

EVALUATION OF THE RELIABILITY OF A FINITE ELEMENT ANALYSIS OF A TURBINE DISC

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Abstract

This paper describes the structural analysis of a turbine disc using the finite element method. In order to check the reliability of the results, the analysis was done on three different, commercially available finite element packages, viz, ASKA, NISA and SAP V. This led to a comparison of the accuracy and reliability of the packages themselves, in terms of element behaviour and the solution of the final problem. The modelling problems encountered are described, as are the finite element models used, and the results of the final analyses are discussed and compared. Possible reasons for the discrepancies found in the NISA results are also given. A brief description of the element tests done, and the results of these tests is presented in the Appendix.

1 Introduction

The primary aim of this work was the structural analysis of the turbine disc of a low cost, 3 kN thrust, turbojet engine. The engine was intended to have a useful life of only 20 hours and as such creep and fatigue were not taken into account in the structural integrity calculations.

The relatively complex geometry of the disc precluded the use of purely analytical analysis methods, so necessitating the use of some approximate numerical technique. The technique chosen in this case was the finite element method.

Experimental verification of the results of the analysis would, under true operating conditions, have been costly and difficult to achieve, and thus a great deal of faith had to be placed in the results of the finite element analysis. In order to check the reliability of these results the analysis was done on three different, commercially available, finite element packages, namely, ASKA, NISA and SAP V. This inevitably led to an evaluation of the accuracy and reliability of the finite element packages themselves.

This paper discusses the results obtained and how the various analyses compare. Some of the modelling problems are described, as are the problems encountered with the actual analysis. A few comments are also made with respect to the possible reasons for the discrepancies found in the final results. Finally, the Appendix contains a description of the basic tests performed to check on element performance.

2 The Finite Element Models

The turbine in question (shown in Figure 1) was designed with 51 blades and two labyrinth seals (for cooling purposes), which were all integrally cast with the disc. Clearances in the seals and at the blade tips were deemed to be critical, and thus the growth of the disc under operating conditions needed to be checked. Furthermore, the yield stress of the material decreases somewhat at elevated temperatures and a detailed stress analysis was required to ensure that the operating stress levels were less than the maximum allowable stresses.

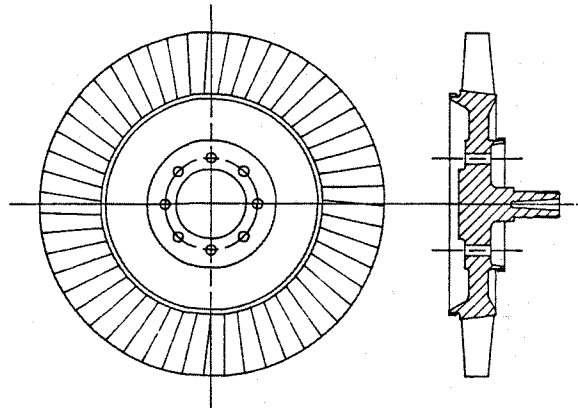


Figure 1. Turbine Configuration

The analyses discussed below concern only the turbine disc, and not the blades i.e. only the cross-hatched section in Figure 1. The main reasons for separating the blades from the disc in the analysis were that, firstly, it greatly simplified the disc analysis and, secondly, a separate blade model was required anyway, for vibration analysis. Furthermore, the stress levels at the blade root were relatively low and so modelling of that part of the structure was not considered to be necessary.

2.1 Load Modelling

Being a disc this part appears to be well suited to an axisymmetric type of analysis. The loading on the disc is, however, not strictly axisymmetric. There are 51 turbine blades (subject to centrifugal and aerodynamic loading) each with a relatively small connecting area on the perimeter of the disc, and there are eight bolt holes for fixing the turbine to the rotor.

These problems were overcome in different ways. The blade leading and trailing edges

overlap each other to a certain extent and thus, despite the small contacting areas, the load transferred from the blades to the disc was assumed to be evenly distributed across the width, and around the perimeter, of the disc. Any inaccuracies resulting from this assumption were likely to be found only in the region immediately adjacent to the blade connecting area and so would not affect the more highly stressed regions closer to the axis of rotation. The final results confirmed these ideas.

