DYNAMIC CONTROL ASPECTS OF DEVELOPMENT OF THREE SHAFT TURBOPROP ENGINE

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Abstract
Three shaft conception of turboprop makes subsequent demand step up of economical power plant especially for commuter and executive aircraft. Significantly higher claims are laid on engine control systems in a field of dynamic parameters and characteristics. Some difficulties are necessary coped with a respect of the demand on engine control in a stage design of proper engine.

The aim of this paper to present some results from investigation dynamic control parameters and characteristics by more perfect, but always approximate calculation and to call on its reverse formulas for acquiring of necessary bases for turboprop compromises, including control field, too.

VÝST - exhaust
R - reduction gear
RS - control system
2R,3R - concerning two, three shaft turboprop engine

f - fuel
c - total conditions
r - reduction on ISA
d - differentiating
c - value on curve of equilibrium modes of operation
Σ - summary

⇒ ensure from
↑ - value increase
↓ - value recede

1.0 Introduction
Applications of turboprops with free power turbine aimed at asserting three shaft design concept of gas turbine engines in one evolutionary direction. However, three shaft engine has some essential distinctive from two shaft design concept if we rate it as controlled and regulated plant. There are the most important differences in a field of dynamic properties, which are limiting for all attainable control modes and actions at operating failures. Meaningfully increasing qualities of many items must be reached, on basic requirements, e.g. minimum weight, specific fuel consumption, final and operating cost, with maximum reliability/maintainability, etc. - /L1/, but good static and dynamic control properties, including easy starting, available and effective thrust reversal, too - /L2/. For engine designeres many of requirements are conflict and they must arrive at best compromise. Some items are more importance than others, depending on the three shaft design concept and its application, as for commuter, utility and executive aircrafts.

Essentially in connection what was written, is necessary to gain the resultant dynamic control characteristics and properties not only the control system, but with "the contribution" of the engine as controlled and regulated plant. The engine could have certain dynamic control characteristics and properties act upon closely connected with - non interaction - invariaction

List of Symbols and Subscripts
/General Nomenclature/>

Q - quantity delivered
n - revolutions
cp - coefficient of propeller power
Δ - angular acceleration
\( I \) - polar moment of inertia
W - specific work
p - pressure
T - temperature
\( ζ \) - calculated air compression, expansion of gases
\( η \) - efficiency factor
P - propeller shaft power
\( π, ω \) - ratio of specific heat capacities of air, gases
\( \delta X \) - ratio of relative differences and absolute value of \( x \)-parameter / \( \delta x / \)
Δ - relative difference /increment/ 
\( t \) - time
\( k_m \) - dimensionless boosting \( l \) parameter with respect \( m \) parameter
\( \tau \) - time constant, dynamic lag, relative temperature gradient
\( s \) - the Laplacian operator
C/\( \Omega, \Omega \), A, B - coefficients of reciprocal influences
K - compressor
T - turbine
V - propeller
SK - combustion chamber
VÝST - inlet

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2.0 Approximate Matrix Calculation of Dynamic Characteristics

Basic block diagrams [fig. 2] and 3 are shown principal differences between three and to the present time two shaft turboprop operated.

Fig. 1 Schematic Diagram of Three Shaft Turboprop

Fig. 2 Basic Block Diagram of 2R Turboprop Control

Fig. 3 Basic Block Diagram of 3R Turboprop Control

If the same laws of control by comparison: 

Qf \rightarrow n_1 \rightarrow n_{10}

c_p \rightarrow n_v \rightarrow n_{v0}

resultant rectangular matrix of linear dynamic coefficients inclusive of three shaft coefficient system can be written as

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<tr>
<th>( \delta Q_f )</th>
<th>( \delta c_p )</th>
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<th>( \delta n_v )</th>
<th>( \delta n_{v0} )</th>
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\( \delta k_{n_1} \) \( \delta k_{n_v} \) \( \delta k_{n_{v0}} \) \( \delta k_{T_e} \)

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Rectangular matrix covers up of two shaft turboprop. Matrix form as the result of the conjecture dynamic characteristics identification, where put all general knowledges from /14/ and especially /15/.

Start equations have frequented forms:

