NEW CONCEPTS FOR DESIGN OF FULLY-OPTIMIZED CONFIGURATIONS FOR FUTURE SUPersonic AircRFT

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I. Introduction

Some previous theoretical considerations\(^1\)\(^\text{(-12)}\) in good agreement with experimental results\(^{13}\) prove that the drag of the supersonic aircraft (S.A.) can be very much diminished (about 20 \%) if the conventional wing of S.A. is replaced with a good suited one in form of a fully-optimized wing (optimum-optimorum wing).

According to\(^2\),\(^3\),\(^4\),\(^5\),\(^6\),\(^9\) and\(^10\) a fully-optimized wing is such a wing for which all its geometrical characteristics i.e. its distributions of thickness, camber and twist and also its plan-projection are optimized in order to obtain a minimum pressure-drag.

As in\(^2\),\(^3\),\(^4\),\(^5\), and\(^10\) the determination of the shape of the surface of the optimum-optimorum wing leads to an enlarged variational problem for the drag-functional \(C_d\) which consists in the simultaneous determination of the equation of the surface of the wing (which enters in the expression of the drag-functional) and of the shape of the planform (which determines its boundary) in order to obtain a minimum for \(C_d\) (at a given supersonic cruising Mach number).

In\(^2\),\(^3\),\(^4\),\(^5\) and\(^10\) an original grafic-analytical method is presented for the approach of the solution of this variational problem for a given set of wings defined through the following properties:

their surfaces are expressed (or appro-
ached) in form of the superposition of polynomes (in two variables);  
their planprojections are polygones;  
all wings satisfy the same imposed auxiliary condition.

The optimal values of the coefficients of the polynomes and of the similarity parameters of the polygones are determined with the help of the lower-limit hypersurface (Fig.1) which is firstly introduced in (2) (for the set of delta wings) and in (5) (for the set of polygonal wings).

Each point of this hypersurface can be analytically determined through the solving of a classical variational problem as for example in (14), (15) and (16).

The position of the minimum of this hypersurface can be graphically (numerically) determined and the corresponding optimum wing for this point of the hypersurface is just the optimum-optimorum wing (of the given set of wing).

The graphic-analytical method was used in (2), (3), (4) and (10) for the effective design of an optimum-optimorum delta wing for the S.A.

For the effective design of this delta wing the results of the high conical flow theory (17) of P. Germain are used.

Further, if the hydrodynamic analogy of R. Carafoli (18), (19) is applied and the minimal singularity principle (20), (21) is taken into account then the exact solution of the boundary problem of the axial disturbance velocity \( u \) of the three-dimensional linearised supersonic flow can be written in a closed form as in (19), (1), (2) and (10) by using of singularities which are placed only along the leading edges of the wing (and, eventually, along some ridges of the wing).

The expression of the axial disturbance velocity \( u \) which satisfies the principle of the minimal singularity fulfils the asymptotic behaviour of the solution in the neighbourhood of the leading edges and also the matching condition with the boundary layer solution of the wing (in the first approximation).

This exact solution of \( u \) satisfies also the linearised boundary conditions at the infinity, of the planprojection (of the wing surface) and of the Mach cone of the apex of the wing.

The solution of \( u \) has been written in closed form in (19), (1), (2) for the flattened triangular wings at small angles of attack if the downwash \( w \) is piecewise approximated in form of superposition of homogeneous polynomes.

A comparison between the theoretical distribution of the pressure coefficients \( C_p \) determined with the help of the present theory and the pressure measured on several holes on the surface of two different models (The optimum-optimorum delta wing model Adela I Aachen and the wave rider *) in two different supersonic wind tunnels shows a very good agreement (for the range of Mach numbers for which the delta wings have subsonic leading edges). This comparison between theoretical curves and experimental points has been done in (13) and (22).

The optimum-optimorum model Adela I Aachen for which the exact solution for \( u \) is used and which is designed with the help of the graphic-analytical method presents the following advantages: reduced drag, increased lift, stabilisation of the boundary layer along the leading and trailing edges, increased ground effect etc. For this reasons the author proposed it for the S.A. of the second generation.

