Abstract

In this paper, results are described of experiments involving the burst of laminar separation bubbles in the flows around an airfoil of a large angle of attack, a circular cylinder and a cone cylinder. First, results of detailed measurements of the change in stalling characteristics of NACA 9324 as they occur with a varying Reynolds number are described, and the aerodynamic mechanism of it is discussed in connection with the occurrence of the burst of bubbles. Secondly, an experiment was conducted on the flow around a circular cylinder involving the burst of bubbles. It was found from the experiment that the drag coefficient decreases discontinuously at the critical values of the Reynolds number, contrary to the description of many textbooks and handbooks where the abrupt change of the drag coefficient at this Reynolds number is continuous. Finally, an experiment was conducted on the force acting on a cone cylinder placed in the flow at an angle of attack of 90°. The side force acting on it is discussed in connection with the existence of laminar separation bubbles.

I. Introduction

The burst of a laminar separation bubble is well-known as a cause of leading edge stall of an airfoil. This phenomenon appears also in the flow around various kinds of cylinders put to practical use, causing a sudden change of forces acting on the cylinders and is thus an important one. In this paper results are described of a line of experiments involving the burst of bubbles in the flows around an airfoil of a large angle of attack, a circular cylinder and a cone cylinder.

The commencement of the works described in this paper was the detailed measurement of the stalling characteristics of an airfoil. The stalling characteristics of airfoil sections are classified into three main types of trailing edge stall, leading edge stall and thin airfoil stall. This experiment was planned in order to understand the aerodynamic mechanism of the change of types from leading edge stall to trailing edge stall. We found that the discontinuous variations of the aerodynamic characteristics of an airfoil and hysteresis are the essential phenomena accompanying the burst of bubbles.

The drag coefficient $C_D$ of a circular cylinder is one of the fundamental subjects of fluid dynamics and has been measured since the 1910's (2)-(6). But the detailed behavior of a cylinder in the critical range of the Reynolds number has not yet been clarified. Many textbooks and handbooks describe the $C_D$ variation of a circular cylinder with the Reynolds number. In these books $C_D$ decreases gradually as the Reynolds number increases, and decreases abruptly but continuously at a Reynolds number value of about $3 \times 10^5$, thereafter increasing gradually.

In 1964, however, Tanii(7) pointed out that for a cylinder the transition from subcritical to supercritical range is much more discontinuous than previously reported. In 1969 Bearman(8) measured the base drag coefficient $C_p$ of a cylinder at Reynolds number values ranging from $2 \times 10^5$ to $7.5 \times 10^5$ and showed that the discontinuity was recorded at $Re = 3.4 \times 10^5$.

Based on the results of the experiment on the stalling characteristics mentioned above, we became convinced that the drag also varies discontinuously at the critical Reynolds number. Then we measured in detail the aerodynamic force acting on a circular cylinder at the critical values of the Reynolds number. We found that the drag really varies discontinuously. Further we confirmed that lift force appeared due to the asymmetry of the flow caused by the burst of the bubble on one surface while the bubble on the other surface remained without bursting, which Bearman had mentioned about.

The authors expected that if this lift force acts on a section of a circular cone at a condition where the Reynolds number based on the local diameter of the section is critical, it could be the origin of the flat-splay phenomenon observed in the air flow around a sounding rocket payload model, which was conducted by the National Space Development Agency of Japan. In order to examine this expectation a measurement has been made on the force acting on a cone-cylinder.

II. Stalling Characteristics of NACA 9324

1. Wind Tunnel and Model

The wind tunnel is the pilot tunnel of the NAL's large low speed wind tunnel(10). The height and width of the test section are 650 mm and 550 mm respectively.

The two dimensional model was mounted spanning the 550 mm width of the tunnel. The airfoil section decided on was an NACA 9324 according to the following principle;

1. $C_L$ max has its maximum value in the Reynolds number range of $0.5 \times 10^5$ to $4 \times 10^5$.
2. The airfoil must be known widely.

