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# AN ANALYTICAL SOLUTION TO THE ELASTIC-PLASTIC BEHAVIOR OF METAL MATRIX COMPOSITES UNDER TENSILE LOADING

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## Abstract

*An analytical approach is proposed for studying the elastic-plastic behavior of short fiber reinforced metal matrix composites under tensile loading. In the proposed method, a micromechanical approach is employed, considering an axi-symmetric unit cell including one fiber and the surrounding matrix. First, the governing equations and the boundary conditions are derived and the elastic solution is obtained based on some shear lag type methods. A plastic deformation is considered for the matrix under each small tensile loading step. Then, applying the successive elastic solutions method, all the plastic strain terms are obtained for the matrix. Thereafter, the elastic-plastic stress transfer behavior of the composite is studied considering this plastic deformation. The results are finally compared with the numerical results obtained from the FE analysis of the considered micromechanical model. The proposed method is capable of predicting all plastic strain terms in the matrix and all the stress terms, as well.*

## 1 Introduction

The use of metal matrix composite materials (MMC) in aerospace structures and engine parts has become more frequent recently. In general, numerous methods have been developed for elastic analysis of MMCs. One of the main approaches to such a problem is the Shear Lag Method [1-9], which due to the good description of the load transfer mechanism from the fiber to the matrix is of major importance and application. But, it should be noted that the various simplifying assumptions involved are considered as the main disadvantage of this model. Since due to such assumptions, the model is not that much accurate, it has been modified by others such as Hsueh [10-16, 21-22], leading to increased efficiency.

The load transfer from the fiber to the matrix is the most important mechanism governing the deflection and fracture response of the various fiber reinforced materials. The Shear Lag method first introduced by Cox [1] was vastly used due to its mathematical simplicity and its good prediction of this mechanism. Although it has been shown that this method can well predict this load transfer mechanism for brittle materials, but for ductile materials such as metal matrix of the MMCs, due to the occurrence of plastic deformations at low strains, it will not be able to well describe the material behavior.

After the elastic analysis of the existing problem, Jiang [23] studied the plastic behavior of short fiber reinforced MMCs, applying the Shear Lag method. In this study, a very simple approach has been used for plastic analysis of the matrix. Since the general shear lag model applied is not capable of predicting the stress distribution in matrix, the problem has been simplified by means of assuming an average axial plastic deformation in matrix, defined by some linear distribution assumptions made later. Furthermore, the plastic strain term considered has been limited to the axial strain only, and the effect of the shear stress has been totally neglected by applying an approximate relation between the axial stress and the average axial plastic deformation in matrix.

In the present study, an analytical approach is proposed for studying the elastic-plastic behavior of short fiber reinforced metal matrix composites under small tensile loading steps. In the proposed research, employing a micromechanical approach, an axi-symmetric unit cell including one fiber and the surrounding matrix is considered. Using a shear-lag based formulation, first, the governing equations and the boundary conditions are derived and the elastic solution is obtained for both fiber and matrix. The governing relations are then obtained, considering all the stress and the plastic strain components. Applying the successive elastic solutions method, the plastic strain terms in matrix are obtained. Thereafter, the effects of this plastic deformation on stress transfer mechanism of the composite are resulted. Some numerical results obtained by FE analysis of the model are shown for comparison, also. It should be added that compared to the previous existing solutions to the current problem, all of which applied many simplifying assumptions, the proposed study is the first of its kind capable of predicting all plastic strain terms in the matrix, and the stress components, as well.

## 2 Mathematical Formulation

In order to study the elastic-plastic behavior of MMCs under simple tensile loading, a cylindrical axi-symmetric unit cell, consisting of a fiber and the surrounding matrix has been considered as shown in Fig. 1 and Figure 2. The cell is subjected to a uniform tensile stress, $\sigma_0$ .

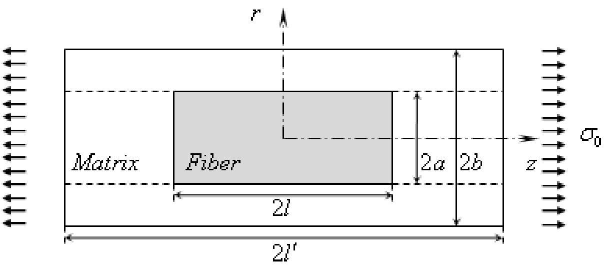



Fig. 1. Micromechanical unit cell.

