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APPROXIMATE FUNCTION FOR FATIGUE CRACK PROPAGATION IN A DESIGN OPTIMISATION PROCEDURE FOR FIBRE METAL LAMINATES

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Abstract

This paper presents a methodology to create a fitness approximation for Alderliesten's fatigue crack propagation prediction model in context of a design optimisation approach for FML, with the fatigue crack growth life as design criteria. An approximate function between the applied stress, the lay-up parameters and the fatigue crack growth life is created to replace the prediction model in the optimisation procedure to eliminate complications related to the high computation time and the implementation of the prediction model. This fitness approximation is verified against the Alderliesten model and good correlation is obtained while immensely decreasing the computation time. This approach establishes the basis for the development of a design tool for an FML structure.

1 Introduction

The application of Fibre Metal Laminates (FMLs) on fatigue sensitive structures is beneficial because FMLs have outstanding fatigue and damage tolerance (F&DT) characteristics. The design of FMLs for fatigue sensitive aircraft structures includes the assessment of fatigue crack initiation (FCI), fatigue crack propagation (FCP), delamination growth and residual strength. These assessments are needed to substantiate the design with a damage tolerance analysis and to prepare an appropriate structural inspection program. This paper focuses on the fatigue crack propagation assessment. Determination of the crack propagation behaviour is important for defining the inspection period in order to detect any crack beyond its detectable length.

Crack propagation is defined as the growth of cracks in the metal layers due to fatigue loading just after cracks have initiated. In FMLs, the crack growth process is regarded as a fatigue process in metal layers influenced by the fibre layers. The fibres transfer a significant amount of load from the metal layers and stay intact over the crack (fibre bridging), which improves the crack growth behaviour of the structure. This phenomenon is modelled by e.g. Alderliesten [1] and Guo and Wu [2] using the crack opening in the compatibility solution and by Wilson [3] using the exact displacement method. The prediction model of Alderliesten is used for further analysis in the current work.

The current design procedure for FMLs is a solution-oriented analysis, where only the corresponding properties of selected lay-ups are determined using a prediction model and where these are evaluated for meeting the design criteria. However, in a design optimization procedure it is desired to have the lay-ups as output for a given design criteria. This means that the current prediction method should be reversed, but two characteristics of the prediction model prevents the methodology to be reversed mathematically. First, the prediction model uses iterative steps to calculate the crack propagation. Secondly, various lay-ups have similar fatigue performance, which means that the design solutions of a reversed approach cannot be unique. Therefore, an optimisation algorithm is needed to search in a given design space for an optimal lay-up (design solution) wherein a prediction model is used as a fitness function. The design solutions are evaluated on their predicted fatigue life and ranked according to their weight and satisfaction of the design requirement.

In [4], an optimisation procedure based on genetic algorithm is described for fatigue crack initiation. A similar procedure could also be applied to FCP. However, in case of FCP, the model is not easily applied as a fitness function: (1) the fitness computational time of a single solution is high and (2) errors in the fitness evaluation show up, caused by the limitation of the prediction model, making the optimisation procedure to fail. The latter point happens when the optimization algorithm tries to evaluate all possible lay-ups for a given load requirement. The model is unable to predict the fatigue propagation life for a laminate if the applied stress is too high. This may be the situation when high load is applied on low thickness laminates in the design space.

For this reason, it may be necessary to forgo an exact evaluation and use a fitness approximation that is more computationally efficient. A fitness approximation is a method for decreasing the number of fitness function evaluations to reach a target solution. Such approximation could be obtained by a regression analysis on the Alderliesten model. In [5], a fitness approximation was created for FCI, wherein a new function was created based on regression analysis that was able to replace the original methodology with a high coefficient of determination within the selected design range. Likewise, a simplified function between lay-up parameters and crack growth life is needed to replace the Alderliesten model while having a good agreement with the model and improving the computation time. This improvement is essential in order to create a fast and reliable optimisation tool for FML structures.

