AIRFOIL OPTIMIZATION

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Abstract

This paper describes the components and applications of a computer program system for the opti-ization of airfoils. The program, characterized by a fast approximate optimization technique, an aerodynamic code for subsonic and transonic viscous flow and the possibility of modifying a desired part of the contour, was developed by Kent Misegades. However, to increase the practical use of the program two additional options have been included. To avoid oscillations in the geometry interpolation, the ordinary cubic spline funtions have been replaced by the taut cubic splines. The design of an airfoil is a problem that requires the consideration of the effects of modification at off-design. Therefore the possibility of implying constraints at off-design has been included.

I. Introduction

During the recent years considerable effort has been spent upon the development of airfoils for high speed flow, supercritical airfoils. Efficient numerical methods now exist for accurate analysis of high speed flows, even such containing shock waves. However, the design of a good airfoil requires the consideration of the effects of a large number of variables and so it becomes natural to automate the process of improvement of airfoil characteristics.

In this report is described briefly a program system that performs an automatic optimization of airfoils intended for subsonic or transonic flow. The system contains

- COPES, an approximate optimization program with a fast convergence characteristic
- DOFOIL, an aerodynamics module for calculation of exterior potential flow and boundary layer characteristics
- GEOM4, a geometry package that interpolates and describes the airfoil contour by means of taut cubic splines

This program system has been used for the optimization of a rear loaded MS-profile in the medium speed range. (Ref. 7)

The idea with the work was to introduce the automated design process with its rapidity compared with the "try and error" method often used in the aircraft industry.

II. Method for Optimization

The problem to be solved can be formulated as the following.

There is a function $F(\bar{x})$ to be minimized, while the conditions $G_j(\bar{x}) \geq 0$, $j=1$, m are to be met. $\bar{x}$ is a vector containing the n independent design variables.

The problem can be approximately solved numerically by the use of a Taylor series expansion for F, using existing data for the evaluation of the partial derivatives. The expansion is made with respect to the best already available design $\bar{x}^o$, characterized by the value F^o.

$$F(x) = F^o + \Delta\bar{x}^T \nabla\bar{F} + \tfrac{1}{2}\Delta\bar{x}^T [H] \Delta\bar{x} \qquad (1)$$

where

$\Delta\bar{x} = \bar{x} - \bar{x}^o$	vector of change of design variables
$\nabla\bar{F}$	vector of first partial derivatives
$[H]$	matrix of second partial derivatives
$\bar{x}$	vector containing the n design variables

Suppose that $F(\bar{x})$ is known for a set of k different designs ($k \geq 2$). Then, in the first step, $F(\bar{x})$ can be approximated by the first k components in the Taylor's series expansion. A system of linear eqs.(1) can be set up and the k-1 first values of ∇F and [H] solved. The conditions G_j are approximated in the same way.

Then the approximating functions are optimized, that is an approximate optimal design $\bar{x}^{k+1}$, using the constrained function minimization program COMMIN. This design is analyzed (F^{k+1}, G_j^{k+1}) and added to the previous set of information. And so new approximating functions containing k+1 components can be defined.

This process is repeated until convergence has been obtained, that is until the value of the optimum of the approximating function within some tolerance agrees with the value analysed for the approximately optimum design. (Fig. 1)

Experience has shown that convergence is seldom obtained before complete second order Taylor's series approximations for F and for G_j have been obtained, which would mean $1+n+(n+1)n/2$ aerodynamic analyses. This can be compared with more time consuming methods used earlier (Fig. 2). For example, the method that updates from a previous design $\bar{x}^o$ iteratively according to

$$\bar{x}^{k+1} = \bar{x}^k + \alpha^* (-\nabla\bar{F}^k)$$

Here the search direction $-\nabla\bar{F}^k$ is computed with finite difference methods, and the search distance, the scalar α^*, with a polynomial fitting procedure. For a typical optimization problem using 10 iterations this would mean 10(n+3) aerodynamic analyses. With two design variables one would have 6 analyses to perform using the first method compared to 50 using the latter method for optimization.

