PREDICTIVE ADAPTIVE CONTROL OF A NON-LINEAR TIME-VARYING AIRCRAFT SYSTEM

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Abstract
An aircraft that is described by a non-linear, time-varying system is transferred from an initial state to a final state in a certain number of steps which result from the discretization of the entire (time-, fuel- or energy-optimized) trajectory in a sequence of points defining elementary trajectories.

The aircraft is guided from one point to another by a finite-time control vector obtained for linear systems with a quadratic performance criterion. The control time interval is chosen such that the set of accessible states of the aircraft almost always lies in the set that may be obtained by freezing the linearization in the initial point of the elementary trajectory.

At the end of the control interval the state is measured and then allows the determination of the control vector, provided that the difference between the aircraft's state and the predetermined state lies inside a tolerable error window. If this condition is not met a parameter identification is carried out.

An application is made to the in-plane, accelerated climb of a hypothetical supersonic aircraft.

I. Introduction
One of the major problems of the flight of an SST is the accelerated climb through the supersonic Mach number range. For such an aircraft flying at high altitude and therefore exhibiting relatively large time constants it may be interesting to investigate the performance of a predictive control system. This will not be done by reducing repetitively the terminal target set of the time-to-go trajectory (by a human pilot), but rather by guiding the aircraft automatically along some nominal trajectory in a certain number of steps.

In this case optimal control methods with finite-time control duration play an important role in anticipating the trajectory for some specified future state of the vehicle: the predicted vehicle control vector is then determined by a fast-time model. For reasons of simplicity and computation time saving optimal control methods for linear systems are used which minimize a quadratic cost functional. The optimal steering function for the model is converted to real time for steering of the actual vehicle.

This can be done by using some control law satisfying Erbsberg's perfect model following conditions or, for low noise vehicle environments and state measurement situations, by feedback the vehicle state only at certain time intervals, i.e., by controlling the vehicle by means of the model control vector in the open-loop manner, provided that certain requirements for the cost functional are satisfied in order to reduce the divergence between the actual trajectory and the predicted one to a minimum. This error-tolerant control system also benefits from the trajectory divergence by using it for parameter adjustment with little computation load.

The guidance and stabilization system consists of an off-line determination of a minimal trajectory used as a stored flight plan and of the on-line fast-time control vector computation and parameter identification, which may be performed by an onboard computer (fig. 1).

II. Problem Formulation
Let the dynamics of the vehicle be described by a non-linear, time- and parameter-varying, twice differentiable, first order differential equation system:

\[ \begin{align*}
    \dot{x}(t) &= f(x,u,a,t), \quad x(t_0) = x_0, \\
    y(t) &= C_1 x(t), \quad u(t_0) = u_0, \\
    t &= [t_0,t_f], (t_f > t_0)
\end{align*} \]

where \( x \) is the n-dimensional state vector, \( u \) an r-dimensional control vector, \( y \) an m-dimensional observation vector (\( m \leq n \)) and \( a \) a p-dimensional parameter vector. If the vehicle dynamics is developed into a Taylor's series about the nominal trajectory

\[ (x,u,t)_{no} \] (2)

then the linear part represents the model dynamics of the vehicle

\[ \begin{align*}
    \delta \dot{x}(t) &= A_k \delta x(t) + B_k \delta u(t) + D_k \delta a \\
    \delta y(t) &= C_2 \delta x(t)
\end{align*} \] (3)

frozen at some point \((x,t)_f\), which lies along the nominal trajectory (2) that extends between the two extremal points \((x,t)_0\) and \((x,t)_f\) from and to which the vehicle has to be transferred. \( \delta x \), \( \delta u \) and \( \delta y \) are the perturbations of the previously defined vectors, \( \delta a \) accounts for some constant parameter variation and \( A, B, C_2 \) and \( D \) are matrices with appropriate dimensions. The problem is now to steer the vehicle with the control law

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from \( (x, u, t)_{no} \) to a neighborhood close to \( (x_\nu(t), u_\nu(t), t)_{nok+1} \) with \( R/U_c = \min ((t_{r+1}-t_r), \min (t_f-t_r)) \).

The vehicle thus remains in its "linear tube" when the weighting matrices of the cost functional (5) are such that \( \min J_k \) reaches a minimum value for some fixed or reference coefficients of the matrix \( R \) as an example.