The objective of this study is to create an approximate function for the fatigue crack growth model by regression analysis. This function is used in the design optimisation approach for FML lay-ups where fatigue crack propagation is considered as design criteria. The model developed by Alderliesten [1] is used to predict the crack growth behaviour of FMLs and to determine the number of cycles to achieve a detectable crack length, thus indirectly the inspection period. As a consequence, a relationship is created to link the lay-up parameters to the fatigue growth life.

First, the theory related to the fatigue crack growth model is briefly explained. Second, the procedure to create the approximate function is presented, and lastly, the approximation is verified against prediction model and an initial computation time comparison is given.

2 Overview of prediction model

The Alderliesten model is an analytical model for constant-amplitude loading fatigue crack propagation of 'through cracks' (same length in all metal layers) in FMLs. The model describes the crack propagation of the fatigue cracks in the aluminium layers and the corresponding delamination growth at the aluminium / fibre interfaces perpendicular to the crack. The model for FML iteratively carries out the steps below to find the number of cycles to reach a certain crack length [1]. Additionally, a schematic overview is given in figure 1.

1. Calculate the crack opening due to maximum and minimum far-field stress.

2. Calculate the deformation of the prepreg layer under maximum and minimum far-field stress.

3. Calculate the bridging stress intensity by applying displacement compatibility at the crack surface at maximum and minimum far-field stress. This displacement is comprised of the results of the two previous steps, the extension of the fibre layer due to a combination of far-field fibre stress and bridging stress, and due to the reduction of crack opening due to the fibre bridging effect. This displacement compatibility requires a discrete solution, because an integration of the non-uniform bridging stress cannot be carried out.

4. Calculate the effective stress intensity ratio from the far-field stress intensity and the bridging stress intensity, again at maximum and minimum far-field stress.

5. Compute the maximum and minimum delamination energy release rates.

6. Calculate the crack- and delamination growth rates from the effective stress intensity ratio and the strain energy release rate ratio, respectively.

7. Apply the calculated crack- and delamination increments to the current coordinates and interpolate to find the updated delamination shape.

3 Definition of variables and constraints

For the regression analysis, it is necessary that the input parameters of the prediction models are identified and decided whether these are constrained or included in the approximate function. The approximate function should include parameters that represent different lay-ups and parameters that control the design process. The remaining parameters are constrained.

In [5], a similar analysis was performed for FCI, in which lay-ups having the same metal volume fraction (MVF) have the same internal stress and thus a similar fatigue initiation life. For this reason, it was possible to come up with an approximate function that was valid for a large range of lay-ups, indifferent of the grade. However, in case of FCP, next to the MVF also the number of interfaces play a role on the crack growth behaviour. Therefore, it is not possible to relate multiple lay-ups to a single parameter. For this reason, it is decided to perform the fitness approximation for a single grade only, since lay-ups with different grades were unable to be related to each other by a function. Thus, for each grade that is defined as variable in the optimisation procedure, a separate fitness function is accessed to determine the lay-up's fitness.

Material properties		**Aluminium 2024-T3**	**S2-glass/ FM94 epoxy**
Young's modulus [MPa]	E_1	72,400	48,900
	E_2	72,400	5,500
Shear modulus [MPa]	G_{12}	26,900	5,550
Poisson's ratio [-]	ν_{12}	0.33	0.33
	ν_{21}	0,33	0,0371
Coefficient of thermal expansion [1/°C]	α_1	$22 \cdot 10^{-6}$	$6.1 \cdot 10^{-6}$
	α_2	$22 \cdot 10^{-6}$	$26.2 \cdot 10^{-6}$
Cure temperature [°C]	T	-	125
Thickness fibre ply [mm]	t_{ply}	-	0.133

Table 1 Mechanical and physical properties of Glare constituents [7]

The grade defines the layer sequence and the composition of the fibre layer, for example, Glare4B has a fibre layer with $< 90/0/90 >$-configuration. This means that the fibre layer consists of three stacked fibre plies 90°, 0° and 90° orientation, respectively [8]. For this analysis, only Glare grades are considered, but the described methodology is generic and can be performed on all types of lay-ups. The material properties of Glare constituents are given in table 1.

