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SINGULARITIES IN LAMINAR BOUNDARY LAYER AND FLOW STRUCTURE NEAR SINK PLANE ON CONICAL BODIES

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Abstract

Singularities in the outer part of the laminar boundary layer on pointed conical bodies and flow structure generated by them near a sink plane are studied. Asymptotic solutions of boundary-layer equations and singularity types are obtained in explicit form. It is shown that in the singularity vicinity the boundary region is formed, in which reduced Navier-Stokes equations describe the flow; regular analytical solutions of these equations are obtained to match with boundary-layer solutions. Two-layer flow model is derived for the region, in which the effect of viscous-inviscid interaction is important. Analysis of equations shows that the interaction weakens the singularity, its type becomes a function of the distance from the nose but this effect does not eliminate the singularity totally.

Introduction

The flow over conical bodies is a simplest and well-studied problem of 3D laminar boundary layer theory. However singularities, which arise in this flow and have direct relation to such problems, as boundary-layer solution unique existence, and 3D separation do not explained until now. Moreover the exact singularity type is not known. Analyses of flows over simple bodies such as round or elliptic cones shown that there are many self-similar solutions in the symmetry plane, and in the sink plane a solution does not exist at some incidence range depending on wall temperature, surface shape, Mach and Prandtl numbers [1-7]. It was assumed that such solution behavior is related with violation of self-similarity in the sink plane. However calculations shown that a non-self-similar solution exist only at well defined blowing velocities [8,9]. These results shown that in the range of the solution nonexistence qualitative flow structure changes are arisen near the wall; however a nature of these changes is not known. Another possible singularity reason is the principle of dependence and influence violation [1,4,6,11]. Numerical integration of self-similar equations in transversal direction reveals the single singularity that is the violation of symmetry conditions in the form of finite transversal velocity in the sink plane [9-12]. Calculations [10,11] point to essential disagreements with experimental data long before the separation appearance. Numerical solutions being in well agreement with data have been obtained on the base of Navier-Stokes equations [8,13]. For a slender round cone singularities in the sink plane have been studied on the base of the explicit asymptotic solution for outer boundary-layer part [14]. For arbitrary conical bodies such analysis is presented in the reference [15,16] and present work. Also the asymptotic analysis of the Navier-Stokes equations is carried out, and the flow structure near a sink plane is studied.

1. Singularities of boundary-layer equations

The laminar boundary layer on a conical surface at $\rho\mu = \Pr = 1$ in the orthogonal coordinate system $xy\varphi$ (Fig. 1) is described by the equations [1]

$$w_{yy} = Aww_\varphi - vw_y + w\left(\tfrac{2}{3}u + Kw\right) - h\left(\tfrac{2}{3} + K\right),$$
$$u_{yy} = Awu_\varphi - vu_y + A_1 w(u-w),$$
$$h_{yy} = Awh_\varphi - vh_y - M_0\left(u_y^2 + \tfrac{3}{2}A_1 w_y^2\right),$$
$$y = 0: u = v = w = 0,\ h = h_w\ \ (h_y = 0);$$
$$y = \infty: u = w = h = 1;\ f_y = u,\ g_y = w. \quad (1.1)$$
$$y = \sqrt{\frac{3\operatorname{Re}(\varphi)}{2x}}\int_0^{y^*}\rho dy^*,\ \operatorname{Re}(\varphi) = \frac{\rho_e u_e \operatorname{Re}_\infty}{\mu_e},$$
$$\operatorname{Re}_\infty = \frac{\rho_\infty u_\infty l}{\mu_\infty},\ M_0(\varphi) = \frac{u_\infty^2 u_e^2}{h_\infty h_e},\ K(\varphi) = \frac{2w_e'}{3Ru_e},$$
$$v = f + \left[K - \tfrac{1}{2}A\left(\ln\left(\rho_e\,\mu_e/u_e\right)\right)'\right]g + Ag_\varphi,$$
$$A(\varphi) = \frac{2w_e}{3Ru_e},\ A_1(\varphi) = \frac{2}{3}\left(\frac{w_e}{u_e}\right)^2$$