The problem of the bolt holes was less easily solved, and in fact a full three-dimensional analysis had to be done of the region around the bolt holes to check on the stress concentration effects of these holes. The axisymmetric analysis obviously had to be done ignoring the bolt holes, as the very nature of such an analysis prevents the modelling of such holes. The three-dimensional model is discussed in more detail in Section 4.

2.2 The Axisymmetric Finite Element Models

The relatively complex cross-sectional shape of the disc necessitated the use of a large number of elements to define the geometry accurately. In order to limit the total number of degrees of freedom (and so the analysis cost) with such a number of elements, four node axisymmetric elements were initially used. This finite element model is shown in Figure 2.

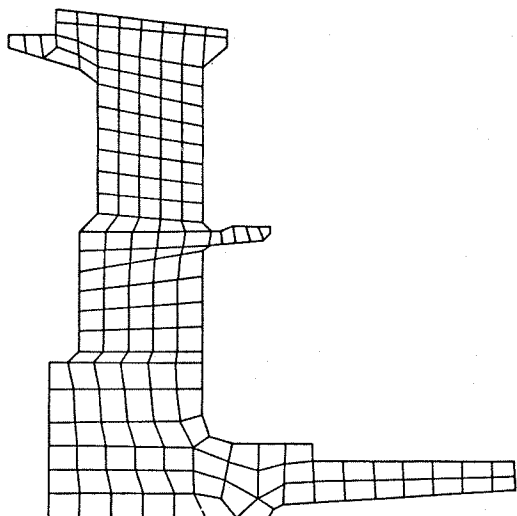


Figure 2. Four node finite element model

Another model, with a decidedly coarser mesh, was created using 8 noded axisymmetric elements. The purpose of this was to check if the generally superior performance of the higher order element would compensate for the large reduction in the number of elements (66 instead of 169). This finite element mesh is shown in Figure 3.

Prior to using these models for analysis purposes, however, a number of element tests were done on the four and eight node axisymmetric elements in ASKA, NISA and SAP in

order to gauge the reliability of these elements. The performance of the various elements is described and discussed in the Appendix.

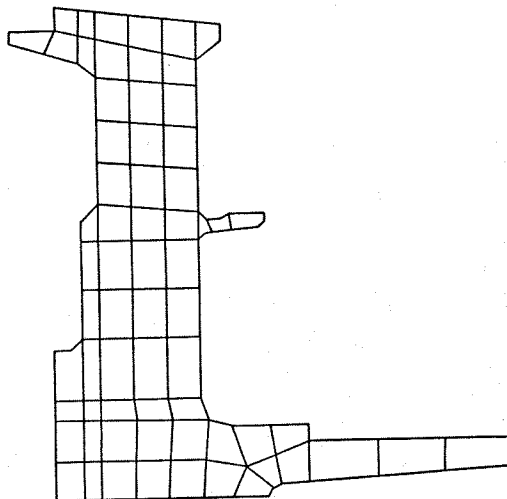


Figure 3. Eight Node Finite Element Model

3. Results of the Axisymmetric Analysis

Based on the results of the element tests that were done it was presumed that the three programs would produce similar results for the full disc analysis. The four node element model was analysed using SAP and ASKA, producing results of maximum stress and deflections within 3% of each other thus confirming the element test results. The same model could unfortunately not be analysed using NISA due to what seemingly was an incompatibility with the NISA software, and the operating system on which it was run. The problem only became apparent when a reasonably large number of elements was used, since the element tests were run with no difficulty.

The eight node element models were successfully run on all three programs but produced some rather disturbing results. The ASKA and SAP runs again gave very similar results, which in turn were within 5% of the four node model results (for maximum stress and deflections). NISA on the other hand gave results for maximum stress and deflection which were 30% and 35% lower respectively, than the ASKA and SAP results. It is interesting to note that when centrifugal loading only was applied in these runs that the results produced by the three programs correlated extremely well, and that it was only when the simulated blade loading was added that the NISA results started differing significantly. The stress contours, and deformed shape plots, however, provided by NISA, ASKA and SAP showed very similar trends, the only difference being that the absolute values in NISA were lower.