Other input variables that relate to the material are the Paris curves (crack growth and delamination). These are given in table 2 for Glare constituents. For FCP, the number of cycles N are defined as the crack propagation between the initial crack length a_0 and the final crack length a_f. In

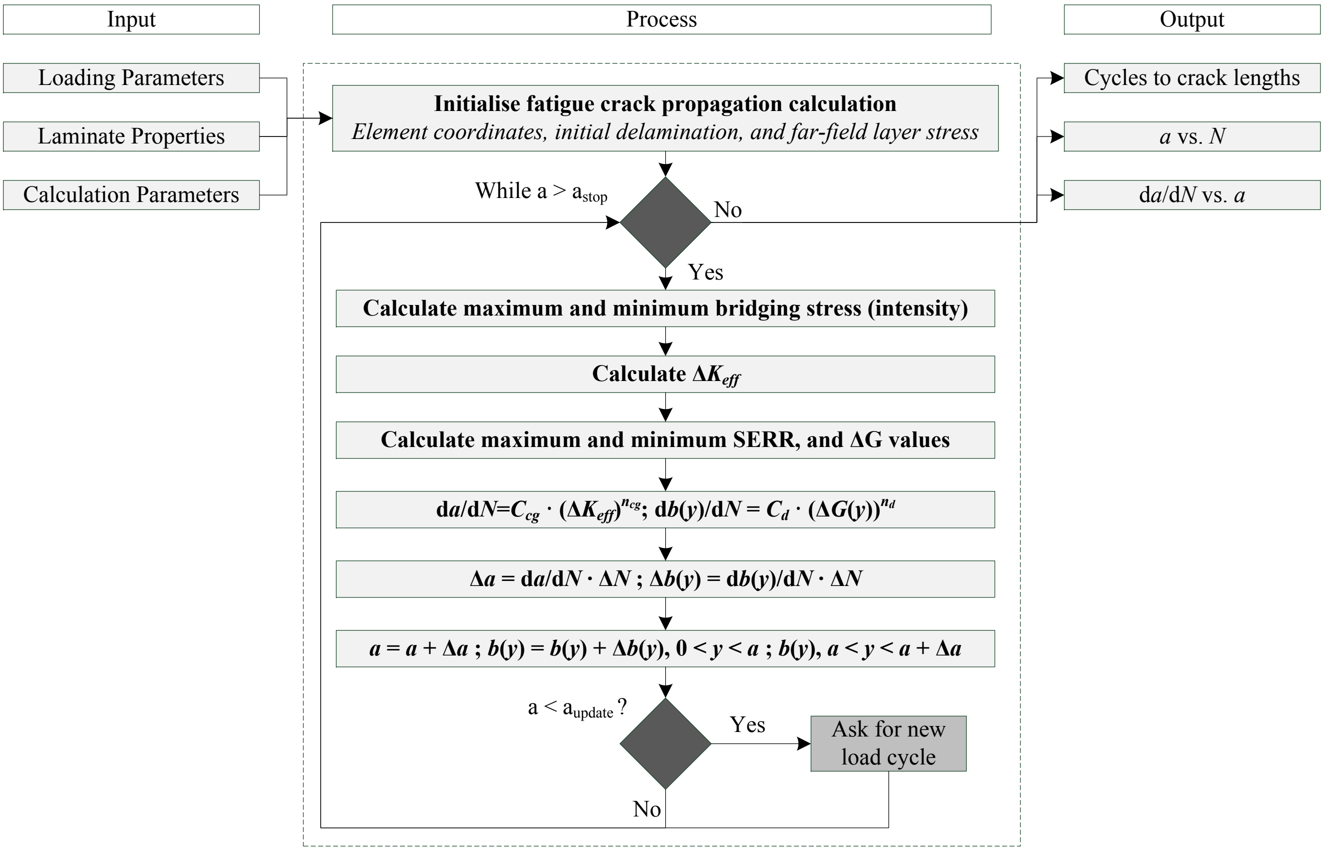

Fig. 1 Overview of Alderliesten FCP model, obtained from [6]

the design process, these values follow from a determined inspection interval, an inspection technique and a residual strength analysis. The initial crack length is $a_0 = 5$ [mm] and the final crack length is $a_f = 45$ [mm] in this particular situation.