The future end point of the nominal trajectory \( (\tilde{y}, \tilde{t})_{nok+1} \) is now taken as the point next to \( \tau_{max k+1} \) and the corresponding vehicle state \( \tilde{x}_{k+1} \) is obtained according to the relation \((t \text{ denotes pseudo-inverse})\)

\[
\tilde{x}_{k+1} = [C_1 t \tilde{y}_{no} + C_2 t \tilde{y}]_{k+1}
\]

where \( \tilde{y}_{k+1} \) is the predicted steady-state vehicle vector corresponding to \( \tilde{y}_{nok+1} \), so that the \( k \)-th initial conditions for the model dynamics (3), with dropped sensitivity matrices, are

\[
\delta x(\tilde{\tau}_k) = \tilde{x}_k - \tilde{x}_{k+1}
\]

Two types of finite-time control vectors are considered for this regulator problem.

1. A continuous unbounded control vector with free terminal state

\[
\delta u(t) = -[R^k B'K(t)]_k \delta x(t)
\]

where \( K(t) \) is the solution of the Riccati matrix equation

\[
\dot{K}(t) = -[K(t)A - A'K(t) + K(t)BR^k B'K(t) - Q]_k
\]

integrated backward from \( K(t_{nok+1}) = F_k \) to \( t = \tilde{t}_k \).

2. A piece-wise continuous control vector which is obtained by the so-called direct convex feedback optimal control method and is a linear function of the initial conditions when the controls are not constrained,

\[
\delta u(t) = -[V(t)Q]_k \delta x(\tilde{\tau}_k)
\]

where the matrix

\[
V_k(t) = [\text{diag } v_1(t), \text{diag } v_2(t), \ldots, \text{diag } v_N(t)]_k, \quad (r \times (N \times r))
\]

has the property

\[
\int_{\tilde{t}_k}^{\tilde{t}_{k+1}} [V(t)V'(t)]_k dt = [r_k I], \quad (r \times r)
\]

and \( t_k = t_{nok+1}/N_k \), i.e., the control interval \( T_k = \tilde{t}_{k+1} - \tilde{t}_k \) is divided (in this case) into
N equal subintervals of length $\tau_k$, and $G_k$ is a constant ($N \times r \times n$) matrix (see Appendix I).

C. The entire trajectory

The terminal point $x_{k+1}$ of the $k$-th elementary trajectory is not an equilibrium state for the vehicle along the nominal trajectory. Thus, a fixed end state for the control vector computation is not required. Furthermore, in order to avoid too large control actions and therefore large acceleration peaks at the connection of two trajectories, the finite-time control vector (which ensures attainability of the intermediate final state at the specified control time) is prematurely interrupted at some time $t_{k+1}^i < t_{k+1}$ at which the next control vector $u_{k+1}(t)$ is applied after a new initialization of the control system at $t_{k+1}^i = t_{k+1}^i - \Delta t_{k+1}$ (where $\Delta t_{k+1}$ is the computation time of $u_{k+1}(t)$, see figure 2).

The entire control maneuver is then realized by the succession of control vectors with incomplete duration provided the vehicle-model errors remain in a tolerable error window.

IV. Parameter Adjustment Method

It is now assumed that the system definition parameters $a$ (which are intimately related to each other to give the elements of the state and control matrices, the latter being known with some accuracy at $t_k$) vary slowly during $T_k$ and remain in a bounded parameter set $A_k$ during the entire observation period $t_0$ to $t_f$. If for $t = [t_k, t_{k+1}]$ the vehicle model error becomes larger than the tolerable error, then a new control vector has to be computed at $t_{k+1}^i$ ($t_{k+1}$) with certain corrected parameter values. Obviously, the time required to perform the parameter adjustment must not exceed too much the control vector computation time. Therefore, a fast adaptation method must be found, based on the solution continuity of the ordinary differential equations with regard to parameters and utilizing only a small number of parameters in the case of a multivariable system.

As shown in Appendix II, the $p$ definition parameters can be reduced to $p_r$ classes of parameters, the variation of any one of the parameters of the same class leading the system state to the same sector. Stating it another way, the variation gives the same sign combination of the state values defining the kinematical relation for the trajectory divergence. Inversely, a certain trajectory divergence can be corrected by changing only one, i.e., the representative parameter of that class. Furthermore, when several of the parameters belonging to at least two of the $p_r$ classes are varied from their nominal values, then a qualitative identification test yields the dominant error parameter $a_q$. Next, the value of the dominant error parameter $\Delta a_q$ responsible for the vehicle-model state error $e_k$ at $t_{k+1}$ is given by the solution of the system resulting from the difference between model differential systems with and without sensitivity terms, where both models are driven by the same last control vector $\delta u_k(t)$: the trajectory divergence is thus compensated by a constant parametric deviation $\Delta a_{k+1}$ by means of the last existing model values.