Static pressure orifices are installed at the center line and the slant orifices in Fig. 1 are installed along a line 21.5 mm from the center line; separation points are detected by measuring the pressure difference between the pair of slant orifices. These slant orifices, however, have not been used for most of the experiments due to the time consumed in this measurement.
3. Results and discussions

(1) $C_L$ - $Re$ Characteristics, hysteresis phenomena

Fig. 2 shows a variation of the lift coefficient with a Reynolds number at an angle of attack of $8^\circ$. As the Reynolds number increases, $C_L$ varies along the curve A B C D, but $C_L$ decreases along the curve D C' B' A as the Reynolds number decreases, so the $C_L$ - $Re$ curve shows hysteresis loop.

![Graph of $C_L$ vs. $Re$ at $8^\circ$](image)

(a) At point A in Fig. 2  (b) At point B in Fig. 2

(c) At point C in Fig. 2  (d) At point C' in Fig. 2

(e) At point D in Fig. 2  (f) At point B' in Fig. 2

![Graphs showing pressure distributions at NACA 9324](image)

Fig. 3 shows pressure distributions at Reynolds numbers corresponding to the points A, B, C, C', D and B'. The discussions in Ref. 11 suggest that there is no bubble at the points A, B and C, and bubbles do exist on the upper surface at the points B', C' and D. The jump from B' to B is due to the burst of a bubble and the jump from C to C' is due to the formation of a bubble by reason of the attachment of the separated boundary layer. The results for several attack angles are shown in Fig. 4.
The $\alpha_{\text{max}} - \text{Re}$ curve is also shown in this figure as the upper limit of all curves except for the portion where the laminar boundary layer separates without reattaching again (line $ABC$ in Fig. 2).

In Fig. 5 $C_L\alpha$ is plotted against $\text{Re}_\alpha$, where $\text{Re}_\alpha$ is a Reynolds number based on the displacement thickness $\delta^*$ of the boundary layer at the laminar separation point. $\delta^*$ is calculated by use of 
Thwaites's(12) method using the experimental pressure distribution, where the laminar separation point is determined from the measurement of the difference of the pressure between the pair of slant orifices.

The value of the Reynolds number corresponding to the jump of $C_L\alpha$ lies within the range of from 350 to 480, the value of which coincides with the criterion of Tani(13), Owen and Klein(14), irrespective of the attack angle and whether the Reynolds number is increasing or decreasing.

Further, the fact that hysteresis phenomena disappear where $\text{Re}_\alpha$ is taken as the abscissa, shows that the formation of a bubble due to the attachment of the separated boundary layer, occurs at the same Reynolds number of about 400 as the bursting of a bubble.

As shown in Fig. 4, the value of the Reynolds number at which a bubble bursts (point $B'$ in Fig. 2) depends on the attack angle. The solid line of Fig. 6 shows the relation between them. In other words this curve shows the locus of the point at which $\text{Re}_\alpha = 400$ holds. We will hereafter call this curve the stalling curve.

In this figure also shown is the locus of the point where the formation of a bubble occurs (point $C$ in Fig. 2) by a dotted line, which is hereby referred to as the recovery curve. The points $A$, $B$, $B'$, $C$, $C'$ and $D$ in Fig. 2 correspond to $A$, $B$, $B'$, $C$, $C'$ and $D$ in this figure.

Fig. 6 Stalling curve and recovery curve for constant angle of attack

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H and I and bubbles do exist at the points F, G, H', I' and J. The jump of $C_L$ from G to G' is due to the burst of the bubble as shown in (b) and (c) of Fig. 7. This fact is in accordance with the explanation on the stalling curve. If one crosses the stalling curve from left to right at the point G, the bubble on the surface should burst, since a Reynolds number of $Re_0$ based on the displacement thickness at the laminar separation point is lower than 400 in the region below this curve.

It is remarkable that $C_L$ increases discontinuously at the point I in Fig. 6. The pressure distributions in Fig. 8 (d) and (e) show that this jump is due to the re-formation of a bubble on the upper surface at this value of the attack angle. In Fig. 8 (f) circles show the pressure distribution at point J and X dots show that at point K in Fig. 7 (b), where the angle of attack and the value of $C_L$ for both cases are the same. The fact that these two distributions are quite different, demonstrate the existence of a bubble at point J. This fact is also in accordance with the explanation on the recovery curve.

