NONLINEAR MODEL IDENTIFICATION OF A SMALL-SCALE UNMANNED ROTORCRAFT

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Abstract
This paper illustrates a non-linear model for the dynamic simulation of small-scale helicopter and the activities carried out for the identification of the unknown parameters. Recently, the University of Pisa has undertaken activities aiming to the development of small-scale unmanned systems starting from small commercial RC model helicopters. In this context, the simulation plays a crucial role, for both performance characterisation and flight control laws design, and accurate mathematical models of such vehicles are required. The simulation model of the helicopter dynamics has been developed in Matlab/Simulink environment by minimising the number of parameters to be identified. To this end, a detailed mathematical model for the actuation system of the blades collective and cyclic pitch has been developed, which takes into account the Bell-Hiller mixer effects. The model relating the three servo-actuators rotations to the blades pitch is based on complex nonlinear equations, while the rotor aerodynamics has been modelled with the blade momentum theory, coupled to the inflow Glauert predictions. Concerning the identification process, a database has been collected by carrying out specific flight tests with the helicopter equipped with a GPS, inertial sensors and a data-acquisition system. Contrary to the literature examples, the proposed identification method operates on a nonlinear model in the time domain, rather than on linear models and (often) in the frequency domain. The proposed approach leads to an identification based on a minimum set of unknown parameter and assures a satisfactory matching between simulation predictions and experimental data on the whole flight envelope.

1 Introduction
Among the Unmanned Aerial Vehicles (UAVs), there is a growing interest in developing unmanned autonomous helicopters. The helicopter has unique capabilities, such as to take-off and land vertically, to maintain hovering for an extended period of time, broad envelope of flight, high maneuverability. These abilities allow a wide range of applications, both in civilian and in military field. In military application, rotary-wing UAVs (RUAVs) have been tested for urban and coast surveillance, search-and-rescue missions, zone patrol, ELINT/SIGINT, spying mission. For civil application, small autonomous helicopters can be used for law enforcement and emergency service (Police, Civil Security, medical transport), firefighting, emergency rescue (e.g. mountain rescue), environmental monitoring, climate monitoring, aerial photography, mapping and surveillance.

On the basis of the skills in the field of the modern flight control sensors [1][2][3][4][5][6][7] and actuators [8][9][10], gained in last ten years, the aerospace division of University of Pisa has begun research activities that aim at developing unmanned rotorcrafts starting from small commercial helicopters models [11]. Among these activities, the simulation plays a crucial role, in particular for the flight control law synthesis. This work deals with the development of accurate mathematical models able to characterize the dynamic response of such vehicles.

There are several reports in literature on system identification of model-scale helicopters [12][13][14][15][16][17], by using an approach based on discrete stability derivatives and
frequency domain identification. In this work, a non-linear dynamic model of the rotor and of the whole helicopter has been developed, without using stability derivatives, and the model parameters have been identified with a time-domain process. This approach has several advantages: the identification should not be repeated for every flight condition, the pilot feedback during experiments does not degrade information about the plant and correlation among inputs (for helicopters, every longitudinal manoeuvre excites lateral and yawing dynamics, and so on) does not bias results. In this way the first-principle modelling approach and the identification process have been used in a complementary way: first-principles modelling provides the basic understanding of the involved physical phenomena, introducing some parameters that cannot be measured directly but that the identification is able to determine using experimental evidence.