Previous and actual theory is more in detail discussed in Ref. 1. The programs COPES and CONMIN are described in Refs. 2 and 3.

III. Aerodynamic method

The aerodynamic code DOFOIL, developed at Dornier, is used to calculate the subsonic and transonic viscous flow over airfoils using the displacement surface concept. (Ref.4)

It mainly consists of two parts:

1. Full potential flow method

The full potential flow field around an arbitrarily shaped airfoil is solved by using a fast multigrid solver. (Ref. 5).

2. Boundary layer method

For a given potential flow field the boundary layer displacement thickness, both laminar and turbulent, is calculated by Stock's laminar method and Horton's turbulent method. (Ref. 4)

The vertical coordinates of the airfoil are iteratively updated by adding the boundary layer thickness to the airfoil. After each coordinate update a new full potential flow field is calculated and the process continues until convergence, based on the lift coefficient, is reached.

On a VAX 11/780-computer, a typical computation time is 3-4 cpu min. for a full aerodynamic analysis.

IV. Geometry modifications

As previously mentioned the optimization process is performed in a design space of n dimensions, the numbers of design parameters.

In this work the n design parameters $(x,\ldots x_n)$ define the thickness distribution in a specified modification region of the airfoil, i.e. the nose region.

Then for each optimization cycle a new airfoil for aerodynamic calculations is defined by:

1. the determination of a cubic spline fit through all points (fixed, modified) defining the airfoil
2. adding points, evaluated from the spline functions, in the modified region

However, cubic spline interpolation often tends to model extraneous inflection points (EIP's) in the region of modification (Fig. 3). (Specially in high curvature regions). Therefor the alternative of using taut cubic splines has been added to the program.

The idea with this is to use a cubic spline with additional knots placed so that the interpolant can make sharp bends where required without breaking out into oscillations as a consequence.

The taut cubic spline is a good alternative to the more, in computational point of view, unpractical exponential spline functions. (Ref. 6)

V. Airfoil design program system

The complete design program system contains the 3 mentioned programs, COPES, DOFOIL and GEOM4. COPES, including the constrained function minimization program CONMIN, serves here as a main program.

Fig. 7 show the organization where COPES gets the "exact" function values from a call to subroutine DOFOIL. The constrained function is approximated and, within a call to CONMIN, minimized. The loop is then ended with a call to subroutine GEOM4, which evaluates a taut cubic spline fit through the points, including fixed points and design variables, describing the airfoil geometry.

Design examples

Example 1: Total drag minimization
$M = 0.41$, $\alpha = 0^{\circ}$, $Re = 13 \times 10^6$

The drag computed using the approach of Squire and Young for compressible flow is here minimized with a constraint on the lift ($C_\ell \geq 0.54$).

The modification region is placed between 50 and 100 % chord length and because of the rather smooth contour only 2 design variables, at 65 and 85 % chord length, were chosen.

Figures 4 - 6 show the results of the optimization.

Convergence can be noticed before 8 optimization iterations, that is in this case with 2 starting designs (A and B) a total number of 10 analyses.

The curvature in the modification region has somewhat decreased, which also can be seened in the pressure distr.

Example 2: Total drag minimization,
($C_\ell \geqq 0.72$) $M = 0.65$, $\alpha = 0^{\circ}$,
$Re = 13 \times 10^6$

Figures 8 - 10 show the great possibilities of optimizing the transonic area of a profile.

Again only 2 design variables, at 17.5 and 25 % of the chord, define the modification region and, because of the rather high curvature, the degree of tension of the taut cubic spline fit is quite high.

The results were satisfactory. In this case the constraint on the lift coefficient was unnecessary. Fig. 8 shows how relative small changes in the design variables increase lift with a decrease in the total drag.

Example 3: Lift maximization,
$M = 0.41$, $\alpha = 0^{\circ}$, $Re = 13 \times 10^6$

This example shows the possibilities of optimization with constraints at off-design, that is another flight condition like a new Mach number or angle of attack.