If one crosses the recovery curve from left to right at the point I in Fig. 6, the formation of a bubble should occur on the upper surface, since a Reynolds number of $Re_0$ is higher than

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**Fig. 7** Lift characteristics

**Fig. 8** Pressure distributions of NACA 9324

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(3) $C_l - \alpha - Re$ characteristics

Fig. 10 shows the bird's-eye view of the variation of $C_l - \alpha$ characteristics with the Reynolds number. Only the cases, where angle of attack is increasing, are shown here.

According to the above discussions and Fig. 10, $C_l - \alpha - Re$ characteristics of the NACA 9324 section is summarized as follows: There are two critical Reynolds numbers; the lower critical Reynolds number $Re_{CU}$ and the upper critical Reynolds number $R_{CL}$. At a Reynolds number below $R_{CL}$ a bubble does not exist on the upper surface of the airfoil and the airfoil does not stall within the range of an attack angle below $40^\circ$. The difference between the $C_l - \alpha$ characteristics at Reynolds numbers above and below $R_{CL}$ is not

![Recovery curve](attachment:recovery_curve.png)

![Stalling curve](attachment:stalling_curve.png)

![Bird's-eye view](attachment:birdseye_view.png)
III. Aerodynamic Force Acting on a Circular Cylinder

1. Wind tunnel and model

The wind tunnel is the NAL 2m x 2m gust tunnel\(^{15}\). The wind velocity ranges from 3 m/s to 60 m/s and the turbulence level is about 0.2 percent.

The model is a circular cylinder whose diameter and span are 0.26m and 2m respectively. It is set up vertically from the floor to the ceiling. The Reynolds number based on the diameter ranges from \(1 \times 10^5\) to \(1 \times 10^6\). Static pressure orifices are installed in the center section at intervals of five degrees. Drag and lift are obtained by integrating the measured pressure on those orifices, and corrections for blockage are made.

Fig. 11 Variation of drag coefficient of a circular cylinder with the Reynolds number

Fig. 12 Variation of lift coefficient of a circular cylinder with the Reynolds number
In the critical range of the Reynolds number, the experiment has been carried out carefully varying wind velocity at intervals of 0.2 - 0.3 m/s monitoring the pressure distribution using multi-tube alcohol manometer in order to catch the Reynolds number where the drag coefficient, $C_D$, varies discontinuously.

2. Results and discussion

Fig. 11 shows $C_D - Re$ characteristics. $C_D$ varies along the curve A C C' E E' F as the Reynolds number increases and varies along the curve F D' D B' A as the Reynolds number decreases. $C_D$ varies discontinuously at CC', EE', B'D and B'B' and hysteresis phenomenon appears in accordance with the expectation mentioned above.

Fig. 12 shows the variation of the lift coefficient $C_L$ with the Reynolds number. $C_L$ is nearly zero except in the region C'E and B'D, in which the value of $C_L$ is of the order of 1. In Fig. 13 the location of the stagnation point is plotted against the Reynolds number, where $\theta_{sep}$ is the angular distance of the stagnation point from the leading edge. Fig. 14 shows the variation of the angular distance, $\theta_{sep}$, of the separation point on the upper surface from the leading edge, where $\theta_{sep}$ is determined from oil flow visualization.
tion. $\theta_{sep}$ varies along the axis of the cylinder. The vertical lines show the range of variation. Since pressure measurement has not been made in the case of oil flow visualization, the symbols in this figure do not correspond one to one with the symbols in Figs. 11-13. However the Reynolds number at which discontinuous variation occurs as shown by $B'B$ and $E'E'$ in Figs. 11-13 is essentially the same as the Reynolds number at which the jump of the separation point occurs in Fig. 14.