Paris coefficients for Glare		
Crack growth Paris coefficient [-]	C_{cg}	$1.27 \cdot 10^{-11}$
Crack growth Paris exponent [-]	n_{cg}	2.94
Delamination Paris coefficient [-]	C_d	0.05
Delamination Paris exponent [-]	ν_{12}	7.5

Table 2 Paris crack growth [9] and delamination [10] coefficients for Glare

For the discussion, the Glare2A grade will be considered, where the number of fibre layers n_f are linked to the metal layers as $n_f = n_m - 1$. The composition of a fibre layer, like the amount, orientation, thickness or stacking order of the fibre plies are defined in the grade. Thus for Glare2A, the fibre layer consist of two fibre plies with a ply thickness of $t_{ply} = 0.133$ [mm] and a ply orientation of $\theta = 0°$ for both plies. The constrained parameters are given in table 3. Once the grade is defined, only two variable are required to generate the lay-ups, namely the number of metal layers n_m and the thickness of the metal layers t_m. The range of the parameters are given in table 4. The applied laminate stress S_{lam} is also varied, since this is the main parameter that controls the crack propagation. On the other hand, the stress ratio R is constrained, since this is defined by the selected test case.

Constrained variables		
Initial crack length [mm]	a_0	5
Final crack length [mm]	a_f	45
Stress ratio [-]	R	0.05
Grade		Glare2A
Orientation fibre plies [°]	θ	$< 0/0 >$
Number fibre layers [-]	n_f	$n_m - 1$

Table 3 The constraints variables of the approximate function

Free variables		
Laminate stress [MPa]	S_{lam}	80 – 200 (step: 20)
Thickness metal layer [mm]	t_m	0.3 – 0.7 (step: 0.1)
Number metal layers [-]	n_m	3 – 11 (step: 1)

Table 4 The free variables of the approximate function

4 Regression analysis

The design optimisation approach uses the crack propagation prediction model of Alderliesten, which is valid for symmetric lay-ups and considers only longitudinal loads

[1]. A regression analysis was performed on the Alderliesten model by analysing a database that contains fatigue crack growth data for a range of lay-ups predicted by the prediction model. An approximate function is created that can replace the model with a high coefficient of determination. Regression analysis is a statistical process for estimating the relationships among variables. This analysis helps to understand how the typical value of the dependent variable changes when any one of the independent variables is varied, while the other independent variables are held fixed. The mechanical material properties, thermal properties and crack geometry are assumed to be constrained, as discussed in the previous section, while variables, like lay-up parameters (layer thickness t_m and layer number n_m) and applied laminate stress S_{lam} are included in this function to calculate the crack growth life N.

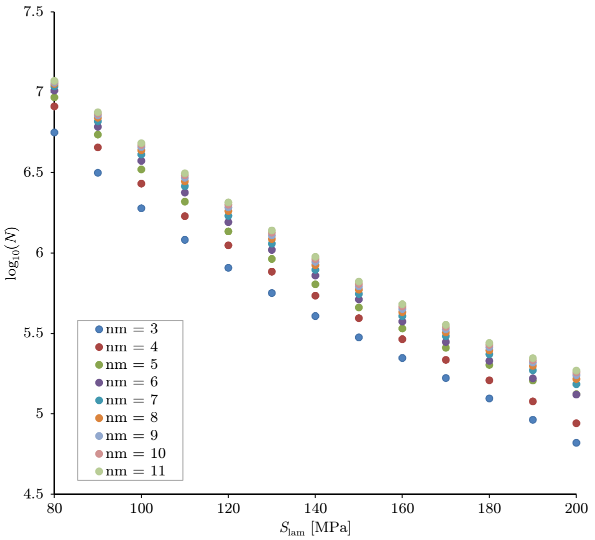

Fig. 2 Data points $\log_{10}(N)$ plotted versus S_{lam} for $t_m = 0.6$ [mm]