2 Helicopter Description

2.1 General Characteristics

The T-REX 500 aerobatic helicopter (Fig. 1) has been used for the identification process. The T-REX 500 is a small rotorcraft popular among hobby pilots for aerobatics: it is highly maneuverable and it is suited for studies on guidance algorithms and high-frequency dynamics. The mass and the moments of inertia have been measured with high accuracy by means of pendulum experiments [18] and verified with a CAD model. The main characteristic of T-REX 500 is the rigid hingeless rotor head with carbon fiber blades. The flapping motion is allowed only by the blade elasticity and by the damper rubber O-ring of the feathering shaft (the first one gives only a small contribution, because of the high rigidity of composite material). The rotorcraft is equipped with a stabilizer bar (also known as Bell-Hiller bar or, more commonly among hobbyists, as flybar) and an Active Helicopter Tail Control System (AHTCS). The stabilizer bar is a secondary rotor consisting in two paddles connected to the main rotor shaft by an unrestrained teetering hinge. It receives only cyclic input from the swashplate, and its flapping motion influences the main rotor blades pitch via the Bell-Hiller mixer bar. The stabilizer bar is used to generate a control augmentation to the main rotor cyclic input and realizes a "mechanical feedback" in angular rates $p$ (roll) and $q$ (pitch). The damping in pitch rate and roll rate derives from the gyroscopic moment acting on the flybar. The Active Helicopter Tail Control System is made of a single-axis gyro (Silicon Micro Machines sensor) that senses the yaw angular rate $r$ and a micro-processor. It assists the pilot to compensate any unintended yaw, induced by helicopter itself during manoeuvres. The tail rotor generates a thrust to counter the main rotor torque. Its tip speed is nearly equal to that of the main rotor.

Fig. 1. RC model helicopter T-REX 500

2.2 Servo-motors

The helicopter is provided with three identical servo-motors (called SA, SP and SE) to move the swashplate through levers, and a servo-motor near the tail for implementing the tail blade collective pitch. Before developing a model that links the servos analog inputs with blade pitch, the identification of the servo-motors has been performed. By setting several values of the command input signal, the rotation of the servo horn has been measured with a digital clinometer. The results showed that the relationship between analog signal and rotation angles of servo horn is close to be linear. For the tail servo, it has not been possible to carry out the same test, so a linear relationship between
input and rotation has been assumed and negligible errors are expected.

2.3 On-board Instrumentation

The T-REX 500 has been equipped with on-board instrumentation able to record high-quality flight data:
- a Digital Signal Processor board, provided with a X-BEE module, able to transmit real-time data to the ground control station during flight, and a SD memory card for data storage
- a MEMS-based Inertial Measurement Unit (IMU) integrating three gyroscopes, three accelerometers and three magnetometers
- a 4 Hz - 50 channels GPS receiver
- a barometric pressure sensor for altitude and a temperature sensor
- a resolver installed on the main shaft
- a rotor speed governor

The IMU is mounted on a rubber suspension, to reduce the level of vibrations on sensors.

3 Helicopter Dynamic Model

3.1 Rigid body equations of motion

The rigid body motion equations have been used for the dynamic simulation of the model-scale helicopter. The first and second Newton-Euler’s equations are:

\[
\begin{align*}
X &= m(\dot{u} + qw - vr) \\
Y &= m(\dot{v} + ur - pw) \\
Z &= m(\dot{w} + pv - uq) \\
L &= I_{xx}\dot{p} + I_{xy}(pr - \dot{q}) + \\
&\quad - I_{xz}(\dot{r} + pq - qr(I_{xy} - I_{zz})) \\
M &= I_{yy}\dot{q} - I_{xy}(\dot{p} + qr) + \\
&\quad - pr(I_{zz} - I_{xx}) + I_{xz}(p^2 - r^2) \\
N &= I_{zz}\dot{r} - pq(I_{xx} - I_{yy}) + \\
&\quad + I_{xy}(q^2 - p^2) + I_{xz}(qr - \dot{p})
\end{align*}
\]

where \( X, Y \) and \( Z \) are the resultant of aerodynamic and gravitational forces, \( L,M \) and \( N \) are the moments of aerodynamic forces, \( I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz}, \) and \( I_{zz} \), are the components of inertia tensor matrix, \( u, v \) and \( w \) are the linear speeds, \( p, q \) and \( r \) the angular rates and \( m \) the helicopter mass.

3.2 Kinematic Model of the Main-rotor Command Chain

A mathematical model for the main rotor command chain has been developed. The inputs of this model are the analog signals of the three servo-motors and the outputs are blade pitch and stabilizer bar pitch, both functions of the blade position angle \( \Psi \). This model allows no additional parameters need to be added to the identification process, differently from the approach used in [12-17]. The developed kinematic model bases on the following assumptions:

- Rods, control arms, swashplate and holders are considered rigid bodies; only in quick and abrupt manoeuvres, the longer rods can bend, but this effect can be considered negligible;
- No backlash is considered; this is a strong hypothesis because the mechanical backlash plays a key role in the kinematics of a hand-assembled machine, but no simple model can be used to successfully represent this phenomena.