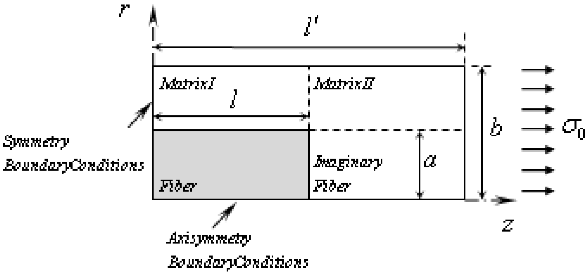



Fig. 2. Axi-symmetric model.

According to FEM results and the governing equations, which can be admitted by common sense also, the plastic deformation in the matrix will start at somewhere in the adjacent region to the fiber, region I shown in Fig. 2, due to the low matrix yield stress and high stress concentrations in that region. Considering the occurrence of plastic deformation in matrix and focusing on region I of matrix from now on, the stress-strain relations for matrix in this region can be re-written as [24]:

$$\varepsilon_r^m = \frac{1}{E^m}\left[\sigma_r^m - \nu^m\left(\sigma_\theta^m + \sigma_z^m\right)\right] + \varepsilon_r^p \tag{1a}$$

$$\varepsilon_\theta^m = \frac{1}{E^m}\left[\sigma_\theta^m - \nu^m\left(\sigma_z^m + \sigma_r^m\right)\right] + \varepsilon_\theta^p \tag{1b}$$

$$\varepsilon_z^m = \frac{1}{E^m}\left[\sigma_z^m - \nu^m\left(\sigma_r^m + \sigma_\theta^m\right)\right] + \varepsilon_z^p \tag{1c}$$

$$\varepsilon_{rz}^m = \frac{1+\nu^m}{E^m}\tau_{rz}^m + \varepsilon_{rz}^p \tag{1d}$$

where, $E^m$ and $\nu^m$ are Young's modulus and Poisson's ratio of the matrix respectively and $\varepsilon^p$ terms define the plastic strain components. It is clear though that as loading is increased, the plastic region expands within the matrix, while the points for which the yield criterion is not satisfied yet, still remain in the elastic region, with $\varepsilon^p$ terms being zero in Eq. (1a) to (1d).

Considering the axi-symmetry of the model, and thus neglecting the derivatives with respect to $\theta$, the general equilibrium equations for both fiber and matrix can be written as [24]:

$$\frac{\partial\sigma_z}{\partial z} + \frac{\partial\tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0 \tag{2a}$$

$$\frac{\partial\sigma_r}{\partial r} + \frac{\partial\tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{2b}$$

Assuming $\frac{\partial \sigma_z^m}{\partial z} = g(z)$ and using Eq. (2a), it can be shown that [9]:

$$\tau_{rz}^m(z) = \frac{a}{b^2 - a^2}\left(\frac{b^2}{r} - r\right)\tau_i(z) \tag{3}$$

Where, $\tau_i(z)$ is the shear stress at the fiber/matrix interface. According to the last of the strain-displacement relations [24]:

$$\varepsilon_r = \frac{\partial u}{\partial r} \tag{4a}$$

$$\varepsilon_\theta = \frac{u}{r} \tag{4b}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \tag{4c}$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \tag{4d}$$

with $u$ being the radial and $w$ the axial displacement, and neglecting the term $\frac{\partial u^m}{\partial z}$ for matrix, according to the assumption that $\left|\partial u^m / \partial z\right| << \left|\partial w^m / \partial z\right|$ , which due to the tensile loading condition and the symmetry of the model is a reasonable assumption, and assuming that the radial and tangential stresses in matrix are much smaller compared to the axial stress term, $\left|\sigma_r^m + \sigma_\theta^m\right| << \sigma_z^m$, and thus neglecting the term $\left(\sigma_r^m + \sigma_\theta^m\right)$ in Eq. (1c), and finally considering the strain-displacement relations, Eq. (4a) to (4d), it can be shown that:

$$\begin{aligned}\sigma_z^m(r,z) &= \sigma_a^m + E^m\varepsilon_z^p(a,z) \\ &- E^m\varepsilon_z^p(r,z) + 2E^m\frac{\partial}{\partial z}\int_a^r \varepsilon_{rz}^p(r,z)dr \\ &+ \frac{b^2\ln\left(\frac{r}{a}\right) - \frac{1}{2}\left(r^2 - a^2\right)}{b^4\ln\left(\frac{b}{a}\right) - \frac{1}{4}\left(b^2 - a^2\right)\left(3b^2 - a^2\right)} \\ &\times \left\{\begin{aligned}&-a^2\bar{\sigma}_z^f + b^2\sigma_0 - \left(b^2 - a^2\right)\left[\sigma_a^m + E^m\varepsilon_z^p(a,z)\right] \\ &-4E^m\frac{\partial}{\partial z}\int_a^b r\int_a^r \varepsilon_{rz}^p(r,z)drdr + 2E^m\int_a^b \varepsilon_z^p(r,z)rdr\end{aligned}\right\}\end{aligned} \tag{5}$$

in which, $\bar{\sigma}_z^f(z)$ is the average axial stress in fiber and $\sigma_a^m$ and $\sigma_b^m$ are the matrix axial stresses at $r = a$ and $r = b$, respectively..

On the other hand, from the first of the equilibrium equations, (1a), for fiber, using the average axial stress introduced in Eq. (5), the following relation, known as Shear Lag equation can be derived:

$$\frac{d\bar{\sigma}_z^f(z)}{dz} = -\frac{2}{a}\tau_i(z) \tag{6}$$

Combining Eq. (6), Eq. (3), the stress-strain relations, Eq. (1a) to (1d), and the strain-displacement equations, Eq. (4a) to (4d), and finally substituting for $\sigma_z^m(r,z)$ from Eq. (5), the following relation will be derived for the fiber average axial stress, $\bar{\sigma}_z^f(z)$:

$$\frac{d^2\bar{\sigma}_z^f(z)}{dz^2} = -\frac{b^2 - a^2}{a^2\left(1+\nu^m\right)\left[b^4\ln\left(\frac{b}{a}\right) - \frac{1}{4}\left(b^2 - a^2\right)\left(3b^2 - a^2\right)\right]} \times \left\{\begin{aligned}&-a^2\bar{\sigma}_z^f + b^2\sigma_0 - \left(b^2 - a^2\right)\left[\sigma_a^m + E^m\varepsilon_z^p(a,z)\right] \\ &-4E^m\frac{\partial}{\partial z}\int_a^b r\int_a^r \varepsilon_{rz}^p(r,z)drdr + 2E^m\int_a^b r\varepsilon_z^p(r,z)dr\end{aligned}\right\} \tag{7}$$

As can be seen, the only unknown term in Eq. (7) is the matrix axial stress at the interface, $\sigma_a^m$, which can be derived from the assumption of a perfect bond between fiber and matrix at the interface. In a same manner, equality of the tangential strains at the boundary will imply the (6) radial stress at the boundary, introduced later as $\sigma^p$.

Finally, solving the ordinary differential equation, Eq. (7) one can easily derive the relation for fiber average axial stress, including the effect of a plastic deformation in the matrix.

$$\bar{\sigma}_z^f(z) = \exp(-\sqrt{AB}z)\begin{Bmatrix} C_1 \\ -\frac{1}{2}\sqrt{\frac{A}{B}}\int \exp(\sqrt{AB}z)[C+F(z)]dz \end{Bmatrix} + \exp(\sqrt{AB}z)\begin{Bmatrix} C_2 \\ +\frac{1}{2}\sqrt{\frac{A}{B}}\int \exp(-\sqrt{AB}z)[C+F(z)]dz \end{Bmatrix} \tag{8}$$

in which:

$$A = \frac{b^2-a^2}{a^2(1+\nu^m)\left[b^4\ln\left(\frac{b}{a}\right)-\frac{1}{4}(b^2-a^2)(3b^2-a^2)\right]} \tag{9a}$$

$$B = a^2 + (b^2-a^2)\frac{E^m}{E^f} \tag{9b}$$

$$C = -b^2\sigma_0$$

$$F(z) = 4E^m\frac{\partial}{\partial z}\int_a^b r\int_a^r \varepsilon_{rz}^p(r,z)drdr \tag{9c}$$

$$-2E^m\int_a^b r\varepsilon_z^p(r,z)dr \tag{9d}$$

Using Eq. (6), fiber shear stress can be derived accordingly.

$$\tau_i(z) = -\frac{a}{2}\begin{Bmatrix} -\sqrt{AB}\exp(-\sqrt{AB}z) \\ \times\left\{C_1-\frac{1}{2}\sqrt{\frac{A}{B}}\int \exp(\sqrt{AB}z)[C+F(z)]dz\right\} \\ +\sqrt{AB}\exp(\sqrt{AB}z) \\ \times\left\{C_2+\frac{1}{2}\sqrt{\frac{A}{B}}\int \exp(-\sqrt{AB}z)[C+F(z)]dz\right\} \end{Bmatrix} \tag{10}$$