Since some of the system definition parameters are identified rather than the system matrix elements, it is of course possible to adjust the vehicle to the model parameters and to keep them in a certain domain or to let them obey a specified law (e.g., the static stability margin) as in the CCV-concept. The error vector being the difference of the vehicle and model state vectors, it is obvious that linearization and freezing operations add to parameter variation effects, so that the true dominant error parameter may not always be identified. But the adjusted one always corrects the trajectory divergence completely, provided conditions III 1) to III 3) hold for the next control period.

V. Application and Results

The theory is applied to the accelerated climb of an SST-type aircraft. The state and control vectors of the in-plane rigid-body aircraft dynamics are respectively:

$$\mathbf{x} = (u, w, \phi, \theta, z), \quad n = 5$$

$$u, w: \text{body-axis speed vector}$$

$$\phi: \text{pitch angle}$$

$$\theta: \text{pitch velocity}$$

$$z: \text{altitude}$$

$$u' = (\delta_t, \delta_z), \quad r = 2$$

$$\delta_t: \text{throttle command position}$$

$$\delta_z: \text{elevator angle}$$

For the supersonic high altitude flight the atmosphere density gradient is taken into account\(^5\) so that the pair $(A, B)$ of the model and control matrices is controllable. Moreover, allowing a certain dynamic pressure error between vehicle and model, the limiting distortion condition becomes (see III B) for a negligible altitude difference

$$2\Delta u_{k+1}/\tilde{u}_k \leq \varepsilon, \quad 0 < \varepsilon < 1$$

and the $k$-th predicted maximum control duration is given by (using $a_{11}$ and $b_{12}$ of the matrices $A_k$ and $B_k$):

$$T_k = a_{11}^{-1} \begin{bmatrix} \Delta \tilde{u}_{k+1} \\ \Delta \delta_{t+1} \end{bmatrix}$$

(14)

where

$$\Delta \tilde{u}_{k+1} = u_{nc} - \tilde{u}_k$$

$$\Delta \delta_{t+1} = \delta_{t+1} - \delta_{t+1}$$

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and $u_{n+1}$ is that speed value in the nominal trajectory point set which lies closest to $x_{k+1}$, as determined by relation (15).

The choice of the weighting matrices of the quadratic functional (5) determines
- the vehicle-model following performances as stated by condition (6),
- the non-violation of the control domain (maximum throttle command) when the control vector is not constrained.

A suitable choice for the guidance task of carrying out the transfer from the initial to the final point of the elementary trajectory is found to be $P = \text{diag} P = 0$ with only $p_1, p_2 > 0$, which are the co-factors of the speed and altitude increases, respectively. The shape of the trajectory depends on the stabilization, i.e., on the vehicle behavior about its center of mass. $Q = 0$ is chosen with an $n-2(=3)$ significant element positive-definite ($Q_{k+2}>0$) core, the elements being the co-factors of the state values defining the short period motion:

$$Q = \text{diag} [0, q_2, q_3, q_4, 0].$$

Although the $Q$ is of the state weighting matrix $Q$ is to minimize the effects of the non-linear and particularly the second order terms of the series development of the vehicle equations of motion (1), $Q$ is not taken to be the Hessian matrix of the system.

The control weighting matrix is set equal to the unit matrix, $R = I$, and has to prevent excessive amplitudes of the control maneuvers as well as the violation of the throttle command limit $\delta_{\text{max}} = 1$.

For any speed and altitude increases, it is found that only $Q$ has to be adapted, $F$ remaining constant over the entire flight path. So $Q$ can be computed off-line together with and according to the nominal trajectory, so that only little changes are needed due to parametric errors between vehicle and model. Furthermore, the weighting matrices are the same for both optimal control methods.

Figure 3a shows three elementary trajectories for the continuous control vector case, starting from the level flight conditions at the altitude of 12 km and at the three Mach numbers $M = 1.2, 1.6, 2$. In the plane $(\delta M, \delta Z)$ for $\delta M = -12$, $\delta Z = -50$ m. Figure 3b shows the corresponding control vectors $(\delta x, \delta z)$ for the control duration of 17.9 s for which a small trajectory divergence is obtained although about 20% dynamic pressure difference between initial model and final vehicle values are tolerated.