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II. The general theory of fully-optimized (optimum-optimorum) wings

According to (1), (14), (15) and (16) a fully-optimized wing (called also optimum-optimorum wing) is such a wing for which all its geometrical characteristics i.e. its distributions of thickness, camber and twist and also its planprojection are optimized in order to obtain a minimum pressure-drag (at the cruising Mach number). The wing must also satisfy some geometrical and aerodynamical auxiliary conditions.

The design of an optimum-optimorum delta wing for the S.A. leads to an enlarged variational problem which consists in the simultaneous determination of the equation of the surface which represents the unknown function entering in the expression of the drag-functional $C_d$ and the contour of the planform which represents its boundary, in such a way that the drag-functional $C_d$ attains its minimum.

In order to solve this enlarged variational problem an original graphic-analytical method is proposed in (2), (3), (4), (10), (11) which allows the approach of the solution by using wings which have:
- their planforms generated by polygones in order to have a finite number of free similarity parameters $(v_1, v_2, \ldots, v_n)$ and
- their surface piecewise generated by finite polynomials expansions in order to have a finite number of free parameters (i.e. the coefficients of these polynomials).

This set of lifting systems is divided into classes. Two of these wings belong to the same class if:
- their planforms are polygone which can be related through an affine transformati-
on. It follows that both planforms are defined by the same number of similarity pa-
rameters with the same significance;
- their surface-expansions contain homo-
geneous polynomes of the same degree.
- both wings satisfy the same auxiliary conditions.

The first step of approach consists in constructing the solution for one class. For a given set of similarity parameters $(v_1, v_2, \ldots, v_n)$ and an arbitrarily chosen area of the planprojection $(S_0)$ the planform is determined and the proposed variational problem with auxiliary conditions reduces to a classical one (with given boundary for the drag-functional) which can be solved analytically in the usual way as in (1), (14), (15), (16). The solution is the optimal drag-functional $(C_d)_{opt}$ for the prescribed planprojection.

Through systematical variation of the set of similarity parameters $a$, what is termed here a lower-limit hypersurface of the drag-functional i.e.

$$(C_d)_{opt} = f(v_1, v_2, \ldots, v_n), \quad (1)$$

can be analytically determined.

The "position" of the minimum of the hypersurface gives the best set of similarity parameters $(v_1, v_2, \ldots, v_n)$ of the planform (Fig. 1).

Figure 1. The lower-limit hypersurface $(\Sigma)$ for two similarity parameters

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This set of similarity parameters determines together with a chosen area \( S_0 \) of the planform the shape of the planform of the optimum-optimorum wing of the class. The optimum-optimorum wing is exactly the optimized wing corresponding to this set of similarity parameters. The set of coefficients of the surface-expansion of this optimized wing represents in the same time the set of the optimal coefficients of the optimum-optimorum wing of the class.

The minimum value of the "ordinate" of the hypersurface represents the drag coefficient of the optimum-optimorum wing of the class.

By comparison of the minimum of the optimized drag coefficients of various classes the lowest drag coefficient of all classes can be found and the corresponding wing is the optimum-optimorum wing of all classes taken into consideration.

III. New concepts for the effective design of the optimum-optimorum delta wing

The above theory can be used for the effective design of the optimum-optimorum delta wing (of a class).

The delta wing must satisfy some auxiliary conditions i.e. to have a given lift coefficient \( C_L \), pitching moment coefficient \( C_m \) and relative thickness \( \tau \).

Additionally the wing must be of null-thickness along its leading and trailing edges and, at cruising Mach number, an equalization of pressure along the subsonic leading edges is required in order to avoid the birth of vortex sheets along these leading edges.

For the cancellation of vortices along the entire subsonic leading edges according to (1) - (15) it is necessary that the planprojections of these leading edges must be straight-lines, i.e. the wing must be delta (i.e. neither gothic nor ogee) and the wing shall be cambered and twisted (i.e. the mean surface of the wing does not be plane). It is also necessary that (at cruising Mach number) the parallel flow has the direction of the tangent to the skeleton of the central section at the apex of the wing (i.e. the flow must have shockless entry) and the camber and twist distribution must be correctly coupled i.e. must satisfy some supplementary auxiliary conditions, as given in (1)-(11).

Let us here consider a class of delta wings which fulfill the following properties:

- the shape of their surfaces is approximated in form of superposition of some homogeneous polynomials in two variables with undetermined coefficients;
- their planprojections are isosceles triangles (with undetermined aspect-ratio and a free chosen area \( S_0 \));
- all wings of the class fulfill the same auxiliary conditions (mentioned above).