It is worth noting that no small angles assumption has been introduced in order to better represent the dynamics also at the maximum blade angle of attack (about 12 degrees). Examples of the simplified schemes used for the main rotor command chain model, with rods’ lengths and angles, are shown in figures 2 ÷ 5. The system of equations so obtained is composed of 21 non-linear equations in 21 unknowns that are solved, at each integration step, with the Newton-Raphson iterative algorithm. Rods and control arms lengths have been measured with a digital caliper, while angles have been measured with a digital clinometers. The inputs of this model are the stabilizer bar flapping angle \( \beta_{fb} \) and the
The analog signal of the three servo-motors; the only outputs useful to simulate blade and flybar dynamics are the blade pitch $\theta_{bl}$ and the flybar paddles pitch $\theta_{fb}$. The others unknowns are not taken into account in the helicopter dynamics simulation model.

3.3 Main Rotor Dynamics

The control forces and moments of a rotorcraft are produced by the main rotor and tail rotor. A rotor is a dynamic system that responds both to control inputs and to helicopter dynamics. This coupling between rotor dynamics and helicopter dynamics is a key characteristic, above all for small vehicles, where high frequency modes are fundamental for an accurate helicopter dynamics simulation. Two separated models, to be coupled to helicopter rigid body dynamics, have been developed for the rotor dynamics: the blade flapping model and the stabilizer bar flapping model. The main hypotheses are:

- rotor blade is rigid in bending and torsion; it can be considered a symmetric body (the shape of the airfoils gives a negligible contribute to centrifugal
moments of inertia), so its inertia tensor is a diagonal matrix;

- command chain and blade elasticity are neglected; this is a weak assumption, but the rod elasticity can be considered important only in abrupt manoeuvres, where the simple linear aerodynamic model is not reliable;
- command chain backlash is neglected; this is a strong assumption because it has been demonstrated experimentally that it has great importance;
- drag coefficient and lift coefficient of airfoil are independent of local blade angle of attack (mean values for NACA 0012 airfoil have been used);
- lead-lag motion due to the Coriolis forces induced by flapping motion, causes small forces on the hub and they have been neglected;
- both the flapping angle and the inflow angle have been assumed to be small;
- the effects of the helicopter dynamics on the blade flapping are limited to those due to roll and pitch angular accelerations, roll and pitch rates, z-axis acceleration and longitudinal and lateral velocities;
- the reversed flow region was ignored, as the compressibility and stall effects;
- the Glauert theory is used for inflow function:

$$v_i = v_{i0} \cdot (1 + K_{ip} \cdot x \cdot \cos \psi)$$  \hspace{1cm} (2)

- Effects of inflow dynamics theory on flapping dynamics are neglected;
- The tip loss factor has been assumed to be 1 (root-cutout effect is neglected).

Because of these assumptions, the results of this analysis are valid only in a limited range of conditions. However, it can be demonstrated that the results are usually valid for rotorcraft simulation up to an advance ratio $\mu$ of 0.2 (examined advance ratios are lower than this value).

3.3.1 Blade Flapping Dynamics

The flapping differential equation of motion has been explicitly derived for a two-blade rotor, starting from the Euler’s equations [19].

Supposing small flapping angles $\beta$ and neglecting second order terms, the j-axis component of Euler’s equation is:

$$\dot{\beta} + \left[ K_\beta + \Omega^2 \left(1 + \frac{m_d x_e R}{B}\right)\right] \beta = -\dot{\psi} \sin \psi$$

$$+ \dot{\psi} \cos \psi - 2\Omega (q \sin \psi + p \cos \psi) \left(1 + \frac{m_d x_e R}{B}\right) + \frac{m_d x_e}{B} (\dot{w} - uq + pv) + \frac{M_j}{B}$$  \hspace{1cm} (3)

where $m_{bl}$ is the blade mass, $\Omega$ the rotor angular speed, $e$ is hinge offset, $K_\beta$ the stiffness of the flapping hinge, $x_e$ the blade CoG position along the i-axis component, $R$ the rotor radius and $B$ the blade moment of inertia in flap.