Fig. 15(a), (b), (c), (d), (e), (f) and (g) show pressure distributions along a stream direction corresponding to the points A, B, B', D, D', F and E in Fig. 11 respectively. It is clear from the discussions of Refs. 5 and 11 that (a) and (b) in this figure show the non-existence of a bubble on both surfaces at points A and B (no bubble flow). In this case the value of $C_D$ is about 1 and the value of $C_L$ is essentially zero. (c), (d) and (g) show that a bubble exists only on one surface. In the laminar boundary layer separates without attaching again (one bubble flow).

Fig. 13 shows that in this case the stagnation point moves to the surface where bubbles do not exist, resulting in the generation of a lift coefficient of the order of one. The flow is stable and the value of $C_D$ is about 0.4. (e) and (f) show the existence of bubbles on both surfaces at points D' and F (two bubbles flow). In this case the value of $C_D$ is about 0.2 and the value of $C_L$ is essentially zero.

The flow around a circular cylinder in the critical range of the Reynolds number may, therefore, be classified into three types: no bubble flow, one bubble flow and two bubbles flow in accordance with the result obtained by Bornman (8).

There exist, therefore, four critical values of the Reynolds number corresponding to the boundary of the types of flow, they are, $Re_{CC'}$ and $Re_{B'B}$ for increasing Reynolds number and $Re_{D'D}$ and $Re_{F'E}$ for decreasing Reynolds number, where $Re_{CC'}$ for example, is the Reynolds number corresponding to the vertical line $CC'$ in Fig. 11. The type of flow is "no bubble flow" if the Reynolds number is lower than $Re_{CC'}$. As Reynolds number increases a bubble forms on one surface at the value of the Reynolds number of $Re_{CC'}$ so that the type of flow changes discontinuously to "one bubble flow". The surface, where the bubble forms, changes from a run of the wind tunnel to another run. However once the bubble forms on a surface, the flow becomes stable so that the bubble continues to remain on the surface during the run. The reason for the stabilization of flow is as follows. If a bubble forms on a surface for some reason the stagnation point moves to the side where a bubble does not exist as shown in Fig. 13. The separation point of the laminar boundary layer on the surface without a bubble, however, does not move so much as shown in Fig. 14, resulting in the decrease of the Reynolds number $Re_{B}$ based on the displacement thickness of the boundary layer at this point. Then the formation of a bubble on this surface becomes difficult, because the $Re_{B}$ is now lower than the critical value of Tani, Owen and Klammer mentioned in Chapter II, resulting in the stable flow. The formation of bubbles, therefore, does not occur on both surface simultaneously, but occurs only on one surface at first. In Bornman's experiment the bubble formed consistently on the same surface, whereas the surface, where the bubble forms, sometimes changes from a run to another in this experiment. This fact means that the generation of lift is not due to the asymmetry of the geometry of the model or the flow in the wind tunnel. As the Reynolds number increases a bubble forms also on the other surface at the value of the Reynolds number of $B'B$ so that the type of flow changes discontinuously to "two bubble flow".
As the Reynolds number decreases the type of flow changes as "two bubbles flow" → "one bubble flow" → "no bubble flow" discontinuously. The critical Reynolds numbers, at which the type of flow changes, in the case of decreasing Reynolds number is different from those of increasing Reynolds number, resulting in the hysteresis phenomenon.

It is remarkable that in the range of the Reynolds number between R1' and Rm' two types of flow correspond to one Reynolds number. The type of flow that is realized depends on the history of the flow. The discontinuous variations of C0 and Cl can be considered as those accompanying the jump from one to the other of the two types which correspond to a Reynolds number.

The reason for the continuous variation of C0 of a circular cylinder in the critical range of the Reynolds number as described in textbooks, would be as follows. In wind tunnels the flow around a circular cylinder is not exactly two-dimensional so that the section drag acting on a section varies along the axis. In the experiments (2)(3)(4) described in textbooks the drag force acting on a cylinder was measured directly by use of a balance so that the section drag force acting on a section was integrated to give continuous variation, although section drag force itself varied discontinuously with Reynolds number as discussed above.

The discussions described so far about the measurement of the aerodynamic force acting on a circular cylinder in the critical range of the Reynolds number are summarized as follows.

