenon of crack initiation in compression can be used as a new procedure for performing accelerated fatigue crack growth threshold tests. While this technique led to the same (at R = 0.75) or more conservative (at R = 0.05) threshold stress intensity range, Kth, compared to conventional “load-shedding” procedures, it also minimized inherent plasticity effects and uncertainties arising from the artifacts of conventional test techniques [12]. Suresh’s work [12] also demonstrated that when a fatigue crack is propagated from a notch tip (until it arrests) under fully compressive far-field stresses, the extent of damage left to its tip after complete crack arrest is not sufficient to affect growth in tension-tension fatigue. More recently, Christman and Suresh [13] used the compression technique to examine the reasons for the differences between the growth rates of physically short (a = ~0.5 mm) and long (a = ~2.5 mm) flaws in center- and edge-notched specimens. They applied this technique to determine the threshold stress intensity level at which short fatigue crack growth in cyclic tension begins at low and high load ratios and to examine the critical crack growth distance over which closure develops [13]. It was found that closure develops after growth over a minimum distance of about 0.5 mm, which is in close agreement with numerical predictions based on an analysis technique similar to the one used in the present paper [14].

The first detailed study on the crack growth during fully compressive cyclic loading was only performed recently by Holm et al. [15]. These authors presented both numerical and experimental results on the initiation, growth and arrest of fatigue cracks under far-field cyclic compression in a fully bainitic 2 1/4 Cr-1 Mo steel and a commercial 7075-T7351 aluminium alloy. Emphasis was put on the evaluation of the stress field ahead of a fatigue crack propagated under far-field cyclic compression in both compact and center-cracked geometries. Also, for the first time, the extent of crack closure and total distance of crack growth under compression was investigated in detail for both plane stress and plane strain conditions. It was found that while the extent of the residual tensile stress field diminishes with an increase in crack length, there is a progressive increase in crack closure during crack advance. Holm et al. [15] also showed that plane stress conditions result in a larger residual tensile stress field and therefore leads to farther crack advance in compression than plane strain. As expected, an increase in the far-field nominal load range, at constant load ratio, caused an increase in the effective load range during which the crack-tip remained open. For small cyclic compressive loads, the crack length at which the plane strain closure loads computed from finite element analyses approach the peak loads of the fatigue cycle, correlated rather well with the measured saturation crack length values. It is interesting to note that the numerical and experimental results on crack closure reported in [15] are also comparable to those observed in another independent study of crack initiation under far-field compression in CCT specimens of BS4360 50B steel (yield strength = 380 MPa) [16].

The purpose of this article is to continue the work referred to above [15]. Specifically, the influence of the very first compressive load cycle on the elastic-plastic notch field, crack initiation, crack propagation and crack arrest, is examined. Both numerical and experimental procedures are used to evaluate such characteristics in center-cracked specimens of the same bainitic 2 1/4 Cr-1 Mo steel as used in our previous study [15].

3. Materials and Experimental Procedures

The material used in this investigation is a fully bainitic 2 1/4 Cr-1 Mo steel (ASTM A 542 Class 3) (yield strength, σ_y = 500 MPa). The nominal composition, in weight percent, is provided in Table 1. The room temperature mechanical properties and heat treatment procedure are described in detail in Refs. [17, 18]. Cyclic crack growth experiments were performed in center-cracked (CCT) specimens, machined in the T-L orientation, at a fully compressive mean stress (R = P_min/P_max = -1), test frequency 25-50 Hz in room temperature (~23°C) moist air (~39-40 % RH) environment. Besides from fatigue testing at a constant positive stress ratio, i.e. fully compressive loading, a series of overload tests was performed. Hereby, the very first compressive load cycle was applied as a single compressive overload with P_min chosen to either 2 or 3 times the magnitude of the minimum load level in the subsequent constant amplitude loading which was always performed with R = -10.

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**TABLE 1.** Nominal Composition (wt %) of A542-3 steel
The test specimens had the following dimensions: thickness, \( B = 2.54 \text{ mm} \), \( 2W = 50.8 \text{ mm} \), \( 2H = 60 \text{ mm} \), total notch length = 12.7 mm, notch-tip angle = 60° and notch-root radius < 0.05 mm. The specimens were loaded using friction grips. Crack length under cyclic compression loading was measured optically on both sides of the specimen surface (as well as on both sides of the notch). Furthermore, through-thickness crack length was examined on the fracture surfaces after the test. The average crack length values were used for the evaluation of fatigue crack growth.