The aerodynamic moment $M_A$ have been evaluated by knowing the velocity components of the blade with respect to the air [19]. The velocity components are evaluated with reference to a particular hub-plane reference system (HP), wherein axis $z_{HP}$ lies on the rotor shaft axis (up-direction) and $x_{HP}$ is direct as the projection on hub-plane of the helicopter velocity $V_\infty$ ($\alpha_{HP}$ is the angle of attack w.r.t. HP).

By neglecting the spanwise component of air velocity, it is usual to define as tangential velocity $U_T$, positive when blows from front to back, the component along $x_{HP}$, and as perpendicular velocity $U_P$ the component along $z_{HP}$, positive it blows from wing underside to the upper surface:

$$U_T = \Omega x + V_\infty \cos \alpha_{HP} \sin \psi$$  \hspace{1cm} (4)

$$U_P = V_\infty \sin \alpha_{HP} - V_\infty \cos \alpha_{HP} \beta \cos \psi +$$

$$- x \frac{d \beta}{d \psi} - \dot{v}_i - p_{\beta_i} x \sin \psi + q_{\beta_i} x \cos \psi$$

$$p_{\beta_i} = p \cos \beta_H + q \sin \beta_H$$

$$q_{\beta_i} = -p \sin \beta_H + q \cos \beta_H$$  \hspace{1cm} (5)

where $x$ is the distance between generic blade section and hinge position, and $\beta_H$ is the helicopter angle of sideslip. The blade pitch changes according to the law:

$$\Theta_{\beta_i} = \dot{\theta}_\beta - A \cos \psi - B \sin \psi$$  \hspace{1cm} (6)
where \( \theta_0 \) is the collective pitch angle, \( A_l \) and \( B_l \) the lateral and longitudinal cyclic pitch.

The elementary aerodynamic flapping moment \( dM_A \) about the hinge is:

\[
dM_A = \frac{1}{2} \rho a U_T^2 \left( \Theta + \frac{U_r}{U_T} \right) c x dx \tag{7}
\]

By using the Eq. (4-7) in Eq. (3) and neglecting the terms containing \( e^3 \), \( e^4 \) and higher terms, the differential equation of blade flapping is obtained, that is a linear equation with periodic coefficients. It is valid only for the advancing region, since in the reverse flow area the lift and flapping moment are incorrectly evaluated, but this is a not relevant error. To obtain a simplified and more practical form of the equation for numerical simulation, the flapping is approximated by the first-harmonic terms with time varying coefficients:

\[
\beta_\psi(t) = a_0(t) - a_{1s}(t) \cos \psi - b_{1s}(t) \sin \psi \tag{8}
\]

where \( a_{0}(t), a_{1s}(t) \) and \( b_{1s}(t) \) are the blade flapping coefficients. Equating, respectively, the constant terms and the terms with \( \sin \psi \) and \( \cos \psi \) in the differential equation of blade flapping and using Eq. (8), the tip-path plane dynamics equations is obtained:

\[
\ddot{a} + \Omega \mathbf{D} \mathbf{a} + \Omega^2 \mathbf{K} \mathbf{a} = \mathbf{f} \tag{9}
\]

where \( \mathbf{a} = [a_0, a_{1s}, b_{1s}] \) is the unknowns vector, \( \mathbf{D} \) is the damping matrix, \( \mathbf{K} \) is the stiffness matrix and \( \mathbf{f} \) is the forcing function vector.

### 3.3.2 Stabilizer Bar Flapping Dynamics

By proceeding in a similar manner as made for main rotor blade flapping, a second order differential equation is obtained:

\[
\ddot{a}_b + \Omega \mathbf{D}_b \mathbf{a}_b + \Omega^2 \mathbf{K}_b \mathbf{a}_b = \mathbf{f}_b \tag{10}
\]

where \( \mathbf{a}_b \) is the flybar flapping state vector, \( \mathbf{D}_b \) is the damping matrix, \( \mathbf{K}_b \) is the stiffness matrix and \( \mathbf{f}_b \) is the forcing term.