Elementary trajectories initiated at the level flight conditions $M = 1.2$ and $Z = 12$ km obtained for a stepwise-continuous control vector with 5 subintervals for different final altitudes ranging from $\delta Z = 0$ to $-150$ m and for the same Mach number increase as in the previous case are shown in figure 4a. With a duration of 17.9 s computed without prescribed final altitude, it appears according to figure 4b that for $\delta Z = -150$ m the first control subinterval lies on the throttle command limit. Straight lines joining the same control vector sub-intervals for the different terminal altitudes show the proportionality with the initial conditions when the control vector is free.

Figure 5a shows an entire time-optimized trajectory starting and ending at the level flight conditions for the initial Mach number $M_0 = 1.2$ altitude $Z_0 = 12$ km and for the final Mach number $M_f = 2$ and altitude $Z_f = 18$ km. The only information the control system gets from the nominal trajectory point set is speed, altitude and flight-path angle values. For an effective total flight duration of 233 seconds (compared to the 224.5 seconds of the predeterminded trajectory duration, the difference is due to the fact that the aircraft lags during the last acceleration phase towards Mach number 2), the transition state is monitored 14 times only. The control time interval for each elementary trajectory is interrupted 5 seconds before completion and the fast-time system computation is set to 1 second. In this case, for which perfect knowledge of the vehicle parameters is assumed, the speed and altitude vehicle-model errors are kept within $\pm 10$ m/s and $\pm 40$ m, respectively (Figure 5b).

In that particular case of a 5th order characteristic equation, three classes of parameters are found (according to the kinematic condition $\delta z$) to which the qualitative test vector $e_a$ is related in the following manner:

$$e_{a1} = 1: \text{static stability margin variation } \Delta x/x \text{ dominates}$$
$$e_{a2} = 1: \text{mass stability margin variation } \Delta m/m \text{ dominates}$$
$$e_{a3} = 1: \text{lift coefficient margin variation } \Delta c_Z/c_Z \text{ dominates}$$

and the logical $e_a$ yields (see Appendix II)

$$e_a = 1/3 \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix} \text{ sgn } e_{yi} \ (15)$$

where (sgn $e_{yi}$) = (sgn $e_w$, sgn $e_q$, sgn $e_z$) denotes the sign vector of the angle of attack, pitch velocity and altitude errors, respectively. Furthermore, $e_{a1}$ accounts for trim value errors, $e_{a2}$ for $\alpha$-related density changes and dynamic pressure measurement inaccuracies, $e_{a3}$ for vertical wind shear as well as for aerodynamic distortion. For the nominal case $M_0 = 1.2, Z_0 = 12$ km, the relative sensitivity ratio is

$$\Delta x/x : -\Delta m/m : \Delta c_Z/c_Z = 1: 35: 4 \times 10^{-9}.$$
(after 20 s) for \( t > 0 \), and which also indicates the zones of the parameter dominances. Finally the identification process is demonstrated for two examples where identification occurred one second before the end of the first elementary trajectory due to an initial mass error (\( \Delta m/m = .008 \); vehicle heavier than the model, figure 7a) and a static margin error (\( \Delta x/x = .001 \), figure 7b). The corrective identification effect becomes evident on the next elementary trajectory.

VI. Concluding Remarks

Concerning the system performances it can be said that 1) elementary trajectory errors are not additive in the case of the entire trajectory, i.e., that the final state errors do not propagate from one elementary trajectory to the next one, 2) control vector dilation inaccuracies (when playing it back in real time) do not induce trajectory divergence but only a difference between effective and predicted elementary final states.

If strong perturbations or too abrupt parameter changes occur, the actual control task can be accomplished with reduced control intervals in order to smooth the trajectory divergence. If the latter can not be kept in the interior of the error window, the predicted control vector has to be played back in closed-loop form or the control interval has to be shortened again until the actual control system degenerates into a dual one, for which it may not be possible to separate control and estimation.

References


Appendix I: Convex Feedback Optimal Control Method

The theory is best explained with the aid of the linear time-varying system

\[
\begin{align*}
x(t) &= A(t) \, x(t) + B(t) \, u(t), \\
x(t_0) &= x_0, \quad t \in [t_0, t_0 + T] \\
x(n \times 1) &= u(n \times 1)
\end{align*}
\]

which has to be transferred to

\[x(t_0 + T) = 0 \quad \text{free (for the purpose of the present study)}\]

by means of the piece-wise continuous control vector of the form

\[u(t) = -V(t)G \, x_0\]

if \( u(t) \) is not bounded, so that the cost functional (5) (with adequate notation) is minimized.