Here the lift coefficient is maximized at 0°, while the drag of Squire and Young is limited to 0.0078 at 2° angle of attack.

The lift is increasing (~ 12 %) with increasing drag (~ 6 %) until the limit of drag is reached at 7 iterations (Fig. 11). That is satisfying, at these flight conditions, but the bump in the pressure distribution (Fig. 12) indicates that the design may not be satisfactory at higher angles of attack (early separation).

VI. Conclusions

A program for the opimization of airfoils was described. The individual components were each described and the manner in which they were coupled together was explained.

To show the applications of the program three examples in optimization of a rear loaded MS-profile have been presented.

The examples contain optimization cases in the medium speed, with or without transonic effects. Also the possibilities of optimization with a constraint at off-design has been shown, like different angles of attack or different Mach numbers. The results seem satisfactory.

However, problems with the optimization process have also been obtained.

In example 1 the drag, integrated from the pressure distribution first served as the objective function. Incorrect results containing big bumps in the pressure distribution were reached. Specially the calculation of small numerical values like drag coefficient in low speed at small angles of attack can be critical.

Therefore the requirement of an aerodynamics code with high accuracy is very important in order to perform realistic optimization cases with, for instance, implying constraints at higher angles of attack.

References

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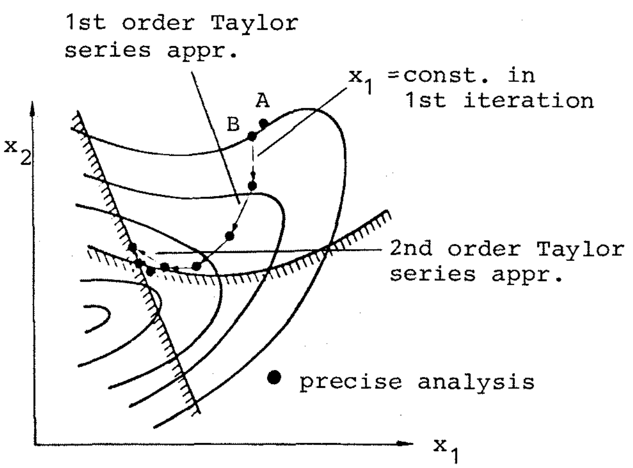