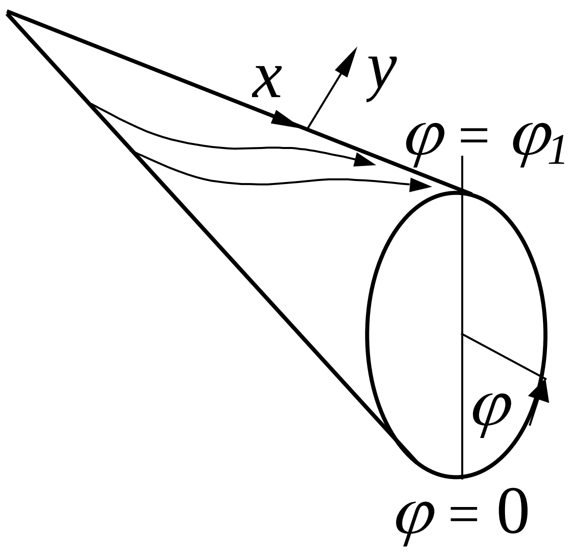


Fig. 1. Flow Scheme and Coordinates

Here x is the distance from the body nose along generator referenced to the body length l, φ is transversal coordinate (Fig. 1), $\Pr$ is Prandtl number, $f(y,\varphi)$, $g(y,\varphi)$ and $v(y,\varphi)$ are flow functions and transformed normal velocity, y^* is referenced to l normal to the body, $R(\varphi)$ is metric coefficient, asterisks denote the differentiation with respect to argument, indexes x, φ and y denote the differentiation with respect to these variables. The density ρ, enthalpy h, viscosity μ, longitudinal and transversal velocities (u and w) are referenced to them values at the outer boundary, $\rho h = 1$. The flow function on the outer boundary-layer edge indexed by "e" are normalized to their values in the freestream indexed by "∞"; they are functions of φ only. The transversal velocity on outer boundary-layer edge $w_e = 0$ in the initial value plane $\varphi = 0$ ($K(0) > 0$), and in the sink plane $\varphi = \varphi_1$ ($K(\varphi_1) = -k < 0$), in which two boundary layer parts came from different sides of the source plane are collided.

Eqs. (1.1) are simplified for slender bodies since in this case $A = O(1)$, $A_1 << 1$, $u_e = \rho_e = \mu_e = 1$. In this case the enthalpy equation (1.1) allows the Crocco integral

$$h = h_w + h_r u - \tfrac{1}{2}M_0 u^2,\ h_r = 1 - h_w + \tfrac{1}{2}M_0. \quad (1.2)$$

We consider the asymptotic form of Eqs. (1.1) at $y >> 1$, so that the flow functions are represented as

$$u = 1 + U(\eta,\varphi),\ w = 1 + W(\eta,\varphi),$$
$$H = -\left(\tfrac{1}{2}M_0 + h_w - 1\right)U,\ v = (1+K)y,$$
$$W_{\eta\eta} = \tfrac{2}{3}a\left[\tfrac{3}{2}AW_\varphi + (1+3K)W + p(\varphi)U\right] - \eta W_\eta$$
$$U_{\eta\eta} = aAU_\varphi - \eta U_\eta,\ \eta = y\big/\sqrt{a(\varphi)} \quad (1.3)$$

Equations (1.3) have the solution

$$U(\eta,\varphi) = C_1 erfc\left(\eta/\sqrt{2}\right),\ W(\eta,\varphi) = -b(\varphi)U$$
$$b' + 2(1+M)(\ln w_e)'b = 2pM(\ln w_e)',$$
$$p(\varphi) = 1 + \left(1 + \tfrac{3}{2}K\right)\left(\tfrac{1}{2}M_0 + h_w - 1\right),$$
$$a' + 2\left[(N+1)(\ln w_e)' - \tfrac{1}{2}\ln\left(\rho_e\mu_e/u_e\right)'\right]a =$$
$$= 2N(\ln w_e)';\ N(\varphi) = 3M(\varphi) = K^{-1}. \quad (1.4)$$