As cleared at the beginning of the formulation, all the above discussions up to this point are valid for region I of the matrix shown in Fig. 2 and the fiber within $0 \le z \le l$ range. Considering the $l \le z \le l'$ area, a same approach as the one proposed by Hsueh [14, 15], known as the Imaginary Fiber Technique has been applied. By other words, the fiber is considered to be continuous along the whole cell length, with the $l \le z \le l'$ region being known as the imaginary fiber. The same treatment as the one discussed above is applied to this fiber, except that at the end of the process, the material properties of this imaginary section will be replaced by the matrix properties. Considering the local plastic deformation in region I of the matrix as discussed, a same argument with [23] has been used as follows. As stated in [23], when the matrix local plastic deformation occurs in region I, it will also occur in the region near the fiber end face with the same magnitude. Therefore, the stress transfer in the fiber end region will not be affected by the plastic deformation in the fiber region. Following such an argument, the governing differential equation for imaginary fiber axial stress can be written as:

$$\frac{d^2\bar{\sigma}_z'^f(z)}{dz^2} = -\frac{b^2-a^2}{a^2(1+\nu^m)\left[b^4\ln\left(\frac{b}{a}\right)-\frac{1}{4}(b^2-a^2)(3b^2-a^2)\right]} \times\left(-b^2\bar{\sigma}_z'^f+b^2\sigma_0\right) \tag{11}$$

Therefore, the axial and shear stress for imaginary fiber can be derived as follows:

$$\bar{\sigma}_z'^f(z) = C_3\exp(-\sqrt{AB'}z) + C_4\exp(\sqrt{AB'}z) - \frac{C}{B'} \tag{12}$$

$$\tau_i'(z) = -\frac{a\sqrt{AB'}}{2} \times\left[-C_3\exp(-\sqrt{AB'}z)+C_4\exp(\sqrt{AB'}z)\right] \tag{13}$$

Where constants $A$ and $C$ are previously defined in Eq. (9a) and (9c), while $B'$ is:

$$B' = b^2 \tag{14}$$

As can be seen in Eq. (8), (10), (12), (13), four unknown constants $C_1$, $C_2$, $C_3$, and $C_4$ are still to be calculated. For this reason the following boundary conditions are applied to the axial and shear stresses previously derived:

$$\bar{\sigma}_z'^f = \sigma_0 \text{ (at } z = l'\text{)}$$
$$\bar{\sigma}_z'^f = \bar{\sigma}_z^f \text{ (at } z = l\text{)}$$
$$\tau_i' = \tau_i \text{ (at } z = l\text{)}$$
$$\tau_i = 0 \text{ (at } z = 0\text{)} \tag{15}$$

After derivation of the stress distribution in fiber, combining the equations (1a), (1b), (2b), (4a), and (4b) for fiber, the following partial differential equation will be obtained for fiber:

$$E^m\left[\frac{\partial^2 u^f(r,z)}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{u^f(r,z)}{r}\right)\right] \quad (16)$$
$$+\nu^m(1+\nu^m)\frac{\partial\sigma_z^f(r,z)}{\partial r}$$
$$+(1-\nu^{m2})\frac{\partial\tau_{rz}^f(r,z)}{\partial z}=0$$

In which, $u^f(r,z)$ is the radial displacement in fiber, as stated before. The following boundary conditions are applied to Eq. (17):

$$u^f(0,z)=0 \quad (17a)$$
$$\sigma_r^f(a,z)=\sigma^p(z) \quad (17b)$$

In which, the radial stress can be easily obtained by combining Eq. (1a), (1b), (4a), and (4b) for fiber. After derivation of the radial displacement in fiber and combining the equations (1a), (1b), (2b), (4a), and (4b) for matrix in a similar way, the following partial differential equation will be obtained for the matrix:

$$E^m\left[\frac{\partial^2 u^m(r,z)}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{u^m(r,z)}{r}\right)\right] \quad (18)$$
$$+\nu^m(1+\nu^m)\frac{\partial\sigma_z^m(r,z)}{\partial r}+(1-\nu^{m2})\frac{\partial\tau_{rz}^m(r,z)}{\partial z}$$
$$-E^m\frac{\partial\varepsilon_r^p(r,z)}{\partial r}-(1-\nu^m)\frac{E^m}{r}\varepsilon_r^p(r,z)$$
$$-\nu^m E^m\frac{\partial\varepsilon_\theta^p(r,z)}{\partial r}+(1-\nu^m)\frac{E^m}{r}\varepsilon_\theta^p(r,z)=0$$