4. Three Dimensional Model and Analyses

As was mentioned earlier, the stress concentration effect of the eight bolt holes had to be evaluated using a full 3-D model. The initial model was merely a quarter segment of a flat disc with no bolt holes (created using 20 node solid elements). Under pure centrifugal loading the stress and deflection results produced by this model, when run on ASKA, corresponded surprisingly well with the theoretical and 2-D axisymmetric analysis results (within 6%). This provided the necessary confidence in the 3-D analysis capabilities to proceed with adapting the model to include the bolt holes and re-do the analysis.

The results of this analysis proved the stress concentration effects to be fairly large ($k = 1.9$). The effect, however, dissipated rapidly on either side of the holes so that within one diameter of the holes the stress fields were practically identical to those in the original analysis. As a result, the model was reduced to a ring shape, three hole diameters in width, symmetric about the hole, and the stresses from the 2-D axisymmetric analysis (which were effectively the same as the 3-D analysis results) were applied at the boundaries of the model. The results of an analysis performed on this model in turn correlated very well with the full 3-D analysis results. The ring-shaped 3-D model (shown in Figure 4) could therefore provide reliable results but at substantially lower cost than the full quarter segment model.

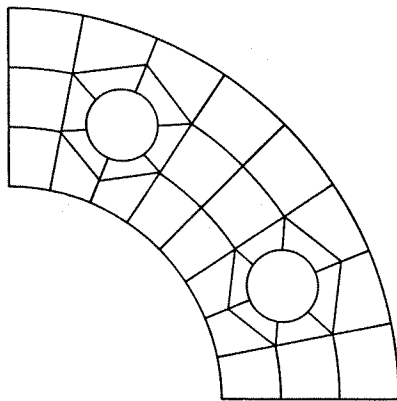


Figure 4. 3-D Model with bolt holes

The time available on this project precluded any 3-D analyses on the other packages, but suffice it to say that the correlation of the results of the 2-D axisymmetric and 3-D analyses on ASKA was quite satisfactory.

5. Discussion of Results

The close correlation of the results produced by SAP and ASKA, when using both four and eight node axisymmetric elements, indicated that a great deal of confidence could be placed

in the maximum stress and deflection values. This confidence was reinforced by the 3-D analysis results which tied in very well with the 2-D axisymmetric analysis.

The stress and deflection results provided by NISA, however, differ by more than 30% from those given by ASKA and SAP. This discrepancy is particularly disturbing in that the NISA results are not entirely unrealistic, and had the analysis been done on this program alone, they would probably have been accepted without question. Analysis errors of this magnitude could however, in many situations, lead to extremely costly design faults.

The fact that NISA provided quite acceptable results under centrifugal loading only, suggests that the major variation in the results comes from applying point loads in an axisymmetric analysis. This perhaps indicates that the element tests should have been more extensive, investigating the performance of axisymmetric elements under point loading as well as centrifugal loading.

Experience with the CSIR installation of NISA, however, points to the problem possibly being rather more involved than merely poor element performance. Difficulties encountered when attempting to analyse models with even relatively few four node axisymmetric elements, and when doing thermal stressing, have indicated that there are some incompatibilities with the operating system software and the NISA software. These incompatibilities appear to have lead to occasional numerical errors being introduced, and this may also have been the cause of the inaccuracies in the analysis of the eight node axisymmetric element model.

These factors emphasize the need for a set of rigorous validation tests for individual elements as well as benchmark tests for assemblies of elements. The element tests would act as a check on element performance over a whole range of element geometry, boundary and loading conditions. The purpose of the benchmark tests for assemblies of elements would be twofold; firstly because certain single element tests may be too rigorous, (some non-conforming elements can perform well with realistic strain gradients) and, secondly, to determine the capability of the system to correctly assemble groups and/or combinations of elements and diagnose errors and incompatibilities. This would hopefully help to detect any of the type of problems experienced on the CSIR installation of NISA.

6. Conclusion

The results produced by ASKA and SAP, when using both four and eight node axisymmetric elements, correlated extremely well. Further-more the three dimensional analysis done on ASKA produced results which tied in very well with the axisymmetric analyses, thus indicating that a great deal of confidence could be placed in these results.

The results of the NISA axisymmetric analysis, however, differed by more than 30% from the ASKA and SAP results.