$$\begin{align*}
\alpha_{10} &= \frac{3p}{J_{1}} \left[ \frac{\eta_{10}}{\eta_{0}} \right]_{1} \left( \delta w_{10} - \delta w_{10} \right) / 2.2 / \\
\alpha_{1} &= \frac{3p}{J_{1}} \frac{1}{\eta_{0}} \left( \delta w_{10} - \delta w_{10} \right) / 2.3 / \\
\delta w_{10} &= \delta w_{10} + \frac{1}{\eta_{0}} \delta w_{10} / i = 1, 2, 3; \ j = 3, 4, 5 / 2.4 / \\
\delta w_{10} &= \delta w_{10} + \frac{1}{\eta_{0}} \delta w_{10} / 2.5 / \\
\delta w_{10} &= \delta w_{10} + A i \eta_{0} / 2.6 / \\
A i \eta_{0} &= \frac{3p}{J_{1}} \left[ \frac{\eta_{10}}{\eta_{0}} \right]_{1} \left( \delta w_{10} - \delta w_{10} \right) / 2.7 / \\
\delta w_{10} &= \delta w_{10} + \delta w_{10} / 2.8 / \\
\delta w_{10} &= \delta w_{10} + \delta w_{10} / 2.9 / \\
\delta w_{10} &= \delta w_{10} + \delta w_{10} / 2.10 / \\
\text{and low pressure compressor K1 is discerning:} & \\
\delta w_{10} &= \frac{1}{\kappa_{10}} \left[ \eta_{10} \left( \delta w_{10} - \delta w_{10} \right) \right] / 2.11 / \\
\delta w_{10} &= \frac{1}{\kappa_{10}} \left[ \eta_{10} \left( \delta w_{10} - \delta w_{10} \right) \right] / 2.12 / \\
\delta w_{10} &= \frac{1}{\kappa_{10}} \left[ \eta_{10} \left( \delta w_{10} - \delta w_{10} \right) \right] / 2.13 / \\
\delta w_{10} &= \frac{1}{\kappa_{10}} \left[ \eta_{10} \left( \delta w_{10} - \delta w_{10} \right) \right] / 2.14 / \\
\text{Continuous functions of high pressure compressor K2:} & \\
\delta w_{10} &= \frac{1}{\kappa_{10}} \left[ \eta_{10} \left( \delta w_{10} - \delta w_{10} \right) \right] / 2.15 / \\
\delta w_{10} &= \frac{1}{\kappa_{10}} \left[ \eta_{10} \left( \delta w_{10} - \delta w_{10} \right) \right] / 2.16 / \\
\text{are rearranged from transformed function} / \text{fig. 4} / \\
\text{Continuous, smooth curve of equilibrium modes of operation of compressor and turbine}.
\end{align*}$$
\[ x_{20} = \left[ 1 - \frac{6.5a}{k_{2a}} \right] \left( 1 - \frac{6.5a}{k_{2w}} \right) x_{20} + \frac{4}{k_{2a}} \rho_{a} + \frac{3a}{k_{4a}}. \]

\[ \dot{x}_{22} = \left[ \frac{\Delta x_{20}(t_c)}{x_{20}} \right]. \]

\[ \dot{\theta}(z) = \frac{[A_{2}(z)]}{[B_{2}(z)]}. \]

The functions, especially defined from equations 2.9 to 2.21, mean a direct effect into proper identification. Matrix form of the results allows to direct physical interpretation. Each element of the matrix means concrete boosting of a column by a row parameters.

If dynamic characteristic of two shaft turboprop are expressed in the same way by laws of control equally and respecting fig. 1, the change of three shaft characteristics against two shaft ones is determined by difference matrix from equation 2.1:

\[ \Delta M = \Delta \theta M - \Delta \theta M. \]

Unzero elements are fully reserved for control column \( z_{2_{21}} \) from point of control it means, that all \( \Delta \theta \) parameters determined in these elements. For this case, central shaft system exites very meaningful failure effects, unfavourable to some points. In the difference matrix \( \Delta M \) the existence of unzero elements different from zeros reflects aggravated conditions for unchanging of control circuits if the external failures are affecting \( (\Delta \theta_{c_{1}}) \) concentrated and removed far more from total self control or government by control circuits of the same transmission responses as ones for two shaft turboprop.

3.0 Comparison of Two and Three Shaft Turboprop Properties

Both design of turboprops represent two and three parameter base system, where the fact the laws of control and regulation are identical. Connecting with the control system, which is securing non interaction and invariance on fig. 5, total invariance of both design is obviously ensured by conditions:

\[ \mathbf{R} = (\mathbf{U})^{-1} \mathbf{Z}. \]

\[ \mathbf{R} = (\mathbf{Z})^{-1} \mathbf{U}. \]

Conditions for total non interaction: product

\[ \mathbf{U} \mathbf{R} \quad \text{and} \quad \mathbf{R} \mathbf{U}. \]

must be "clear" diagonal matrices. After transformation of \( \Delta \theta M \) and \( \Delta \theta M \) matrices to transfer express of elements and filling total invariance and non interaction for two shaft turboprop:

\[ \Delta \theta_{2_{21}} \]

Fig. 5 Common Matrix Block Diagram of 2R and 3R Turboprop Control

\[ \Delta \theta_{1_{21}} \]

\[ \Delta \theta_{2_{21}} \]

\[ \Delta \theta_{3_{21}} \]

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and for three shaft turboprop:

\[
\frac{k_{Eg} \left( c_{1} s + 1 \right)}{s^2 + c_{1} s + 1}
\]