For this class of delta wings the planform depends only of one similarity parameter \( \nu = B \) (here is \( B = \sqrt{\frac{V}{V \infty}} - 1 \), \( \nu = \frac{l_1}{h_1} \) with \( l_1 \), the half span and \( h_1 \) the maximal depth of the delta wing) and the lower-limit hypersurface reduces to a curve which is here called lower-limit line (Fig. 2) i.e.

\[
(C_d)_{opt} \leq C_d, \quad C_m = C_{m_0}, \quad \tau = \tau_0
\]

\[
(C_d^e opt) \leq C_d, \quad V_{opt} = V
\]

Figure 2. The lower-limit line.
\[ (C_d)_{opt} = f(\nu) \]  

For a given \( \nu \) the corresponding \( (C_d)_{opt} \) which represents, the drag of the optimal wing for a given value of the similarity parameter \( \nu \), can be analytically determined as in (1), (14), (16) by the solving of a classical variational problem. The position \( \nu = \nu_{opt} \) of the minimum is graphically (numerically) obtained. The optimum-optimorum delta wing is exactly the optimized delta wing corresponding to this value of \( \nu \).

Using this method a systematical analysis of the influence of auxiliary conditions of the variational problem (i.e. \( M_\infty \), \( C_l \), \( C_m \) and \( \gamma \)) on the shape and drag of about 1000 fully-optimized delta wings has been done in (6), (7), (8) and (12).

The analysis indicates that all the fully-optimized delta wings have a convex shape in the neighbourhood of the apex and take a wave shape in the neighbourhood of the trailing edge. All wings are angular along the leading edges and sharp along the trailing edge (as in Fig. 3) i.e.

*The maximal thickness is located in the central section at the distance \( x_1 = h_1 / 4 \) from the apex.*

*In the vicinity of the trailing edge the transversal sections are narrower as in the vicinity of the central section and the maximal thickness in this transversal section is reached in two lateral points, which are symmetrically placed with respect to the central section.*

*The magnitude of the pressure drag is strongly influenced from the correct choice of:*
- the angle of aperture \( \gamma \) (of the mean profile-section at the apex of the wing); (Fig. 4) and (Fig. 5).

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![Figure 3. The fully-optimized delta wing Model Adela I Aachen](image)

![Figure 4. The dependence of \( \gamma_{opt} \) versus \( M_\infty \) (by \( \tau = 0.035 \))](image)
the aspect-ratio \( \lambda \) (Fig. 6).

In the (Fig. 4) and (Fig. 5) the dependence of the optimal aperture (\( v_{\text{opt}} \)) versus the Mach number \( M_\infty \) (by \( \tau = 0.035 \)) and the relative thickness (by \( M_\infty = 2 \)) is presented.

The optimal aperture increases when the cruising Mach number \( M_\infty \) decreases and the thickness \( \tau \) increases (for given \( v = v_{\text{opt}} \)) thickness is neglected. It results the following range for \( v_{\text{opt}} \):

\[
0.75 < v_{\text{opt}} < 0.85
\]  

(3)

The pressure drag decreases when the distance \( k \) of the pressure center (with respect of the apex of the wing) increases.

Figure 5. The dependence of \( v_{\text{opt}} \) versus \( \tau \) (by \( M_\infty = 2 \))

In the (Fig. 6) is given the dependence of the optimal value \( v_{\text{opt}} \) of the similarity parameter versus the pressure center \( k = C_m/C_\ell \). (Hereby the influence of

Figure 6. Influence of pressure center \( k \) on \( v_{\text{opt}} \).

The extensive analysis of the influence of the cruising Mach number \( M_\infty \), the relative thickness \( \tau \) the lift coefficient \( C_\ell \), and the pitching moment coefficient \( C_m \) on
the drag coefficient $C_d$ shows the importance of the correct choice of the angle $\gamma^\circ$, the position $k$ of pressure center and of the similarity parameter $\nu$ of the plan-projection.

IV. Design of a fully-optimized Model Adela I Aachen

The present theory was used in (5), (6), (9) and (10) for the design of a fully-optimized delta wing model given above (Fig. 3), called Adela I Aachen. This wing was fully-optimized for the cruising Mach number $M_\infty = 2$.