In which, $u^m(r,z)$ is the radial displacement in matrix, as stated before. The following boundary conditions are applied to Eq. (18):

$$u^m(a,z)=u^f(a,z) \quad (19a)$$
$$\sigma_r^m(a,z)=\sigma^p(z) \quad (19b)$$

Deriving the radial displacement for matrix, one can easily find the radial and tangential terms of stress in matrix combining the equations (1a), (1b), (4a), and (4b).

But, as can be seen, Eq. (18) includes the unknown plastic strain functions $\varepsilon_r^p(r,z)$ and $\varepsilon_\theta^p(r,z)$ directly, while the other two components of $\varepsilon_z^p(r,z)$ and $\varepsilon_{rz}^p(r,z)$ will appear through equations (3) and (6) for matrix stresses.

In order to calculate the plastic deformation and its effect on stress transfer in the composite, first of all, the yield criterion should be determined, in order to indentify critical loading at which the plastic deformation occurs at the first place. Considering the von-Mises yield criterion, $\sigma_e=\sigma_y$, yielding will occur as soon as the equivalent stress, $\sigma_e$, in a certain point reaches the yield stress, $\sigma_y$, where the equivalent stress is defined as:

$$\sigma_e=\frac{1}{\sqrt{2}}\sqrt{(\sigma_r-\sigma_\theta)^2+(\sigma_\theta-\sigma_z)^2+(\sigma_z-\sigma_r)^2+6\tau_{rz}^2} \quad (20)$$

As described before, and will be shown in the numerical results, it is evident that the yielding in the matrix will start at the interface, somewhere at the end of the fiber. Thus, substituting for the stress terms at $(r,z)=(a,l)$ in Eq. (20), the critical stress, $\sigma_0$ for which yielding will start for the first time in the composite, will be calculated. Thereafter, for stress values above this critical threshold, the plastic strain components are calculated using the successive elastic solutions method as below. The Prandtl-Reuss relations are considered as follows [24]:

$$\Delta\varepsilon_r^p=\frac{\Delta\varepsilon_p^e}{2\sigma_e}(2\sigma_r-\sigma_\theta-\sigma_z) \quad (21a)$$
$$\Delta\varepsilon_\theta^p=\frac{\Delta\varepsilon_p^e}{2\sigma_e}(2\sigma_\theta-\sigma_z-\sigma_r) \quad (21b)$$
$$\Delta\varepsilon_z^p=-\Delta\varepsilon_r^p-\Delta\varepsilon_\theta^p \quad (21c)$$
$$\Delta\varepsilon_{rz}^p=\frac{3}{2}\frac{\Delta\varepsilon_p^e}{\sigma_e}\tau_{rz} \quad (21d)$$

In which $\Delta\varepsilon_p^e$ is the equivalent plastic strain.

The application of the successive elastic solutions method is as follows now. After deriving the critical loading for which, the yielding will start in the matrix for the first time, the loading path is divided into increments. For the initial small loading step, an initial distribution is assumed for the plastic strain increments, $\Delta\varepsilon_r^p$, $\Delta\varepsilon_\theta^p$, $\Delta\varepsilon_z^p$, and $\Delta\varepsilon_{rz}^p$.

Thereafter, the differential equation Eq. (18) is solved as an elastic problem, and the radial displacement function for the matrix is derived. With $u^m(r,z)$ being known, the first approximation for the plastic strains can now be derived through Eqs. (21), by substituting these assumed values in all corresponding stress terms. It should be added that the equivalent stress term involved in these equations is found from the stress-strain curve. The procedure will now be repeated for these new obtained plastic increments, until the results converge to a constant value. The loading step is then increased and the process is repeated for the next loading increment.

For the above purpose, a MAPLE code has been developed, and the plastic strain terms, and therefore the stresses have been derived for a loading path, as will be described in the next section.