1. The C0 = Re and C1 = Re characteristics has been obtained more in detail than those obtained so far.
2. It has been confirmed that C0 and C1 varies discontinuously as the Reynolds number varies.
3. The variations of C0 and C1 in the case of increasing Reynolds number are different from those of decreasing Reynolds number resulting in hysteresis phenomenon.
4. Two types of flow correspond to a Reynolds number. The type which is realized depends on the history of the flow. The discontinuity in the variation of C0 and C1 is considered to be the phenomenon accompanying the jump from one to the other of the two types of the flow.

**IV. Aerodynamic Force Acting on a Cone Cylinder**

1. **Wind tunnel, model, and test conditions**

The wind tunnel is the NAL large low speed wind tunnel(10). The height and width of the test section are 6.5M and 5.5M respectively.

Fig. 16 shows a schematic diagram of the model. The model consists of a circular cylinder, the diameter of which is 260mm, and a circular cone having the length to diameter ratio of 5 to 1. Static pressure orifices are installed at five sections, S1 - S5, as shown in this figure. In order to measure lift distributions along the axis of the cone, static pressure orifices are installed along two general lines, the angular distance from the leading edge being 80° as shown in Fig. 16(b). Each orifice is referred to as left or right as shown in the same figure.

The model is mounted vertically on the floor. The angle of attack can be varied by a hinge installed at the lower section of the model.

Measurements have been made for angles of attack of 90°, 60° and 30°. However only the results for 90° are discussed in this paper. The Reynolds number based on the diameter of the cylinder is increased from 1 x 10⁵ to 1 x 10⁶. There is no experiment with a decreasing Reynolds number.

2. **Results and Discussion**

Fig. 17 shows the variations of the side force coefficient, Cv, at each section with the Reynolds number, Rn, based on the diameter of each section. At all sections except S4 the side force coefficient is essentially zero except in a small region of the Reynolds number. At sections S3 and S4 the side force appears at small ranges of Reynolds numbers of about 4 x 10⁵ which is close to the critical value of the circular cylinder mentioned in Chapter III (see Fig. 11).
The variations of the drag coefficient, $C_D$, are plotted in Fig. 18 on the axes representing Reynolds number, $Re$, and local diameter of a cone, $D_L$. An abrupt decrease of $C_D$ appears at the same range of Reynolds numbers which suggest that the formation of the bubbles on the right and left surfaces occurs successively at small intervals of the Reynolds number in this range of Reynolds number.

Pressure distributions of section $S_3$ at the Reynolds numbers corresponding to points A, B, C, D, E and F, are shown in Figs. 19(a), (b), (c), (d), (e) and (f) respectively. It is clear from the pressure distributions that there is no bubble on the surface at points A and B. At point C a bubble forms on the right surface while the left surface remains without a bubble resulting in a large side force coefficient of 0.560 as shown in Fig. 19(c). At point D the formation of a bubble on the left surface also occurs resulting in the vanishment of the side force as shown in Fig. 19(d). At point E there are bubbles still on both surfaces so that the side force coefficient is nearly zero as shown in Fig. 19(e). The variation of $C_D$ of this section is so far very similar to that of $C_D$ on the circular cylinder mentioned in Chapter III. At point F, however, a fairly large $C_D$ appears although bubbles exist on both surfaces as shown in Fig. 19(f).

In Fig. 19 the symbols representing the pressure on the two general lines are located at $x/D = 0.41$. The pressure coefficient on these lines is hereby referred to as $C_{Pgg}$. Fig. 19 shows that $C_{Pgg}$ represents approximately the minimum pressure coefficient on the surface. From the value of $C_{Pgg}$, therefore, one knows whether a bubble exists or not on the surface, since the minimum value of $C_D$ on the surface having a bubble is much higher than that without a bubble. Fig. 20 shows the variation of $C_{Pgg}$ with the Reynolds number, $Re$, on the local diameter. It is clear that the formation of bubble occurs in the range of $Re$ around $4 \times 10^5$ in accordance with the case of a circular cylinder mentioned in Chapter III.