3.6

4.0 Reverse Procedure of Calculation.

Deriving of Advisory Trends and Values

Realisation of the transfer functions $3\text{P}_{d}^{\text{Z}}, 3\text{R}_{d}^{\text{U}}$, and $3\text{R}_{d}^{\text{Z}}$ is simultaneously connected with t-optimal control. Problem t-optimal control is generally described f.e. in [15], applicable papers are too numerous, but without counter control plant contribution. Not only in general, which are the possibilities in this field?! Conventionally this question has not applicable solution. Considering, in every particular problem is always possible:

- the matrices $3\text{U}^{\text{Z}}, 3\text{R}^{\text{U}}, 3\text{P}^{\text{Z}}$ and $3\text{R}^{\text{P}}$ by formulas for $2\text{.1}$, $\ldots$, $3\text{.7}$, and $3\text{.10}$ / numerical appointed for full device of operation modes

- to assess the advisable control trends, like f.e. for more perfect t-optimal control

\[
\begin{align*}
\tau_{d}^{\text{Q}} & = \min \left( \tau_{d}^{\text{Q}} \right) \\
\frac{\tau_{d}^{\text{Q}}}{\tau_{d}^{\text{Q}}} & = 1
\end{align*}
\]

Because these coefficients are inured from
the elements of the matrices \( \mathbf{M} \):

\[
\begin{align*}
\eta_{g} & = \frac{n_{g}}{k_{g}} \left( 1 - \frac{n_{1} k_{g}}{k_{g1}} \right)^{-1} \rightarrow \min. \quad / 4.3 /
\end{align*}
\]

\[
\begin{align*}
\eta_{s} & = \frac{n_{s}}{k_{s}} \left( 1 - \frac{n_{1} k_{s}}{k_{s1}} \right)^{-1} \rightarrow \min. \quad / 4.4 /
\end{align*}
\]

\[
\begin{align*}
\eta_{f} & = \left( \eta_{s} + \eta_{g} \right) \left( 1 - \frac{n_{1} k_{s}}{k_{s1}} \right)^{-1} \rightarrow \min. \quad / 4.5 /
\end{align*}
\]

\[
\begin{align*}
\eta_{2} & = n_{2}^{2} \left( n_{g} q_{g} + \frac{1}{n_{1} k_{g}} \right)^{-1} \rightarrow \min. \quad / 4.6 /
\end{align*}
\]

\[
\begin{align*}
\eta_{1} & = \left( n_{g} q_{g} + \frac{1}{n_{1} k_{g}} \right)^{-1} \rightarrow \min. \quad / 4.7 /
\end{align*}
\]

\[
\begin{align*}
\alpha_{1} & = \lambda_{1}^{-1} \rightarrow \min. \quad / 4.8 /
\end{align*}
\]

where

\[
\begin{align*}
1 - \frac{k_{g}}{k_{g1}} & = n_{1} k_{g} \left( 1 - \frac{n_{1} k_{g}}{k_{g1}} \right)^{-1} \quad / 4.9 /
\end{align*}
\]

\[
\begin{align*}
2 - \frac{k_{g}}{k_{g1}} & = n_{1} k_{g} \left( 1 - \frac{n_{1} k_{g}}{k_{g1}} \right)^{-1} \quad / 4.10 /
\end{align*}
\]

\[
\begin{align*}
2 - \frac{k_{g}}{k_{g1}} & = n_{1} k_{g} \left( 1 - \frac{n_{1} k_{g}}{k_{g1}} \right)^{-1} \quad / 4.11 /
\end{align*}
\]

Full analyses of the values and trends bear many antagonisms.

5.0 Conclusion

This paper considers only trivial form of problems. In full extent using of the reverse procedure - analyse of the values and trends, which bear many antagonisms just under the given conditions, quantitative calculations afford predominant work states for advantageous and reasonable compromises.

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It is self evident to apply reverse, the other way round procedure and to derive the advisable trends and values of:

\[ \mathbf{M} \] matrix coefficients are the important plant parameters, including control form of the K1 and K2 compressor characteristic analogically it is possible to extend for turbine/Compendiously and concisely f. e.:

For the urgent need to reduce the degenerative action \( n_{2} \) on \( n_{1} \) excepting

\[ \eta_{g} \rightarrow \min. \quad \text{and also } \eta_{s} \rightarrow \max. \quad / 4.12 / \]

\[
\begin{align*}
K_{n_{1}} & \quad K_{2s} & \quad K_{f} & \quad \Lambda_{2}, \quad / 4.13 / \n\end{align*}
\]

\[
\begin{align*}
I_{f} & \quad [\bar{r}_{g} \bar{c} \bar{c}], \quad [\bar{r}_{2g} \bar{c}], \quad / 4.14 / \n\end{align*}
\]

but also

\[
\begin{align*}
K_{20} & = \frac{\bar{r}_{c}}{\bar{c}} \rightarrow \text{sheerer course} \quad / 4.15 / \n\end{align*}
\]

\[
\begin{align*}
K_{f} & \rightarrow [\bar{r}_{2c}], \quad \text{higher temperature of gases in front of nozzle guide vanes } T_{1}
\end{align*}
\]