To extract singularities in the sink plane we represent solutions for the functions $a(\varphi)$ and $b(\varphi)$ in the form of quadratures using the integration by parts

$$m \neq 1: b = \frac{Mp}{M+1} -$$
$$-Ew_e^{-2(M+1)} \int_0^{\varphi} \frac{p'M(M+1) + pM'}{E(M+1)^2} w_e^{2(M+1)} d\varphi,$$
$$E = \exp\left(2\int_0^{\varphi} M'(t) \ln w_e(t) dt \right),$$
$$m = 1: b = 2Mp \ln w_e -$$
$$-2\int_0^{\varphi} \frac{(pM)' w_e + 2(M+1)Mpw_e'}{E} w_e^{2M+1} \ln w_e d\varphi,$$
$$n \neq 1: a = \frac{N}{N+1} -$$
$$-E_1 w_e^{-2(N+1)} \int_0^{\varphi} \frac{N'}{E_1(N+1)^2} w_e^{2(N+1)} d\varphi,$$
$$E_1 = \exp\left(2\int_0^{\varphi} N'(t) \ln w_e(t) dt \right),$$
$$n = 1: a = 2N \ln w_e - 2E_1 w_e^{-2(N+1)} \cdot$$
$$\cdot \int_0^{\varphi} \frac{N'w_e + 2(N+1)Nw_e'}{E_1(N+1)^2} w_e^{2N+1} \ln w_e d\varphi. \quad (1.5)$$

The constant $C_1(k)$ is calculated from matching condition with a numerical solution inside the boundary layer. At $n \neq 1$ and $m \neq 1$ Eqs. (1.5) satisfy to the initial conditions at $\varphi = 0$ for regular at $K(0) \to 0$ solution branch [14], the expressions for $n = m = 1$ are true at $\varphi > 0$ only. Near the sink plane $\zeta = \varphi_1 - \varphi << 1$ and (1.5) are represented as

$$w_e = \tfrac{3}{2} kR\zeta + O(\zeta^2),\ k = -K(\varphi_1),\ R = R(\varphi_1),$$
$$p_1 = p(\varphi_1) = 1 + \left(1 - \tfrac{3}{2}k\right)\left(\tfrac{1}{2}M_0 + h_w - 1\right);$$
$$m \neq 1: b = \frac{mp_1}{m-1} - b_m \zeta^{2(m-1)};$$
$$m = 1: b = -2p_1 \ln \zeta + b_1,\ m = M(\varphi_1);$$
$$n \neq 1: a = \frac{n}{n-1} + a_n \zeta^{2(n-1)};$$
$$n = 1: a = -2\ln \zeta + a_1,\ n = -N(\varphi_1). \quad (1.6)$$

The coefficients a_n and b_m are determined by integrals (1.5) with $\varphi = \varphi_1$. For a slender round cone they are expressed in explicit form [14]. The formulas (1.6) are true for non-slender bodies also since in this case $A_1 = O(\zeta^2)$.

The results obtained show that in the sink plane two singularity types are in the outer boundary layer part. For $k < 1$ the function $U(\eta, \zeta)$ exists at $\zeta = 0$ but it reaches this limit irregularly, the behavior of this function is studied in details for the slender round cone [14]. For $k \geq 1$ the function $U(\eta, \zeta)$ is singular at $\zeta = 0$, at these parameter values the boundary layer infinitely growths as $\sqrt{a(\zeta)}$; at $k = 1$ the singularity is of logarithmic type. At $k \geq 1$ the flow separation is observed in experimental and numerical studies, this phenomenon leads to change not only the outer part but also the inner boundary-layer structure. Therefore the above analysis is not enough to describe the total flow structure however the obtained results have an interest since give an insight to a new possible singularity type in the 3D boundary layer.