### 3.4 Rotor Forces and Inflow Ratio Evaluation

The calculation of the thrust coefficient \( t_c \) and the inflow ratio \( \lambda_{i0} \) is performed by following the model proposed by [19], in which the inflow on the rotor disk is assumed to be uniform:

\[
t_c = \frac{T}{\rho s A_l \Omega^2 R^2} = \frac{a}{4} \left[ \theta_0 \left( \frac{2}{3} + \mu^2 \right) + \mu \lambda_{i0} - \lambda_{i0} \right] \tag{11}
\]

\[
s = \frac{2c}{\pi R} \text{ (rotor solidity)}
\]

\[
\mu = \frac{w}{\Omega R} \text{ (normal airflow component)}
\]

\[
\lambda_{i0} = \frac{st_c}{2\eta_w \sqrt{\mu^2 + (\mu - \lambda_{i0})^2}} \tag{12}
\]

where \( A \) is the main rotor area, \( \mu \) is the advance ratio and \( \eta_w \) is the coefficient of non-ideal wake contraction (according to the momentum theory, the rotor wake far downstream contracts by a factor of two and this parameter accounts for non-uniform velocity and pressure distribution in the wake).

In this work, at each integration step, the system composed of the Eqs. (11÷12) is solved with the Newton-Raphson iterative algorithm to evaluate the \( \lambda_{i0} \) and \( t_c \). Known the inflow ratio, it is possible to evaluate the rotor force component perpendicular to thrust axis, the \( H \)-force, whose coefficient is given by [19]:

\[
h_c = \frac{H}{\rho s A_l \Omega^2 R^2} = \frac{1}{4} \mu \delta + \frac{a_{1s}}{4} \left[ \frac{1}{2} (a_{1s} + B_1) - \mu \delta \right] \tag{13}
\]

### 3.5 Fuselage Forces

The drag forces can be modeled by means of constant coefficients, both when the forward speed is higher than the rotor induced velocity and when the forward velocity is below the induced velocity. In this last case, the rotor down-wash is deflected by the forward velocity and this deflection creates a force opposing the movement [12]. The drag forces have been modeled as follows:


\[ X_{\text{fus}} = -\frac{1}{2} \rho V_{\infty}^2 \frac{u}{V_{\infty}} S_{x_{\text{fus}}} \]

\[ Y_{\text{fus}} = -\frac{1}{2} \rho V_{\infty}^2 \frac{v}{V_{\infty}} S_{y_{\text{fus}}} \]

\[ Z_{\text{fus}} = -\frac{1}{2} \rho V_{\infty}^2 \frac{w + \lambda_0 \Omega R}{V_{\infty}} S_{z_{\text{fus}}} \]

\[ V_{\infty} = \sqrt{u^2 + v^2 + w^2} \]

where \( S_{x_{\text{fus}}} \), \( S_{y_{\text{fus}}} \) and \( S_{z_{\text{fus}}} \) are the effective drag areas of the helicopter to be identified.

3.6 Tail Rotor Model

Contrary to the approach used for the main rotor, a linear model has been used to evaluate the tail rotor thrust in order to avoid complicating excessively the helicopter simulation software. The tail rotor thrust is calculated as follows:

\[ T_t = K_{\text{HLS}} S_{T_{\text{HLS}}} + K_v v + K_\mu \mu + K_p p + K_q q \]  

(15)

where

- the tail rotor command gain \( K_{\text{HLS}} \) takes into account the contribute of tail rotor command \( S_{T_{\text{HLS}}} \), i.e. the pilot requested heading, modified by the head-lock system; \(^1\)
- the sideslip velocity gain \( K_v \) adds the contribute of the sideslip angle to the local tail rotor inflow;
- the advance ratio gain \( K_\mu \) takes into account the variation of \( \mu \) during flight;
- the rolling and pitching gain \( K_p \) and \( K_q \) add the contributes due to tail rotor offset from the center of gravity.

All these parameters, included those of the head-lock system controller, have been determined during the identification process.