Fig. 1 Approximate optimization history

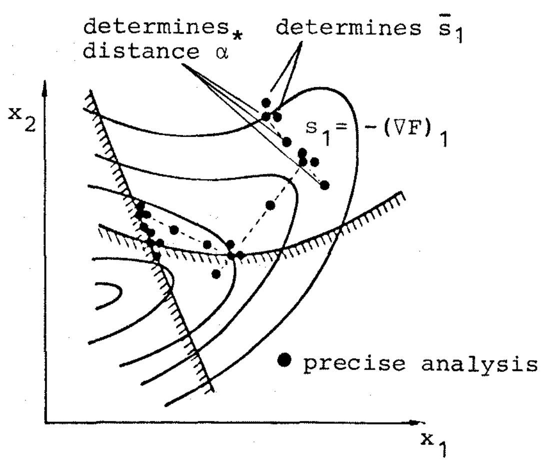

Fig. 2 Previous optimization method

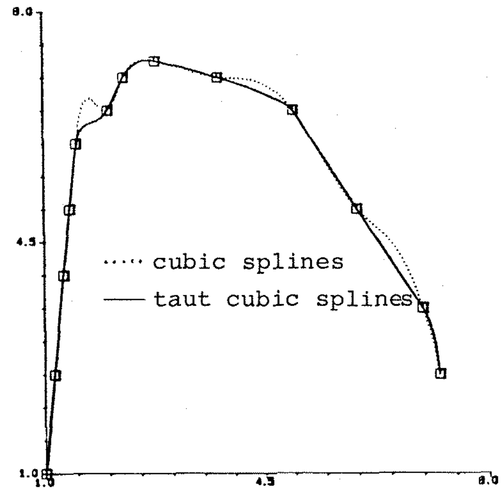

Fig. 3 Curvature discontinuity test case

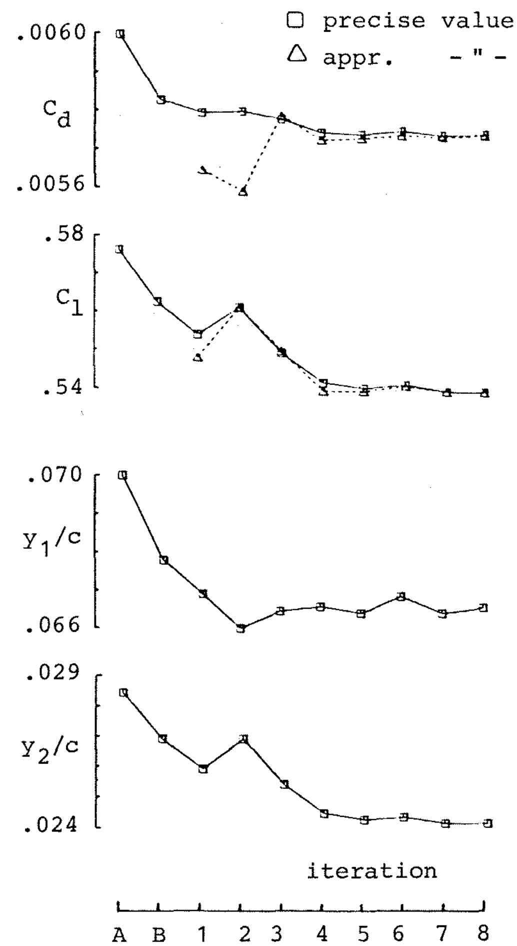

Fig. 4 Example 1 - Total drag minimization. $M = 0.41$, $Re = 13.0 \times 10^6$, $\alpha = 0^0$
(A, B are starting designs)