The function $W(\eta, \zeta)$ is irregular at $\zeta \to 0$; it has finite limit in this plane at $k < 1/3$ and is singular at $k \geq 1/3$; for $k = 1/3$ the singularity is of logarithmic type. The 3D boundary-layer singularity in the sink plane at $1/3 \leq k < 1$ is related with the behavior of transversal flow only. This singularity leads to the longitudinal vortex component strengthening in the outer part of the viscous region. We note that the singularity takes place as at negative ($k \leq 2/3$) and positive ($k > 2/3$) pressure gradient; hence this effect is not determinative, the singularity is formed under action of inertia and centrifugal forces. The critical value $k_c = 1/3$ is undependable on Mach number and wall temperature. However at specific conditions if the numerical value $k_c(h_w, M) \geq 1/3$ [1-7] the flow structure is determined in particular by the singularity in the outer boundary layer part and the presented analysis has actual interest.

2. Boundary-region flow

The singularity of the boundary-layer equations leads to vortex boundary region formation in the sink-plane vicinity with the transversal dimension of the order of boundary layer thickness. In this region, the transverse diffusion is the effect of the first order, and to describe it we introduce the variables

$$z=\sqrt{kx}R\zeta/\varepsilon,\ u=u(y,z),\ h=h(y,z),$$
$$w=w(y,z),\ \varepsilon=\left[\tfrac{3}{2}\mathrm{Re}(\varphi_1)\right]^{-\frac{1}{2}}.$$

Using these variables and Eqs. (1.3) in Navier-Stokes equations [13] at $\varphi\to\varphi_1$, $\varepsilon\to 0$, and $y>>1$ we obtain the self-similar equations for the outer part of the boundary region in the form

$$W_{yy}+(1-k)yW_y+kW_{zz}+\left(\frac{2}{z}+kz\right)W_z+$$
$$+2k(m-1)W+\tfrac{2}{3}p_1U=0,$$
$$U_{yy}+kU_{zz}+(1-k)yU_y+kzU_z=0. \quad (2.1)$$

For $k<1$ these equations has the solution

$$U(y,z)=C_1 erfc\left(y\sqrt{(1-k)/2}\right)erf\left(z/\sqrt{2}\right),$$
$$W=-B(z)C_1 erfc\left(y\sqrt{(1-k)/2}\right),$$
$$B_{zz}+\left(\frac{2}{z}+z\right)B_z-2(m-1)B=-2mp_1F(z),$$
$$F(z)=erf\left(z/\sqrt{2}\right). \quad (2.2)$$

The function $B(z)$ is expressed by Kummer function $\Phi(a,b,x)$ [16]

$$B=mp_1B_0(z)+B_m\Phi\left(1-m,\tfrac{3}{2},-\tfrac{1}{2}z^2\right),$$
$$B_m=b_m\left(R\sqrt{kx}/\varepsilon\right)^{2(1-m)}. \quad (2.3)$$

Here $B_0(z)$ is particular solution of inhomogeneous equation (2.2). The coefficient B_m is determined from matching condition of (2.3) with (1.6). In the particular case $m=1$, Eq. (2.2) is integrated in the explicit form

$$B=B_1-2p_1F_1(z),$$
$$B_1=b_1+p_1\left(2\ln\left(R\sqrt{kx}/\varepsilon\right)+C+\ln 2-1\right),$$
$$F_1(z)=F(z)\left(\ln z-\tfrac{1}{2}\right)-\sqrt{\tfrac{2}{\pi}}\int_0^z e^{-1}(t)\ln t\,dt-$$
$$-\int_0^z e^{-1}(x)x^{-2}\int_0^x e(t)F(t)dt,$$
$$e(z)=\exp\left(z^2/2\right).$$

where C is Euler constant. The number 1 in Fig. 2 denotes the function $F_1(z)$; the number 2 denotes its asymptote at $z>>1$, $\ln z-0.54$, corresponding to the boundary-layer solution (1.6). Another explicit solution corresponds to $m=1/2$,

$$B(z)=B_{1/2}F(z)/z-$$
$$-2p_1\left\{F(z)+2\sqrt{\tfrac{2}{\pi}}\left[e^{-1}(z)-1\right]/z\right\}.$$

The numbers 3 and 4 in Fig. 2, respectively, denote the function $F(z)/z$ and its boundary-layer asymptote $1/z$.