4 Identification of the Dynamic Model

The identification methodology followed for the T-REX 500 is essentially a fitting process between outputs data of simulation tests carried out with the helicopter dynamics simulation model and data recorded during flight tests. An analysis of the flight tests has been performed in order to select portions of the time histories to be used in the identification process. The guide line has been to use the time histories samples characterized by command inputs “exciting” the main helicopter dynamics, so to be representative of pilot commands used for the rotorcraft control. Flight test portions lasting 10 ÷ 20 seconds have been chosen, with particular attention to flight phases characterized by transitions between the hovering flight and forward flight (or lateral flight), with the final flare to take back the helicopter in the hovering condition. As far the cost function is concerned, it includes the helicopter accelerations \( a_x, a_y \) and \( a_z \), and helicopter angular rates \( p, q \) and \( r \) because they are strongly connected with forces and moments, which in turn depend on the parameters to be identified. The local cost function for the generic time \( t \) is:

\[ f(t) = \sum_{k=1}^{6} \frac{\varepsilon_k(t)}{\max_t(\varepsilon_k)} = \sum_{k=1}^{6} \frac{|y_k(t) - z_k(t)|}{\max_t(\varepsilon_k)} \]

(16)

where the \( y_k \) is the model outputs vector and \( z_k \) is the measurements vector. The components of both vectors are \( a_x, a_y, a_z, p, q \) and \( r \).

In this work the identification problem has been solved in Matlab/Simulink environment by using the pre-defined Matlab routine lsqnonlin.m in order to evaluate the parameters minimizing the cost function \( f \) over a defined time range.

4.1 Identification Results

The parameters identified for the T-REX 500 are reported in Tab. 1. As shown in the results illustrated in this section (figures 6 ÷ 9), the time responses predicted by the model with the identified parameters show a very good agreement with the time histories of the flight tests.

\(^1\) The head-lock system has been modeled as an heading hold autopilot, having a PID controller (\( K_p \), \( K_d \) and \( K_i \) are the proportional, derivative and integrative gains respectively)
It is worth noting that the matching between model response and flight time histories is kept on period longer than 5 sec, target value considered in [15], where the identification process is applied in the frequency-domain. Actually, the identified stability derivatives are valid only for flight conditions near to the initial conditions. If this hypothesis is not verified, frequency-domain model could present rapid deviation from expected data model if the identification process has not been repeated for several flight conditions (i.e. hovering, forward flight, take off, etc.). On the contrary, in our case, the identified parameters of the non-linear model are valid without restriction in the entire rotorcraft flight envelope. It has been demonstrated that the developed model simulates correctly the helicopter dynamics also for 50 ÷ 100 sec.

5 Conclusions

This work has been focused on the development, identification and validation of a dynamic model for a small-scale rotorcraft with aerobatic capabilities. A new approach is proposed and it bases on the following points:

- the command chain has been studied with a kinematic model, differently from how is made in a frequency-domain identification approach where command derivatives are used and needed to be identified;
- the first-principles modeling approach and the system parameters identification has been used in a complementary way: the first one provides the necessary understanding of the physics involved in the rotorcraft dynamics; the second one provides a means to identify the introduced parameters that cannot be measured directly.
- the identification process has been performed in time-domain; this has several advantages and in particular it has been demonstrated that the identification results are valid for a wide range of flight conditions (identification is not to be repeated for the different flight phases);

The developed non-linear mathematical model predicts with high accuracy the rotor and helicopter dynamics, including the effects of stabilizer bar and the Active Helicopter Tail Control System (AHTCS). It has been demonstrated that the first-order blade flapping differential model predicts with good accuracy the rotor dynamics by reducing the computational time with respect to that needed for the unreduced model. No inflow dynamic model is used because the influence on rotor dynamics for small-scale rotorcraft is negligible. As far as the model accuracy is concerned, the maximum errors are about 10 deg/sec for angular rates and 5 m/sec² for body-axes accelerations. These results can be considered satisfactory for the scopes of the project, where the model will be used as a test bench to validate the control laws, before using them in flight.
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Fig. 6. Roll rate comparison

Fig. 7. Pitch rate comparison

Fig. 8. Yaw rate comparison

Fig. 9. Linear accelerations comparison

References


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