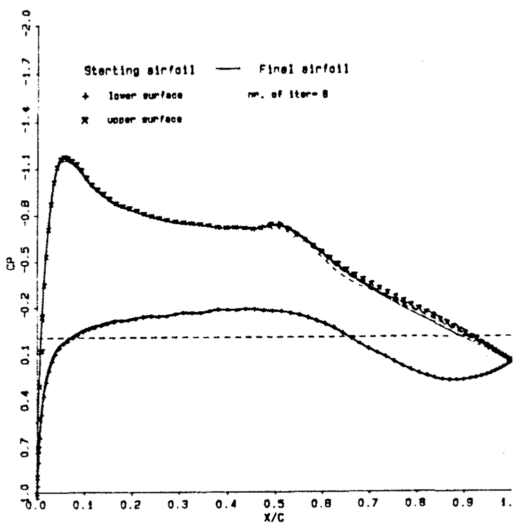

Fig. 5 Example 1 - History of pressure distribution

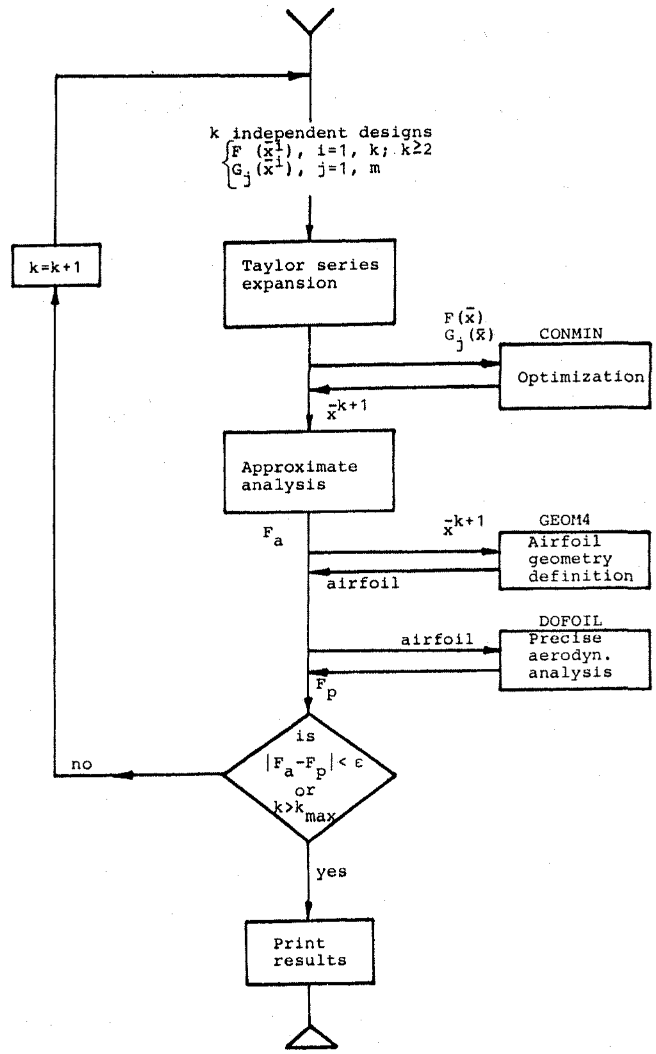

Fig. 7 Optimization program system organization

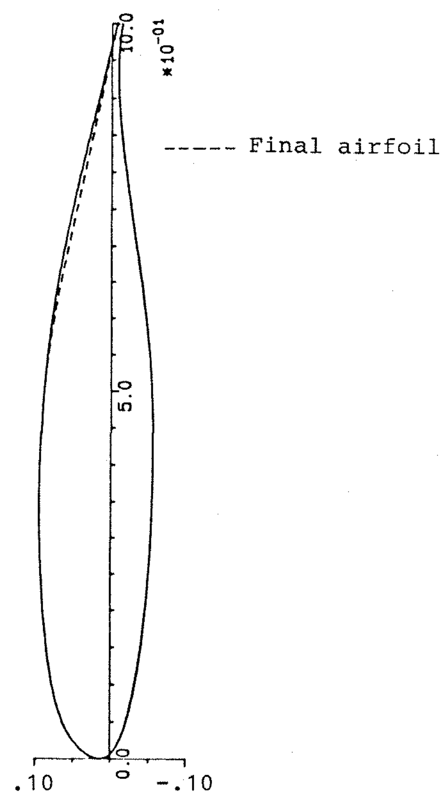

Fig. 6 Example 1 - Basis and final airfoil

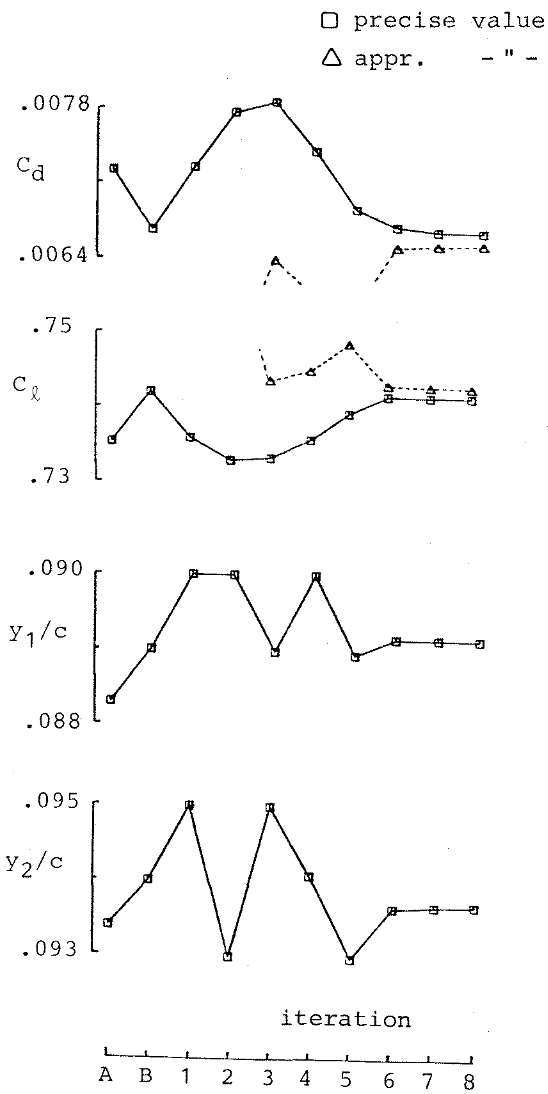

Fig. 8 Example 2 - Total drag minimization, $M = 0.65$, $Re = 13.0 \times 10^6$, $\alpha = 0^\circ$ (A and B are starting designs)