Let all \( u_i(t) \), \( i = 1, r \), be decomposed simultaneously into \( N \) constant subintervals

\[u_i(t) = \sum_{j=1}^{N} \gamma_{ij} \quad \text{for} \quad [t_0, t_0 + T], \quad i.e.,\]

1) \( v_k(t) v_j(t) = \begin{cases} 1 & \text{for } t_{j-1} < t < t_j \\ 0 & \text{otherwise} \end{cases} \)

so that

\[u(t) = \sum_{j=1}^{N} \gamma_{ij} \quad \text{or} \quad u(t) = V(t)U\]

where

\[V(t) = [v_1(t)I_r, v_2(t)I_r, \ldots, v_N(t)I_r] \quad \text{and} \quad U = [U_1, U_2, \ldots, U_N] . \]

If \( u(t) = V(t)U \) is introduced into the solution of the differential system, then

\[X(t) = X_0(t, t_0)x_0 + X(t, t_0)u\]

where

\[X_0(t, t_0) = A(t - t_0)^{T} \quad \text{for} \quad (n \times n) \quad \text{and} \quad (n \times rN) \quad \text{for} \quad t \in [t_0, t_0 + T] \]

\[X(t, t_0) = \int_{t_0}^{t} \Phi(t, t') B(t') V(t') dt' \]

these two matrices are continuous over \( t \in [t_0, t_0 + T] \).

The cost functional becomes

\[J(u(t)) = x'_0 J(0)x_0 + 2u'Bx_0 + u'Cu\]
1) J(u(t)) is quadratic in U.
2) $\beta$ and C do not depend on the initial conditions $x_0$.
3) In the actual case $F$, $Q > 0$ and $R > 0$ ($\beta$ can be identically zero, provided that one of the matrices $F$ or $Q$ does not vanish entirely), the symmetrical matrix $C$ ($rN \times rN$) is non-singular.

Therefore $J(u) = 2(CU - 8x_o) = 0$
and $u(t) = -V(t) G x_0$

where $G = C^{-1} \beta$ , ($rN \times n$).

The expressions for $\beta$ and $C$ are

$$\beta = X'(t_o + T, t_o)FX_o(t_o + T, t_o) + \int X'(t, t_o)Q(t)X(t, t_o)dt = J(F) + J(Q) + J(R),$$

$$C = J(F) + J(Q) + J(R),$$

with

$$J(F) = X'(t_o + T, t_o)FX_o(t_o + T, t_o)$$
$$J(Q) = \int X'(t, t_o)Q(t)X(t, t_o)dt$$
$$J(R) = \int V'(t)R(t)V(t)dt$$

For a bounded control vector of the type $|u(t)| , |U| \leq M$

if, after computation of $U$ it is found that

$$|U_{i,j}| > M_i , i = 1, r , j = 1, N$$

the computation is performed again with

$U_{i,j} = M_i$

until all

$$|U_{i,j}| < M_i.$$
Fig. 1 Block diagram of the predictive control system

Fig. 2

Fig. 3a Elementary trajectories for $Z_0 = 12$ km and $\delta t_{no} = 0.9$

Fig. 3b Continuous control vectors
Fig. 4a Elementary trajectories for different $\delta Z_0$ at $Z_0 = 12$ km, $M_0 = 1.2$ and $\delta t_{no} = .9$

Fig. 4b Step-wise continuous control vectors ($N = 5$, $T = 17.9$ s)

Fig. 5a Entire time-optimized trajectory
($\delta t_{no} = .9$; $M_0 = 1.2$, $Z_0 = 12$ km; $M_f = 2$, $Z_f = 18$ km; final vehicle state after 14 steps: $M = 2.011$, $Z = 18.074$ km; continuous control vector)

Fig. 5b Speed and altitude vehicle-model errors at the end of each step for the entire time-optimized trajectory
Fig. 6 Visible part of the iso-error plane ($\delta z, t = \text{const}, > 0$) in the $p_1$-class parameter space

Fig. 7a
Examples of trajectories with parameter adjustment occurring at the end of the first step due to initial mass error (fig. 7a) and static margin error (fig. 7b) for $Z_o = 12$ km, $M_o = 1.2$ and $\delta t_{no} = .9$