## 3 Results and Discussion

In order to understand the effects of the plastic deformation in matrix on stress transfer behavior of the composite, the effective stresses contributing in this mechanism, $\bar{\sigma}_z^f$ and $\tau_i$, will be studied. The calculations are done for Al6061/SiC20% composite, with the following specifications:

Table 1. Geometrical Specifications of Al6061/SiC20% Composite [25]

| $s$ | $k$ | $f'$ | $f$ |
|---|---|---|---|
| 5 | 1 | 0.342 | 0.2 |

Table 2. Mechanical Properties of Al6061/SiC20% Composite [25, 26]

| Material | $E\,(GPa)$ | $E^p\,(GPa)$ | $\upsilon$ |
|---|---|---|---|
| **Al 6061** | 68.3 | 5.667 | 0.345 |
| **SiC** | 470 | _ | 0.17 |

Fig. 3 shows the equivalent stress distribution in the matrix. It is clear that this stress is maximum at the interface, at the fiber end point, $(r,z)=(a,l)$, as claimed before. As shown in this figure, for an applied loading of $\sigma_0 = 278MPa$, the equivalent stress at the critical point of $(r,z)=(a,l)$ has reached the yield value, $\sigma_y = 276MPa$, for the first time.

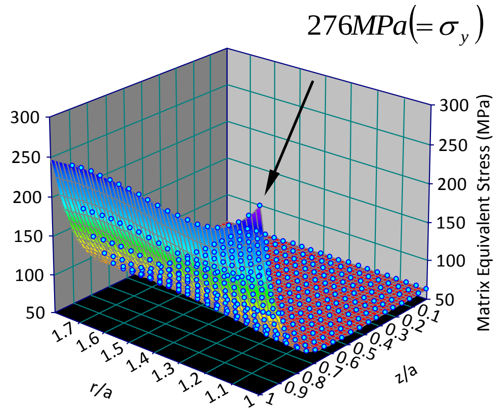



Fig. 3. Matrix equivalent stress distribution for critical loading of $\sigma_0 = 278MPa$ .

Thereafter, the unit cell has been subjected to loading of $\sigma_0 = 280MPa$ . The results for $\sigma_0 = 280MPa$ are shown by details in Figures 4. Fig. 4 shows the shear stress distribution at fiber/matrix interface. Average fiber axial stress distribution is shown in Fig. 5 for $\sigma_0 = 280MPa$ . Finally, the matrix average axial stress distribution can be found in Fig. 6, all of which have been compared with the FEM analysis results of the axi-symmetric model. The calculated distributions for plastic strain components $\Delta\varepsilon_r^p$ , $\Delta\varepsilon_\theta^p$ , $\Delta\varepsilon_z^p$ , and $\Delta\varepsilon_{rz}^p$ are illustrated in Fig. 7 for the subject loading. Also, the numerical results at the interface are shown in Fig. 8 for a general comparison.

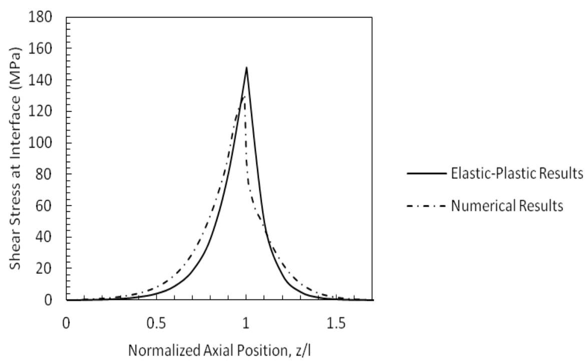



Fig. 4. Shear stress distribution at interface for $\sigma_0 = 280MPa$ .

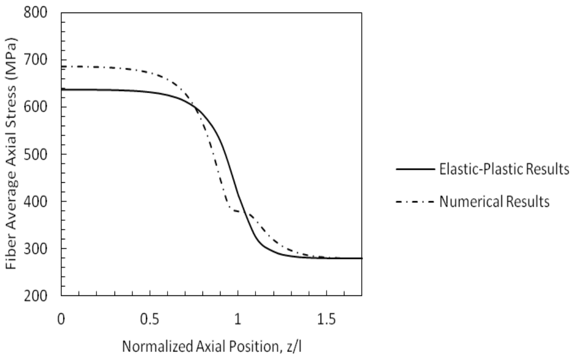



Fig. 5. Fiber average axial stress distribution for $\sigma_0 = 280MPa$.