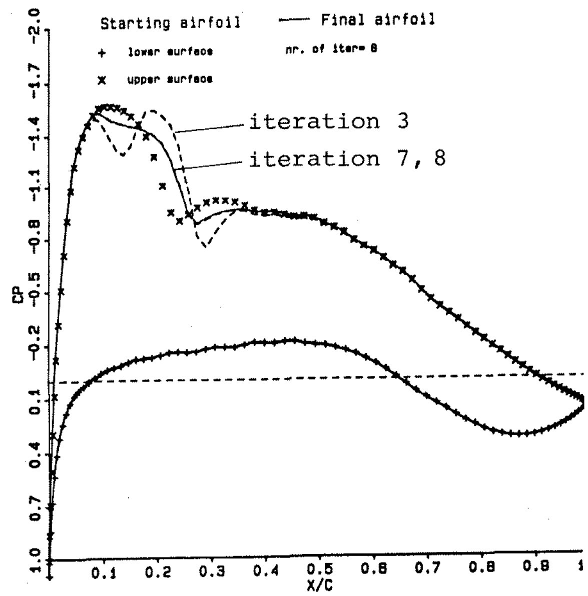

Fig. 9 Example 2 - History of pressure distribution

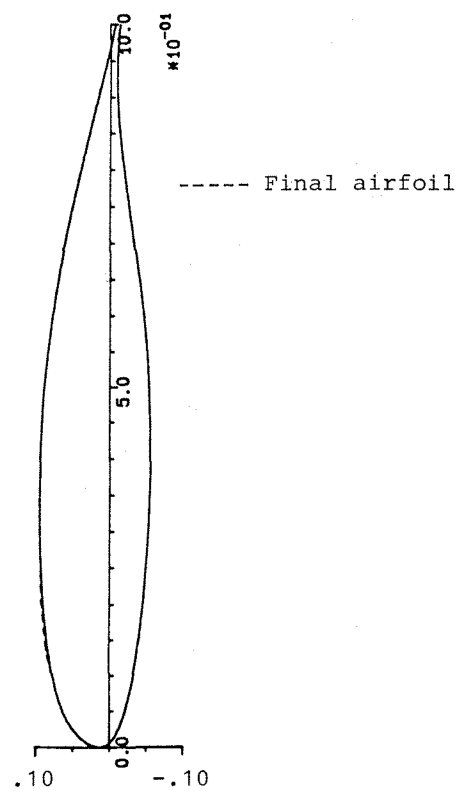

Fig. 10 Example 2 - Basis and final airfoil

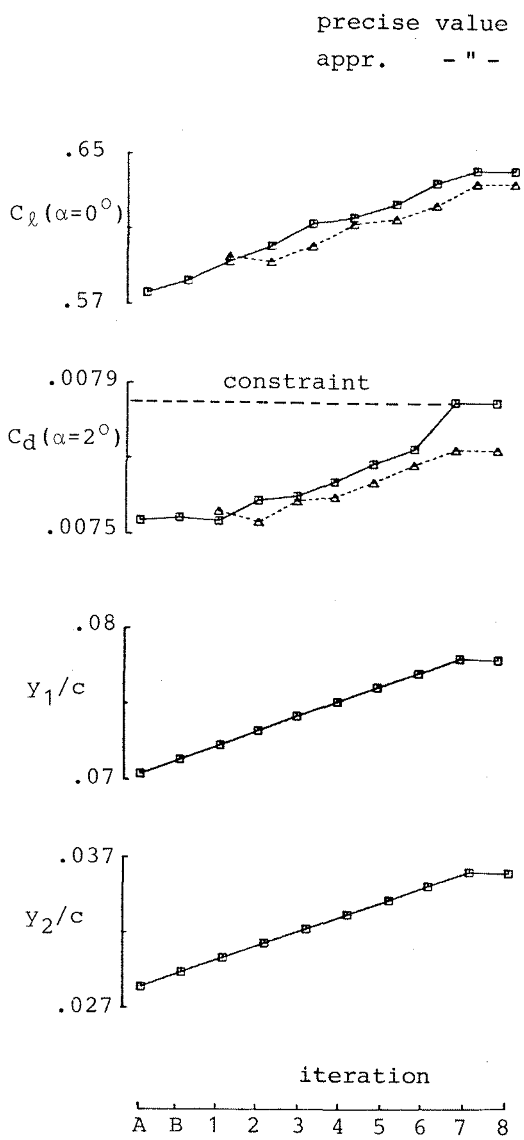

Fig. 11 Example 3 - Lift maximization, $M = 