At a Reynolds number above $4 \times 10^5$, based on the diameter of the cylindrical part, there are bubbles on the surfaces of the cone near the corner. Close to the apex, however, there is no bubble due to a small local Reynolds number, $Re_L$, based on a local diameter. Therefore there exist boundaries between the regions with a bubble and those without. We assume that these boundaries are defined as the location, $y_p/L$, where $C_{Pgg} = -1.6$ holds, where $y_p$ is an axial distance from the apex to the boundary and $L$ is the length of the cone.

In Fig. 21 the location, $y_p/L$, of the boundaries on the right and left surfaces are plotted against the Reynolds number, $Re$, based on the diameter of the cylindrical part. The curves representing the relationship between $Re$ and $y_p/L$ for the right and left surfaces are hereby referred to as $y_p/L$ loci. The locus of the point where $Re = \frac{1}{2} \times 10^5$ holds is also shown by a solid curve in this figure, where $Re_L$ is the local Reynolds number based on the local diameter.
Fig. 20 Variation of $C_{p80}$ with the Reynolds number, $Re$, based on the local diameter

If the flow around a section of the cone behaves like a flow around a circular cylinder of the same diameter, the points which represent $y_b/L$ should move approximately along the curve $Re = 4 \times 10^5$ as $Re$ increases. The points obtained by experiment, however, move slower except for a small region of $Re$ of about $5.3 \times 10^5$ where the movement is abrupt and large.

At the Reynolds number represented by vertical line b in Fig. 21 the boundary on the right surface locates at the apex side of section $S_3$, while the boundary on the left surface locates at the opposite side. This fact means that at section $S_3$ a bubble exists on the right surface whereas a bubble does not exist on the left surface. Fig. 22(b) shows the distributions of $C_{p80}$ along the axial distance $y/L$ at this Reynolds number. The figure confirms the above description concerning the existence of a bubble at section $S_3$.

The side force coefficient $Cy$ of sections from $S_2$ to $S_3$ is plotted against the difference, $\Delta C_{p80}$, between $C_{p80}$ on the right and on the left general lines in Fig. 23. There is approximately a 1 to 1 correspondence between $Cy$ and $\Delta C_{p80}$ so that $Cy$ can be approximately evaluated from $\Delta C_{p80}$.

Fig. 22(b) shows that a side force acts to the right in the region of the cone between $y/L = 0.5$ and $y/L = 0.85$ and it acts to the left in the region where $y/L$ is greater than 0.9, where $y$ is the axial distance from the apex and $L$ is the length of the cone.

Figs. 22(a) and (c) show the $C_{p80}$ distributions at Reynolds numbers represented by vertical lines a and c in Fig. 21 respectively. In both cases the side force does not act except for the region where $y/L$ is greater than 0.9.

In Fig. 21 the regions of the cone where $|C_{p80}| \geq 0.2$, i.e. the side force is appreciable (say, $|Cy| \geq 0.2$), are shown. In the areas simply surrounded by closed curves without shade, the side force acts to the right and in the shaded areas, vice versa. The side force appears in the sections close to the shoulder of the cone cylinder ($y/L = 1$) and the sections close to the $y_b$ locus shown in Fig. 21.

At $y = y_b$, namely at $C_{p80} = -1.6$, $dC_{p80}/d(y/L)$ is very large as shown in Fig. 20, so that small
V. Concluding Remarks

Several results have been described of experiments on the phenomena including the burst and the formation of laminar separation bubbles. These phenomena are characterized by discontinuity, hysteresis and asymmetry, they are:
1. The type of flow changes discontinuously as the Reynolds number varies.
2. The Reynolds number at which the abov discontinuous change occurs is different depending on whether the Reynolds number is increasing or decreasing, resulting in hysteresis phenomena.
3. The flow sometimes becomes asymmetric even if the geometry of the body is symmetric.

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References

2. Helf, F.F., "Discussion of the results of measurements of the resistance of wires, with some additional tests on the resistance of wires of small diameter", ARC R&M 102, (1914).