Fatigue crack closure loads were obtained using compliance techniques (Fig. 1). Strain gages were mounted near the notch tip, above and below the crack plane on both side faces of the test specimens. The inflection point in the load displacement curve was taken as an indication of the crack closure load. Since the change in compliance due to crack closure occurs over a range of loads, the scatter in the measured closure loads is also noted in the results (see Fig. 1b).

**FIGURE 2.** Finite-element discretization of a quadrant of the centernotched specimen (a) and details near the notch-tip (b)

**FIGURE 1.** Schematic showing the location of strain gages (denoted by rectangles) on the compression specimens and the procedure for obtaining closure loads

4. Numerical Procedures

Calculations were performed both for plane stress and plane strain situations using the finite element mesh shown in Fig. 2. The constitutive equation for the A542-3 steel, used in the numerical model is shown in Fig. 3.

**FIGURE 3.** Stress-strain plots showing the constitutive behaviour of the materials used in the numerical procedures
The numerical procedures, described in detail elsewhere [19], consist of elastic-plastic finite element calculations where crack propagation is modeled by releasing the crack-tip node, changing the boundary conditions and solving the contact problem at the crack faces by noting that the (mesh) elements do not interpenetrate. This enables numerical simulation of crack growth and calculation of crack closure load $P_{cl}$ for different combinations of stress ratio, stress state and material properties.

The computer program is a modified version of the elastic-plastic finite element program for static loading (LUCAS) [20], rewritten to include cyclic crack growth behavior. An initially isotropic strain hardening model is used (as the materials studied have fairly low strain hardening exponents) and the finite elements are two-dimensional triangles with cubic base functions [19]. No embedded singularity is used in the elements adjacent to the crack tip. By using a very fine mesh size close to the crack-tip, however, the correct crack tip singularity is believed to be obtained. It is found that an element size of less than about 5 percent of the plastic zone size results in convergent values of the computed crack closure load.

5. Results and Discussion

Crack closure under cyclic compression loading

In the earlier work [15] on crack closure under cyclic compression loading, it was found that computational results for plane strain conditions correlated well to the experimental measurements. In that study the thickness of the steel specimens was 12.7 mm whereas in the present investigation a thickness of 2.54 mm is used. Hence, it may be expected that plane stress conditions are prevalent in this study. Plane stress will also be promoted by the initial overloading.

In Fig. 4 both plane strain and plane stress calculations for the 100% single overload case are shown. The plane strain loading condition causes higher closure loads (and hence a lower effective cyclic load range) than plane stress during the entire growth of the fatigue crack under far-field compression.

Such closure behavior is different for tension- tension or tension-compression loading where plane stress yields higher closure loads (and lower effective load range) than plane strain [14, 19]. The larger closure level and the smaller crack-opening range under far-field compression can be attributed to greater triaxiality and smaller plastic-zone size as compared to plane stress. In this latter case the greater extent of plasticity at the minimum compressive loads leads to enhanced residual tensile stresses over larger distances ahead of the crack tip. The experimentally measured crack closure loads are also plotted in Fig. 4. The closure levels indicated by such measurements are in good agreement with the numerically predicted values for plane stress. Consequently, further calculations were only performed for the plane stress situation.

The influence of the magnitude of the first overload on the subsequent crack extension is shown in Fig. 5. It is seen, as expected, that the greater extent of plasticity at large compressive overloads causes the crack to grow at a higher rate and also over a larger distance before crack arrest occurs. Both the numerically predicted variation of crack closure under plane stress and the experimentally measured closure loads are shown in Fig. 6 for the compression fatigue crack growth tests shown in Fig. 5. In Fig. 6 it is seen that the plane stress computations correlate rather well with the experimental data for the crack extensions covered by the numerical analyses. It is also noted that an increase in the magnitude of the compressive overload leads to an increase in the effective load range and hence in the distance of crack growth.