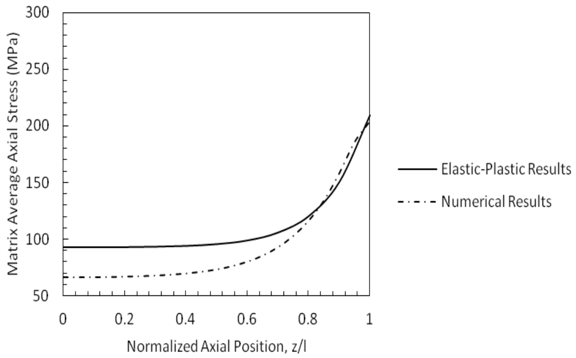
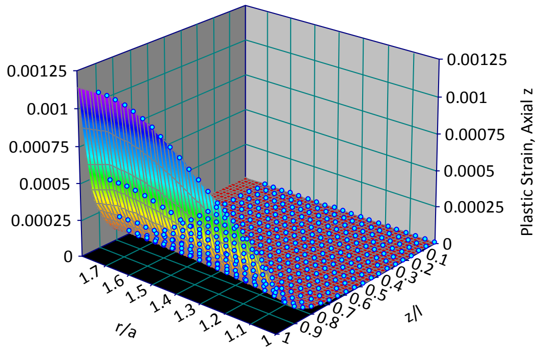
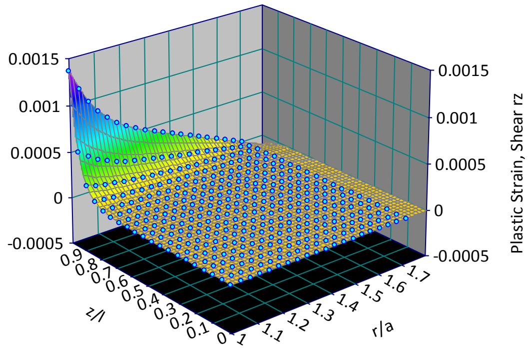


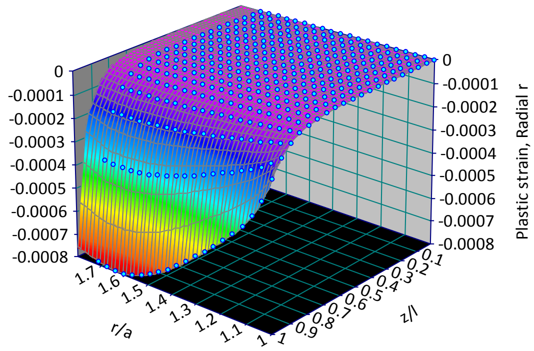

Fig.e 6. Matrix average axial stress distribution for $\sigma_0 = 280MPa$.