For this wing:

\[ S_u = 145 \text{ cm}^2 \] (3)
\[ \tau = 0.035 \] (4)
\[ C_d = 0.2 \text{ (by } M_\infty = 2 \text{ and } \alpha=0^\circ) \] (5)
\[ C_m = 0.157 \text{ (" " " " " " } \) (6)

Additionally the delta wing must be of null-thickness along the leading and trailing edges and the axial disturbance velocity $u$ (of the thin component of the thick-lifting delta wing) must be cancelled along the subsonic leading edges of the wing (at cruising Mach number).

In (Fig. 7) a perspective view of the optimum-optimorum delta wing model Adela I-Aachen is given.

For flattened delta wing, at small angle of attack, the downwash $w$ is related to the geometry of the wing in the following form

\[ w_a = \frac{w}{U_\infty} = \frac{2\alpha}{\delta x_1} \] (7)

Here is $w_a$ the dimensionless downwash, $U_\infty$ is the velocity of the undisturbed flow and $\alpha$ is the equation of the

Figure 7. Perspective view of the optimum-optimorum delta wing model Adela I- Aachen
wing surface.

The pressure coefficient $C_p$ is proportional to the axial disturbance velocity $u$

$$C_p = \frac{2u}{U_\infty} \quad (8)$$

The thin and the thick-symmetrical wing components of the thick lifting delta wing are considered separately as in (11) - (16) and (21).

For the thin component the dimensionless downwash $w_a$ is expressed under the form of superposition of homogeneous polynomials of order $n = 2; 3$ i.e.

$$w_a = \frac{w}{U_\infty} = x_1(w_{10} + w_{01}|y|) + x_1^2(w_{20} + w_{11}|y| + w_{02}y^2) \quad (9)$$

and for the thick-symmetrical delta wing component the corresponding dimensionless downwash $w^*$ is represented under the form of superposition of homogeneous polynomials of order $n = 1; 2; 3$ i.e.

$$w^* = \frac{w^*}{U_\infty} = w^*_{01} + x_1(w^*_{10} + w^*_{01}|y|) + x_1^2(w^*_{20} + w^*_{11}|y| + w^*_{02}y^2) \quad (10)$$

The design of this optimum-optimorum delta wing model leads to the determination of the unknown similarity parameter $\nu$ and the unknown coefficients $w_{1j}$ and $w_{1j}^*$ in order to obtain a minimum drag. The wing must satisfy in addition the auxiliary conditions (4), (5) and (6). The wing must also be of null-thickness at the leading and trailing edges and the axial disturbance velocity $u$ (due to the thin component) must be cancelled along the leading edges (at $M_\infty = 2$).

In the (Fig. 8) the lower-limit lines are plotted separately for the thin, the thick-symmetrical delta wing (component wings of the thick lifting delta wing model) and for the thick lifting delta wing. The lower-limit line of the thick lifting delta wing is to be obtained through the addition of both lower-limit lines of the component wings.

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Figure 8. The lower-limit lines of the optimum-optimorum delta wing model Adela I Aachen and for its component wings.
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The position of the minimum of the lower-limit line of the thick lifting wing determines the optimal value of the similarity parameter $\nu = \nu_{opt}$ i.e.

$$\nu_{opt} = 0.833 \quad (11)$$

and the optimal thick lifting delta wing which corresponds to this value of $\nu_{opt}$ is just the optimum-optimorum delta wing model Adela I Aachen.

The theory predicted that the here proposed fully-optimized delta wing has half frictionless drag as an equivalent planar delta wing which is flying at the same cruising speed and has the same planprojection and the same lift.

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That is the reason for which I proposed it for the S.A. of second generation.

V. The agreement between theory and experimental results

The model Adela I Aachen was tested in two supersonic wind tunnels i.e.
- the supersonic wind tunnel of the Aerodynamisches Institut of RWTH-Aachen (*)
  (test section 40 x 40 cm²);
- the supersonic wind tunnel of DFVLR-Institut für Experimentelle Strömungsmechanik
  (test section 60 x 60 cm²).

This model has 17 holes which allowed the measurement of the pressure distribution on its surface.