![Crack length vs. cycles](FIGURE 5)

**FIGURE 5.** Variation of fatigue crack length (measured from the notch tip) with number of compression cycles. $P_{max} = 1763$ N and $P_{min} = 1763$ N during constant amplitude cyclic compression

The effect of the compressive overload (or underload) on the closure behaviour and the crack growth was also investigated by repeating the application of the underload as indicated in Fig. 7.
the first compressive load cycle, at $P_{\text{min}}$, and also at $P_{\text{max}}$ after unloading, are shown in Fig. 8 (a)-(c) for the cases with no underload, 100% underload and 200% underload, respectively. The influence of the underload ratio on the plastic zone size is also given in Fig. 9 for both the compressive and tensile plastic zones.

**FIGURE 6.** Influence of underloads on crack closure variations with crack extension in cyclic compression. Experimental results for A542-3 steel and plane stress computations. $P_{\text{max}} = -1763 \text{ N}, P_{\text{min}} = -17 630 \text{ N}$

**FIGURE 7.** Influence of repeated underload on crack closure levels in cyclic compression. $P_{\text{max}} = -1763 \text{ N}, P_{\text{min}} = -17 630 \text{ N}$

These experimental results suggest that the underloading is only important at the application of the very first underload which gives rise to notch tip plasticity over a large area. Once the crack starts to grow in this residual tensile stress field subsequent underloads are of less importance as the crack tip plastic zone size is contained in the already existing notch tip plastic field.

**Crack-tip Plastic Zones and Stress Fields**

As discussed earlier, the growth of cracks (subjected to remotely applied compression cycles) is engendered by the existence of a residual tensile stress in the vicinity of the notch tip (when the load is reversed at $P_{\text{min}}$). The plastic zone sizes (assuming plane stress conditions) for

**FIGURE 8.** Plastic zone size at $P_{\text{min}}$ (a) and $P_{\text{max}}$ (+) during first load cycle
(a) No underload, i.e., constant amplitude compression, with $P_{\text{max}} = -1763 \text{ N}, P_{\text{min}} = -17 630 \text{ N}$
(b) 100% underload
(c) 200% underload

With crack advance, the extent of residual tensile deformation at the crack tip decreases due to a progressive increase in crack closure. The initial compressive plastic zone formed due to the stress concentration at the notch tip, however, remains largely uninfluenced. In Fig. 10 (a)-(d) isostrain plots of the normal strain component $\epsilon_y$ are shown for the case of both 100% and 200% underloading. The strain distributions are shown both at the very first load cycle (in compression) and after $0.98 \text{ mm}$ crack extension.
FIGURE 9. Influence of underload ratio on compressive and tensile plastic zone size. $P_{\text{min}} = -17.630 \text{ N}$, $P_{\text{max}} = -1763 \text{ N}$

The distribution of normal stress at and around the crack tip during a load cycle is shown in Fig. 11 (a)-(e) for a 0.98 mm long crack subjected to constant amplitude cyclic compression with $P_{\text{min}} = -17.630 \text{ N}$ and $P_{\text{max}} = -1763 \text{ N}$. In Fig. 11 (a) is shown the stress distribution at the maximum load, a peak tensile stress of 584 MPa exists at the crack tip. The change in stress distribution during loading is apparent in Fig. 11 (b)-(e) where stresses are shown for the closure load, $P_{\text{cr}} = -2408 \text{ N}$, in Fig. 11 (b) and then at $P = -7482 \text{ N}$, $P = -12556 \text{ N}$ and $P = P_{\text{min}}$ in Figs. 11 (c)-(e), respectively.

6. Conclusions

Numerical predictions (plane stress) of crack closure development with crack extension during cyclic compression correlated well with experimental measurements. The very first underload cycle is found to be of major importance as it forms the notch tip plastic field which remains rather unaffected by subsequent loading. Large underloads give rise to greater extent of plasticity which in turn causes the crack to grow at a higher rate and also over a larger distance before crack arrest occurs.

FIGURE 10. Isostrain plots of normal strain along the crack plane.
(a) 100% underload, strain distribution at $P_{\text{min}}$ during underload cycle.
$P_{\text{min}} = -17630 \text{ N}$, $P_{\text{max}} = -1763 \text{ N}$ and subsequent constant amplitude loading
(b) As in (a) but at $P_{\text{min}}$ after 0.98 mm crack extension
(c) As in (a) but for 200% underload
(d) As in (b) for 200% underload

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7. Acknowledgements

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8. References