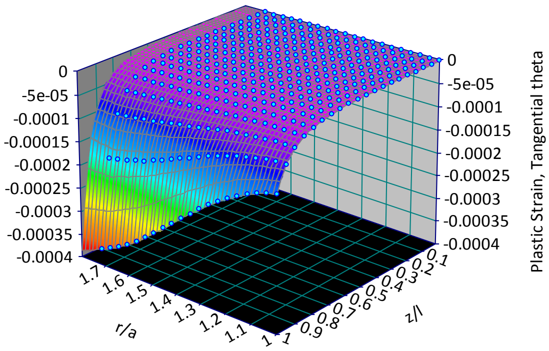

(a) Axial




(b) Shear

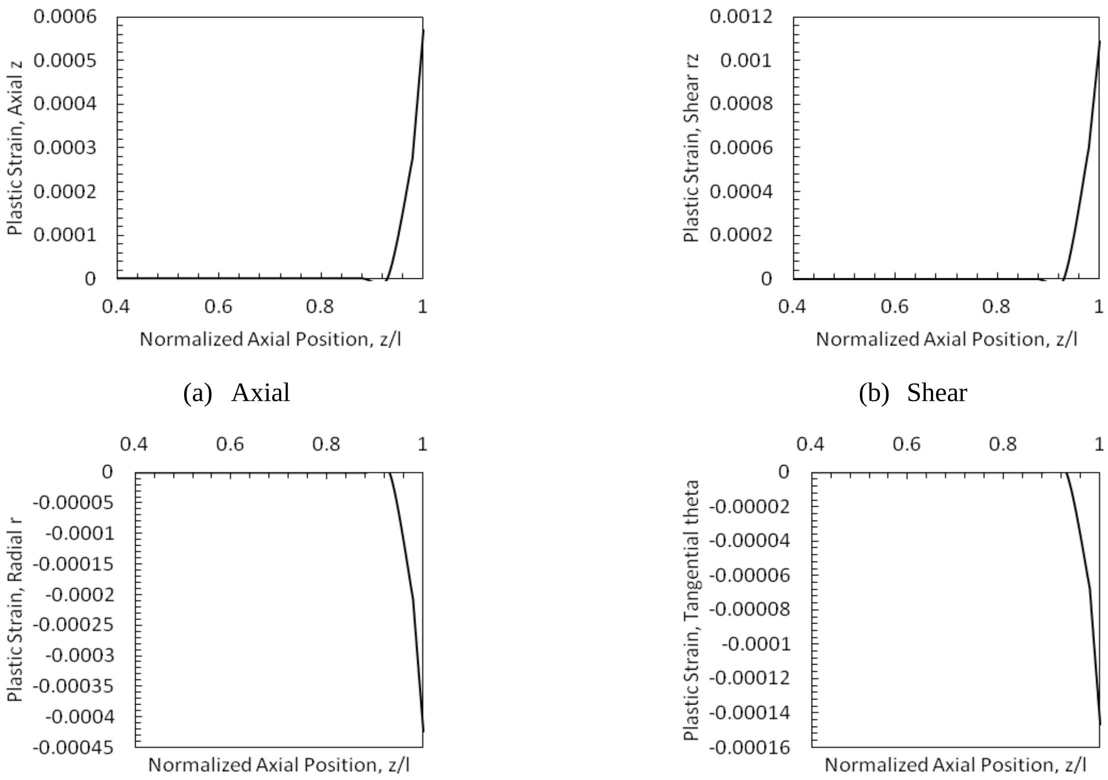



(c) Radial




(d) Tangential

Figure 7. Plastic Strain at Interface for $\sigma_0 = 280MPa$.




(a) Axial (b) Shear

(c) Radial (d) Tangential

Fig. 8. Plastic Strain at Interface for $\sigma_0 = 280MPa$ - Numerical results.

As can be seen, the results show a good compatibility with the numerical solutions. However, the differences found between the calculated and the numerical results are expected to be acceptable due to the simplifying assumptions involved in the analytical method applied, as discussed by details. Also, the numerical results inevitably affect from the errors due to the singularity of the stress distribution at $(r,z)=(a,l)$. But since no detailed work has been previously performed on this subject in the shear lag model framework, concerning all the strain and stress components in the matrix, and since most of the previous analytical solutions to the subject model involved major simplifying assumptions and rough estimations, mostly considering the axial direction only, and not capable of predicting the matrix behavior, it is believed that the current study is the first of its kind, having considered the governing equations to the entire model, avoiding as much as possible the major simplifying assumptions involved in previous studies. As a final note, it should be added that the present errors are the results of the assumptions made in the elastic solution, and if the existing elastic solutions can be modified, the proposed elastic-plastic analysis will lead to much accurate results.

## 4 Conclusion and Remarks

In the present study, an analytical shear lag based model was proposed to study the effects of the plastic deformations in matrix on overall stress transfer behavior of a fiber reinforced metal matrix composite. For this reason, a cylindrical unit cell consisting of a fiber and the surrounding metal matrix was considered under tensile loading. Writing the shear lag based relations for this model, the stress terms for both matrix and fiber were calculated, considering the occurrence of plasticity in the matrix due to the ductility of the metal matrix compared to the brittle characteristics of the hard fiber. Unlike a relatively similar study performed by Jiang [23] based on the shear lag approach, the present work has taken into account the stress distribution in the matrix and has derived the governing relations for that region. Also, the effect of the equivalent stress, i.e. all the stress terms have been considered, rather than the mere axial stress applied in that work. Moreover, all of the four plastic strain terms, i.e. axial, shear, radial, and tangential strain components have been taken into account. Furthermore, the performed study is able to predict the plastic strain terms and the stress distributions in both fiber and matrix with an increase in the loading.

To summarize, having obtained the shear lag based relations for the problem, the plastic strain distribution in the matrix has been derived for a given loading. Thereafter, the effects of these plastic strains on the interface shear and the fiber average axial stress have been obtained. It was also verified that the occurrence of a local plasticity in the matrix has an adverse effect over the stress transfer efficiency of the composite, via a reduction in these two critical load transfer mechanisms compared to the stress distribution in the absence of such deformations. Moreover, it was shown that as a local plastic deformation occurs in the matrix, the increase in the stresses occurs more slowly compared to the elastic case, which is another evidence for the efficiency loss in the presence of the matrix plasticity.

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