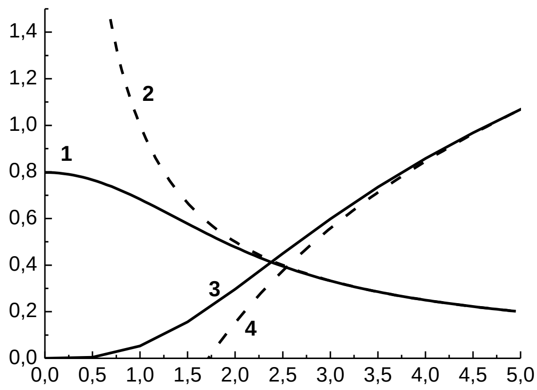


Fig. 2. Navier-Stokes and Boundary-Layer Equations Solutions

Therefore, at $k\geq 1/3$ in the boundary layer near the sink plane the vortex region is formed; in this region, reduced set of Navier-Stokes equations describe the flow. Obtained solutions are regular and matched with solutions of the boundary-layer equations.

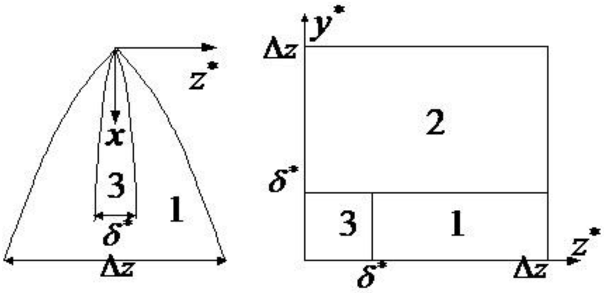
3. Viscous-inviscid interaction

Another effect generated by the singularities in the sink plane at $k \geq 1/3$ is the boundary layer growth at $\zeta \to 0$ and appearance of the viscous-inviscid interaction. From (1.4) and (1.6) the estimations for the boundary-layer thickness $\Delta(\zeta)$ can be obtained

$$k = 1/3: \Delta \sim \sqrt{\ln\ln\left(1/\zeta^2\right)};$$
$$1/3 < k < 1: \Delta \sim \sqrt{(1-m)\ln\left(1/\zeta^2\right)/(1-k)};$$
$$k > 1: \Delta \sim \zeta^{n-1}\sqrt{\ln\left(1/\zeta^2\right)};$$
$$k = 1: \Delta \sim \ln\left(1/\zeta^2\right). \qquad (3.1)$$

In order to the viscous-inviscid interaction effect would be of principal order the transversal velocity w_e from (1.1) would be of the order of the velocity w_{ei} induced by the boundary-layer growth. This condition allows estimating the transverse dimension of the interaction region $\Delta\varphi$ as

$$\Delta\varphi \sim \zeta \sim \sqrt{m\varepsilon}x^{-\frac{1}{4}},\ w_e \sim w_{ei} \sim kRu_e\sqrt{m\varepsilon}x^{-\frac{1}{4}},$$
$$w_{ei} \sim \frac{2\varepsilon u_e}{R\sqrt{x}}\zeta\frac{\partial\Delta}{\partial\zeta} \sim w_e = \tfrac{3}{2}kRu_e\Delta\varphi, \qquad (3.2)$$



a) Top View ; b) Back View
Fig. 3. Flow Structure Near the Sink Plane

In the boundary-layer region 1 (see Fig. 3) $y \sim 1$, $z^* \sim \Delta z = x\Delta\varphi$; to describe the flow in this region we introduce the new variables and obtain the equations:

$$s = R\zeta/\sqrt{\varepsilon},$$
$$u_e = u_e(\varphi_1) + O(\varepsilon),\ h_e = h_e(\varphi_1) + O(\varepsilon)$$
$$w_e = \tfrac{3}{2}u_e\sqrt{\varepsilon}W_e(x,s),\ \ A = W_e,\ \ K = W_{es}$$
$$A_1 = O(\varepsilon),\ R = R(\varphi_1) + O(\varepsilon),$$
$$u = u(x,y,s),\ h = h(x,y,s),\ w = w(x,y,s),$$
$$v = f + Kg + Ag_s + \tfrac{2}{3}xf_x,$$
$$u_{yy} = W_e w u_s - v u_y + \tfrac{2}{3}xuu_x,$$
$$w_{yy} = W_e w w_s - v w_y + w\left(\tfrac{2}{3}u + W_{es}w\right) -$$
$$-h\left(\tfrac{2}{3} + W_{es}\right) + \tfrac{2}{3}xuw_x,$$
$$h_{yy} = W_e w h_s - v h_y - M_0 u_y^2 + \tfrac{2}{3}xuh_x. \qquad (3.3)$$

The boundary conditions for Eqs. (3.3) have the form (1.1), and the enthalpy equations is integrated in the form (1.2). Solutions of Eqs. (3.3) will be matched with (1.6) at $s \to \infty$. Initial conditions are needed also for Eqs. (3.3) an some $x = x_0$, which can be obtained from a solution of the Navier-Stokes equations in the body nose vicinity; this feature peculiar to 3D equations does the problem more complicated.

In the region 2 in Fig. 3 ($y^* \sim z^* \sim \Delta z$), the flow is inviscid and is determined by interaction due to growth of the displacement thickness $\delta^* = \varepsilon\delta(x,s)$. At moderate supersonic Mach numbers the flow in this region is nonvortical and we introduce the flow potential Φ^* and local variables as

$$Y = y^*/\sqrt{\varepsilon},\ Z = z^*/\sqrt{\varepsilon},$$
$$\Phi^*(x,y^*,z^*) = u_e\left[x + \varepsilon\Phi(x,Y,Z)\right]$$
$$\frac{\partial^2\Phi}{\partial Y^2} + \frac{\partial^2\Phi}{\partial Z^2} = 0,$$
$$\left.\frac{\partial\Phi}{\partial Y}\right|_{Y=0} = \sqrt{\varepsilon}\frac{\partial\delta}{\partial x},$$
$$\left.\left(\frac{\partial\Phi}{\partial Y}\right)^2 + \left(\frac{\partial\Phi}{\partial Z}\right)^2\right|_{Y^2+Z^2\to\infty} \to 0$$

Using the local symmetry property, $\delta(x,\phi) = \delta(x,-\phi)$, on the body surface we obtain:

$$\Phi=-\frac{1}{\pi}\frac{\partial}{\partial x}\int_{-\infty}^{\infty}\delta(x,t)\ln|Z-t|dt,$$
$$W_e(x,s)=-ks[1+r],$$
$$r(x,s)=\frac{4m}{\pi}\frac{\partial}{\partial x}\int_0^{\infty}\frac{\delta(x,t)dt}{s^2-t^2},$$
$$W_{es}(x,\phi)=-k(1-q),$$
$$q(x,s)=-r_s=-\frac{4m}{\pi}\frac{\partial}{\partial x}\int_0^{\infty}\frac{\delta_t(x,t)tdt}{s^2-t^2}. \quad (3.4)$$

In the outer boundary layer part at $y>>1$ the solution of (3.3) has the form

$$t=y/\sqrt{d(x,s)},\ u=1+U(x,t,s),$$
$$w=1-c(x,s)U,$$
$$v=y[1-k(1+r)],\ p_0=\tfrac{3}{2}\left(\tfrac{1}{2}M_0+h_w-1\right)$$
$$U_{tt}+tU_t+k(1+r)dsU_s-\tfrac{2}{3}dxU_x=0,$$
$$U=C_1erfc\left(t/\sqrt{2}\right),$$
$$(1+r)sd_s-2mxd_x-2(n-1+q)d=-2n,$$
$$(1+r)sc_s-2mxc_x-2(m-1+q)c=$$
$$=-2m(p_1-qp_0)$$