This discrepancy is particularly disturbing since the NISA results are not entirely unrealistic, and may well have been accepted at face value had no comparative analysis been done. The reasons for the inaccuracies encountered may be some obscure incompatibilities with the NISA software and operating system on which it was run, rather than merely poor element performance.

In order to avoid such problems, it is therefore, recommended that, prior to doing any analysis, an extensive set of tests should be performed to check on the behaviour of individual elements as well as assemblies of elements. These tests should be done for a variety of boundary and loading conditions.

Appendix - Element Tests

None of the manuals provided with ASKA, NISA or SAP V provided anything like a full description of the theoretical formulation of the elements or the numerical techniques used in their implementation. While this knowledge is not generally sufficient for proper element evaluation anyway, it certainly emphasized the need to verify the technical capabilities of the elements that were to be used.

Tests were therefore done on the four and eight node axisymmetric elements available in ASKA, NISA and SAP.

These tests were:

- (i) Convergence tests
- (ii) Element Aspect Ratio tests
- (iii) Structure Aspect Ratio tests (fixed number of elements)
- (iv) Element Distortion tests.

The aim of these tests was to compare the stress and displacement values (under centrifugal loading), given by the three programs, to the analytical solution for rotating discs given by Timoshenko⁽¹⁾.

The simple flat rotating disc shown below (Figure A1) was used as a basis for the tests.

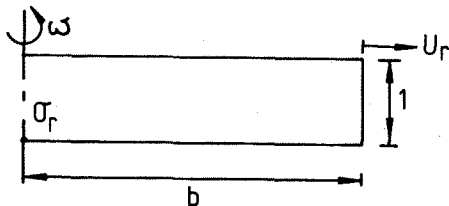


Figure A1. Flat Rotating Disc

The results of the element tests are summarized in graphical form below. It should be noted that on all the graphs the % error is given with respect to the analytical solution, the subscript r on σ_r denotes the radial stress at the axis of rotation, and on u_r the radial displacement at the outer edge of the disc.

A.1 Convergence Tests

The convergence tests were done using the model in Figure A1 with $b = 6$ and a variable number of elements. The results are summarized in Figure A2.

Although the behaviour of the eight node elements seems to be generally superior, it is probably true to say that with a large number of elements little difference will be found in the results. The same comment holds for the differences in the results of the three programs.

A.2 Element Aspect Ratio Tests

For these tests the value of b (Figure A1) was varied from 0.5 to 12 while only one element was used. The Element Aspect Ratio Test Results are presented in Figure A3.

The absolute values of error are of no real significance since the convergence tests showed that single element performance was poor, and thus it is only the trends that should be noted. Furthermore, the analytical results used as a basis for these comparisons are strictly only valid for "thin" discs, i.e. aspect ratio of about 2:1 or larger ($b > 2$). Element behaviour appears to be reasonably consistent where $b > 2$ for both four and eight-node elements in all three programs.

A.3 Structure Aspect Ratio Tests

The structure (Figure A1) was divided into three elements for these tests and the value of b varied from 1.5 to 18. The superior behaviour of the eight node elements is relatively marked in this case (Figure A4) although, the performance of the four node elements can be regarded as being quite acceptable considering that only three elements were used.

A.4 Element Distortion Tests

The element distortion tests indicated that both the four and eight node elements in all the programs perform very well. The error in the results was so small, even for highly distorted elements, that the results were not considered to be worth presenting here in graphical form.

References

- 1 Timoshenko, S and Goodier, J N, "Theory of Elasticity", McGraw-Hill, New York, 1970.
- 2 SAP V Manuals
- 3 ASKA Manuals
- 4 NISA Manuals