The pressure measurements have been done in the supersonic wind tunnel of the Aerodynamisches Institut of RWTH-Aachen.

A very good agreement between the theoretical distribution of the pressure coefficients \( C_p \) determined with the help of present theory and the pressure coefficients measured on the surface of the model (see Fig. 9) is obtained over the entire surface of the wing and for whole range of Mach numbers for which the delta wing has subsonic leading edges, i.e. in this case the Mach number of the supersonic flow doesn't exceed the following upper limit

\[
M_\infty < 2.307 \tag{12}
\]

*) Experiments done with the amiability of Prof. Egon Krause, Ph.D., Director of this Institute, together with Ing. A. Scheich and Techn. L. Simons and a group of students (H. Reimerdes, G. Bozinis and P. Bürger).

**) Experiments done with the amiability of Prof. Dr.-Ing. A. Heyser, and Dr.-Ing. G. Maurer, together with Dipl.-Ing. Esch, Dipl.-Ing. H. Riedels and Ing. H. Emunds.

Figure 9. Variation of \( C_p \) versus \( M_\infty \) for the central holes

The experimental results concerning the variation of lift coefficient and pitching moment coefficient with Mach number and incidence have been done in supersonic wind tunnel of the Institut für Experimentelle Strömungsmechanik.

The comparison between the theoretical aerodynamic characteristics (i.e. lift- and pitching moment coefficient) determined with the help of present theory and the aerodynamic characteristics measured on the model (with the strain gauge balance) are also in good agreement with each other.

The hypothesis, that there are no flow detachments along the trailing and leading edges of the fully-optimized delta wing model (at cruising Mach number \( M_\infty = 2 \) and for the range of the angle of attack \( \alpha \).
(-2 < α < 2), is also experimentally confirmed with the help of oil pictures.

In conclusion, there is a good agreement between the theoretical results obtained by using the high conical flow theory and the experimental results for the whole range of Mach numbers for which the delta wing has subsonic leading edges.

VI. New concepts for the design of second generation of fully-optimized supersonic aircraft (S.A.)

The principal conclusion of this paper is that the rentabilty of a S.A. can be achieved if the classical gothic wing of the S.A. is replaced with a good suited wing i.e. with a fully-optimized delta wing. Such a replacement has been done in (Fig. 10) for the S.A. of type Concorde.

The Concorde with fully-optimized delta wing has the same fuselage and the same wing area as the classical Concorde.

The wing was fully-optimized for the same cruising Mach number.

A geometrical comparison between the both wings provides the following differences:
- it is delta shaped (i.e. the projections of the leading edges on the planform are straight lines) and not gothic;
- it has angular leading edges and not rounded leading edges;
- the mean surface is not plane but cambered and twisted;
- all geometrical parameters of the new wing (camber, twist, thickness, planform) are optimized in order to obtain minimum drag (at the same cruising Mach number).

The Concorde with fully-optimized delta wing presents the following aerodynamical advantages:
- its pressure drag is reduced because the wing is fully-optimized (at cruising speed); it results a reduced fuel-consumption

![Comparison at take-off](image)

**Figure 10.** Comparison between a classical Concorde and the Concorde with fully-optimized delta wing, by start
(for the same trajectory, by the same cruising speed);
- its lift is increased (at the same cruising speed and for the same wing area) because of the condition of pressure equalisation along the leading edges (at cruising speed); it results in an increased possibility of transport of payload;
- its lift is also increased at all regimes of flight because of its twist, therefore this S.A. needs shorter run-ways for take-off and landing;
- its induced drag is cancelled (at cruising Mach number) and is reduced at all other regimes of flight because of the twist of the wing.
- its twist produces a lateral stream which transports additional central energy in the central part of the back region of the upper side of the wing and therefore a stabilisation of the boundary layer in this region is achieved; the detachment at the trailing edge is retarded. It results in an increased lift and a supplementary reduction of the drag.

A better concept for the design of S.A. of second generation is to optimized the whole configuration of the aircraft i.e. wing-fuselage-machineries and to design an integrated fully-optimized wing which contains inside its thickness the fuselage and the machineries.

The mean surface of the integrated wing looks like the fully-optimized isolated wing, but its thickness-distribution must be modified in order to allow the introduction of the fuselage and the machineries inside its thickness.

Literature
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