Along characteristics $\xi(x,s)=const$, which are streamlines of the inviscid flow, the equations for $d=d(\xi,s)$ and $c=c(\xi,s)$ are reduced to the equations similar to (1.4)

$$(1+r)s\xi_s-2mx\xi_x=0,$$
$$r=r(\xi,s),\ q=q(\xi,s),$$
$$(1+r)sd_s-2(n-1+q)d=-2n,$$
$$(1+r)sc_s-2(m-1+q)c=-2m(p_1-qp_0)$$

The solutions of these equations are represented by the quadratures:

$$c=CE_2s^{L-1}+Q(\xi,s)+s^{L-1}E_2\int_s^{\infty}Q_tE_2^{-1}t^{1-L}dt,$$
$$E_2(\xi,s)=\exp\left(-\int_s^{\infty}L_t\ln tdt\right),$$
$$Q=\frac{m(p_1-p_0q)}{m-1+q},\ L(\xi,s)=\frac{m-1+q}{1+r},$$
$$d=DE_3s^{I-1}+\frac{n}{n-1+q}-ns^{I-1}E_3\int_s^{\infty}\frac{q_tE_3^{-1}t^{1-I}dt}{(n-1+q)^2},$$
$$E_3(\xi,s)=\exp\left(-\int_s^{\infty}I_t\ln tdt\right),$$
$$I(\xi,s)=\frac{n-1+q}{1+r}. \quad (3.5)$$

At $s\to\infty$ the interaction is became weaker, $r\to 0$ и $q\to 0$, and the integrals in Eqs. (3.5) tend to zero. Comparing the limit expressions for d and c with Eqs. (1.6) for a and b we find:

$$C=b_m\varepsilon^{m-1},\ D=a_n\varepsilon^{n-1}.$$

The functions $c(\xi,s)$ и $d(\xi,s)$ have infinite values at the points $(\xi_c,0)$ and $(\xi_d,0)$, in which $L(\xi_c,0)=0$ and $I(\xi_d,0)=0$; in both points the singularities are of the logarithmic type and near these points the functions $c(\xi,s)$ and $d(\xi,s)$ are expressed by equations that are similar to (1.5) and (1.6) for $m=n=1$. At $L(\xi,0)<0$ and $I(\xi,0)<0$ the singularity is of power type. If the boundary layer thickness growth at $s\to 0$ along the streamline $\xi=const$ than $q+r>0$ and the viscous-inviscid interaction does the singularity weaker. The singularity appearance depends on the longitudinal coordinate, in the nose region the interaction is stronger and the singularity cannot arise however it can arise downstream where the interaction is weaker.

4. Conclusions

Solutions of equations for the outer part of the boundary layer on slender conical bodies are obtained in the form of quadratures; its asymptotic expressions near a sink plane are true for non-slender bodies. The singularity of the solutions leads to formation of multi-layer flow structure near the sink plane that is shown in Fig. 3. In the boundary region 3 of the dimension $z^*\sim y^*=O\left(\varepsilon\sqrt{x}\right)$ the flow is described by reduced Navier-Stokes equations with the constant pressure gradient. The obtained solutions of these equations are regular and matched with solutions of boundary layer equations. In the region 2 ($z^* \sim y^* = O\left(\sqrt{\varepsilon} x^{\frac{3}{4}}\right)$), the flow is inviscid and determined by interaction with boundary layer. The 3D interacting boundary layer lies at the bottom of the region 2 (the region 1: $y^* = O\left(\varepsilon\sqrt{x}\right)$, $z^* = O\left(\sqrt{\varepsilon} x^{\frac{3}{4}}\right)$). In the whole singular region including regions 1, 2, and 3, the flow is described by parabolized Navier-Stokes equations, which are composite equations from the point of view of asymptotic theory. Although in this work the conical bodies are considered results of item 3 show that singularities of regarded type can occur in solutions of the 3D boundary layer equations on arbitrary bodies.

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