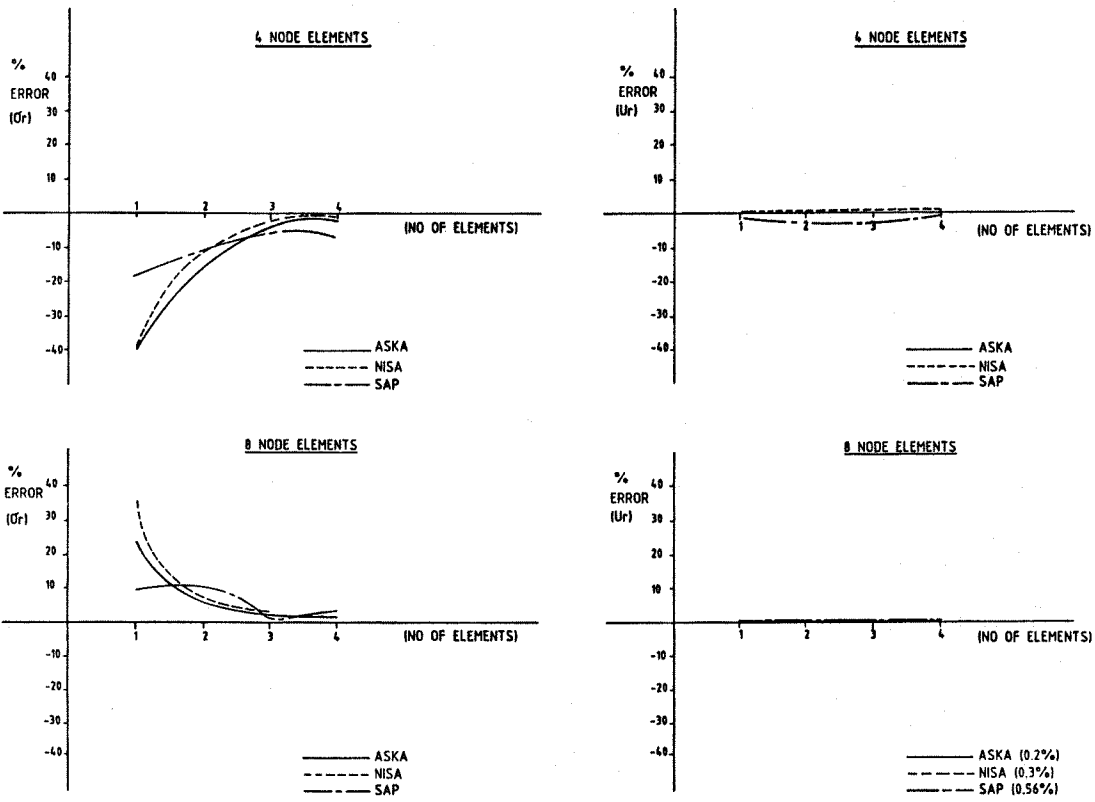


Figure A2. Convergence Test Results

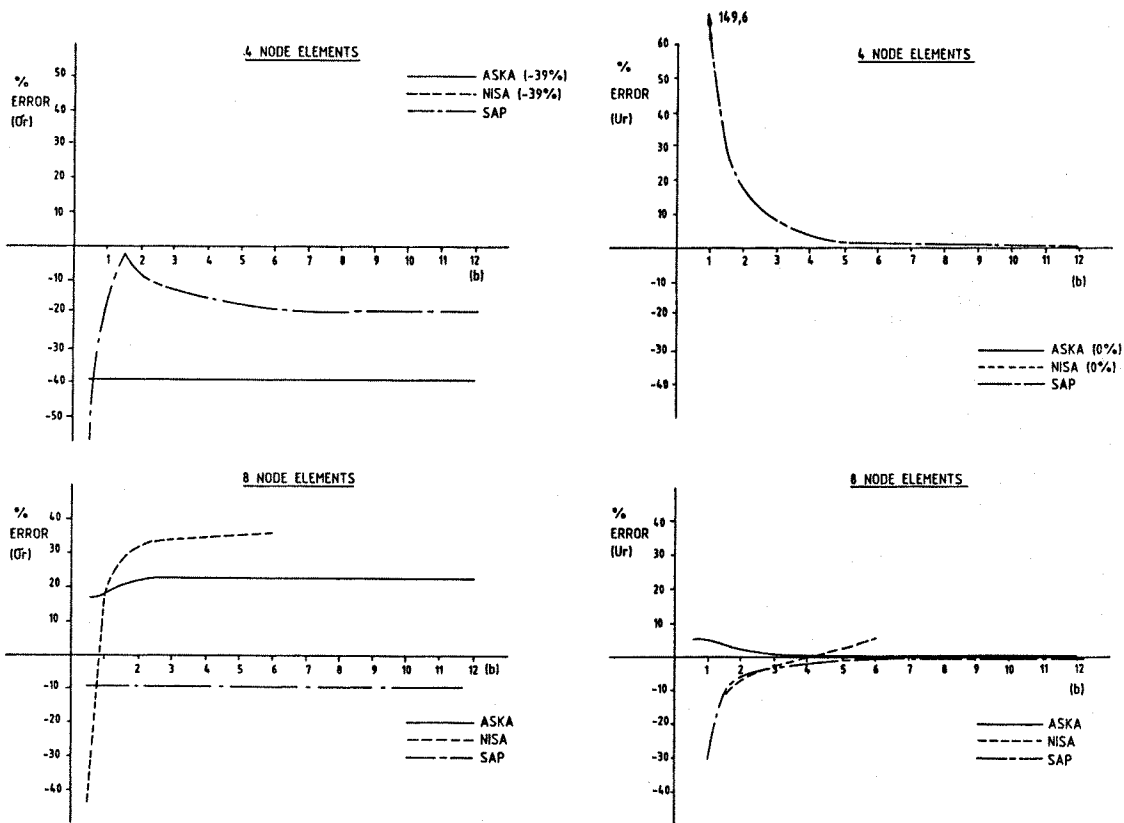


Figure A3. Element Aspect Ratio Test Results

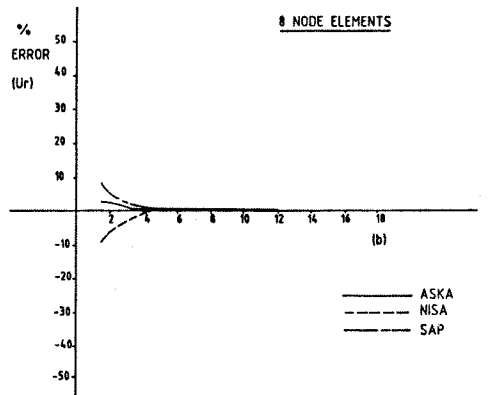