0.41$, $Re = 13 \times 10^6$, $\alpha = 0°$ (A and B are starting designs)

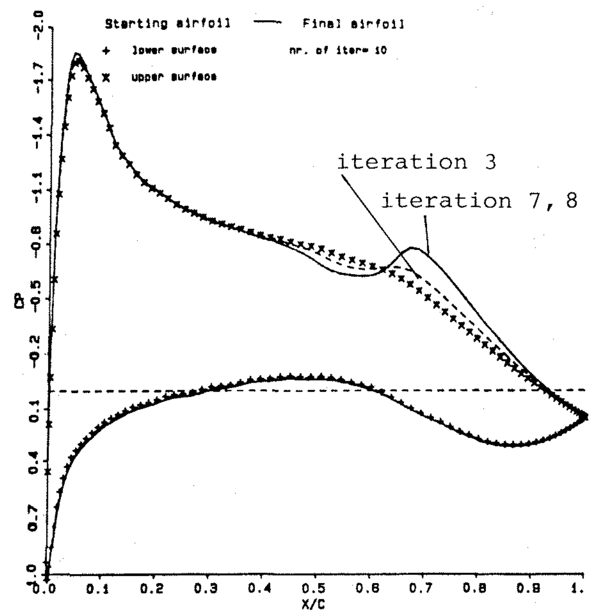

Fig. 12 Example 3 - History of pressure distribution at $\alpha = 2°$

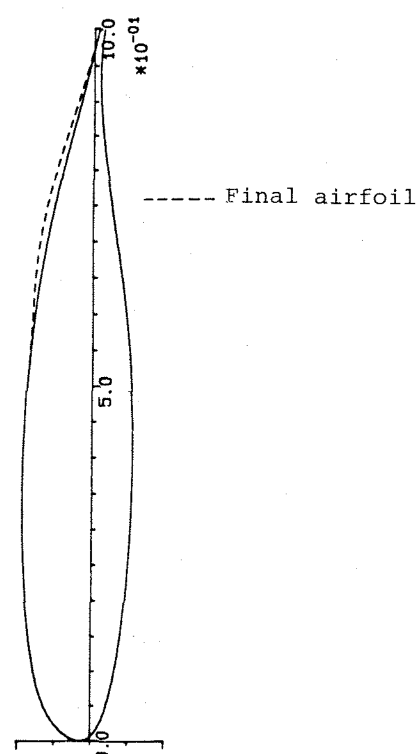

Fig. 13 Example 3 - Basis and final airfoil