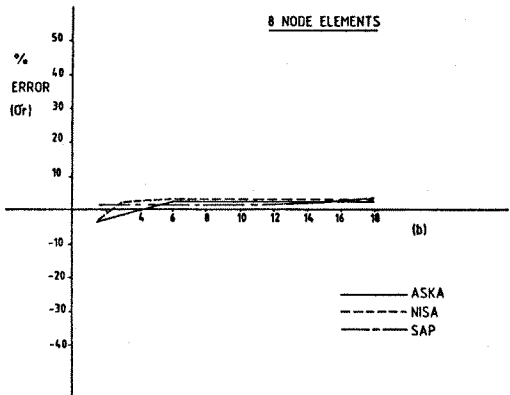
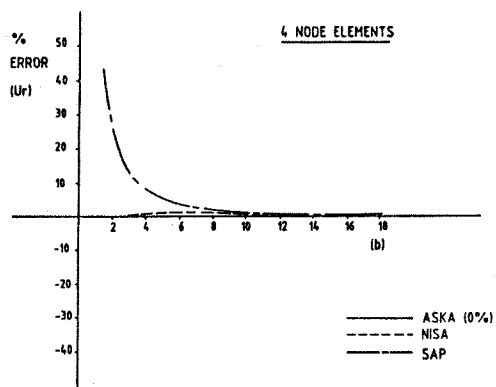
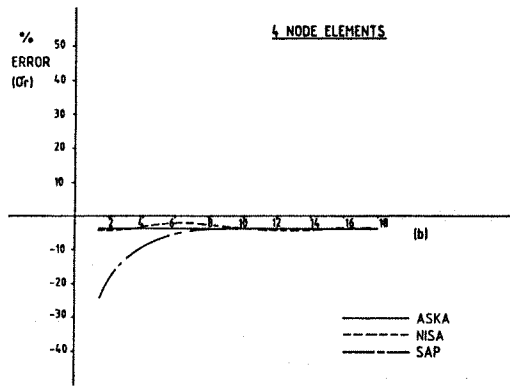


Figure A4. Structure Aspect Ratio Test Results