In order to clarify the regression analysis, the approximate function is manually constructed by stepwise fitting a curve to the crack growth data for a small range of lay-ups. The first step is creating a database of fatigue crack propagation life N by varying applied laminate stress S_{lam} for a range of lay-ups that are generated for the range of thickness t_m and number of layer n_m given in table 4. Figure 2 is generated by plotting the $\log_{10}(N)$ against S_{lam} for variable n_m. The plot shows cycles to propagation data points for Glare2A for an applied laminate stress between $S_{lam} = 80-200$ [MPa], the number of metal layers is varied between $n_m = 3-11$ layers, and the thickness of metal layer is $t_m = 0.6$ [mm]. Similar plots are also generated for other t_m value.

Next, the data points are structured to prepare for a good fit. As can be seen in figure 3, the $\log_{10}(N)$ is multiplied by the corresponding n_m to distanciate the curves from each other, and additionally, a natural logarithm of the S_{lam} is taken in order create linear curves in the log scale.

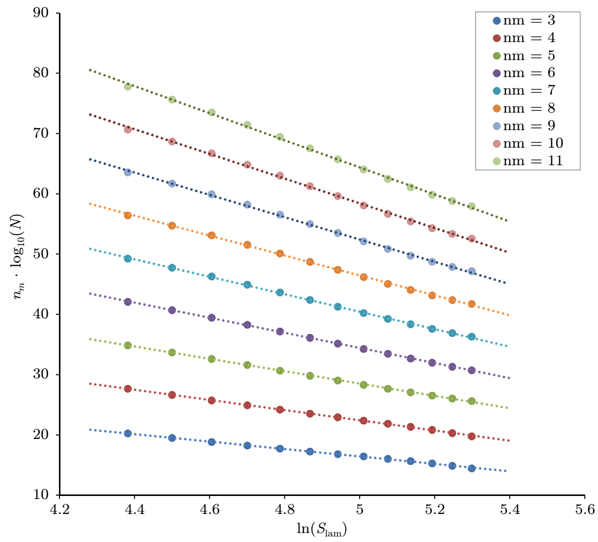

Fig. 3 $n_m \cdot \log_{10}(N)$ plotted versus $\ln(S_{lam})$ for $t_m = 0.6$ [mm]

The fit is described by the following equation:

$$n_m \cdot \log_{10} N = A_1 \cdot \ln(S) + A_2 \tag{1}$$

The coefficient A_1 and A_2 of each curve is plotted against n_m. This process is then repeated for all thicknesses in the range of $t_m = 0.3-0.7$ [mm] and for each them a linear curve is obtained. The obvious point is that these curves intersect at $n_m = 1$ (physically impossible, since a minimum of two metal sheets are required, but a regression has no physical meaning). For this reason, the x-axis is shifted by 1 $(n_m - 1)$ to have the intersection point on the y-axis, see figure 4 and 5. The following equations describe the fit:

$$A_1 = B \cdot (n_m - 1) + a_1 \tag{2}$$

$$A_2 = C \cdot (n_m - 1) + a_2 \tag{3}$$

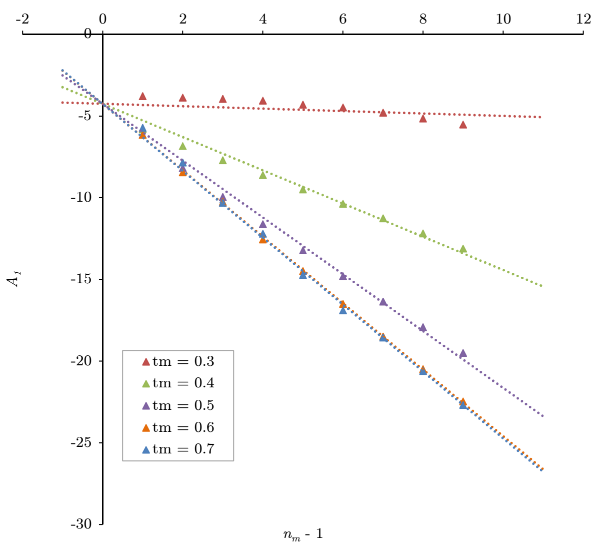

Fig. 4 A_1-coefficient plotted versus n_m

Looking at the plots of coefficient A_1 and A_2, it is now observable that all data point cross each other at $(n_m - 1) = 0$, meaning that only the slope B and C of the curves are changing depending on t_m. For this purpose, it is decided to fix the a_1 and a_2 coefficients to only have the slopes B and C as function of t_m. The best value for the a_1 and a_2 coefficients is determined by fitting every curve (for each t_m) with the best available linear fit $A_x = D_x \cdot (n_m - 1) + a_x$. Consequently, the average of all a_x is taken and assumed to be equal to a_1 or a_2. Once, the a_1 and a_2 coefficients are determined, the B and C coefficient (slope) of the curve is calculated using the best fit principle.

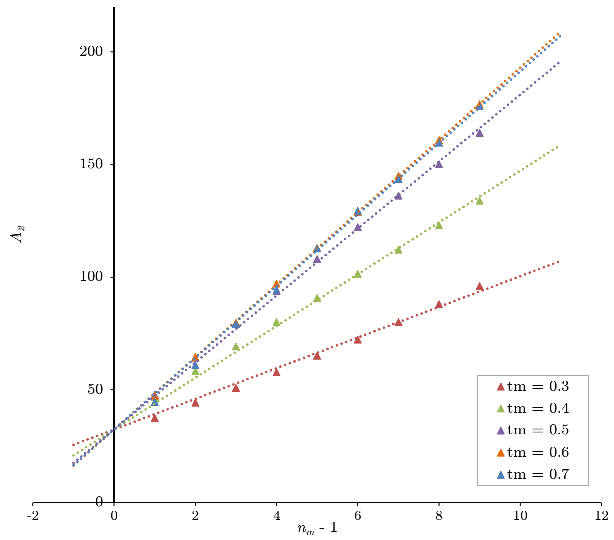

Fig. 5 A_2-coefficient plotted versus n_m

In the next step, t_m is incorporated in the equation. The B and C coefficient are plotted against t_m, like in figure 6 and 7, and approximated by a third order polynomial function given by:

$$B = b_1 + b_2 t_m + b_3 t_m^2 + b_4 t_m^3 \quad (4)$$

$$C = c_1 + c_2 t_m + c_3 t_m^2 + c_4 t_m^3 \quad (5)$$

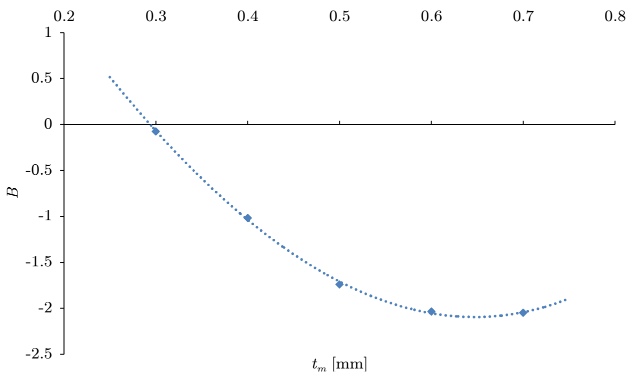

Fig. 6 B-coefficient plotted versus t_m

Finally, the cycles to crack growth N is linked to the applied laminate stress S_{lam} and the lay-up parameters n_m

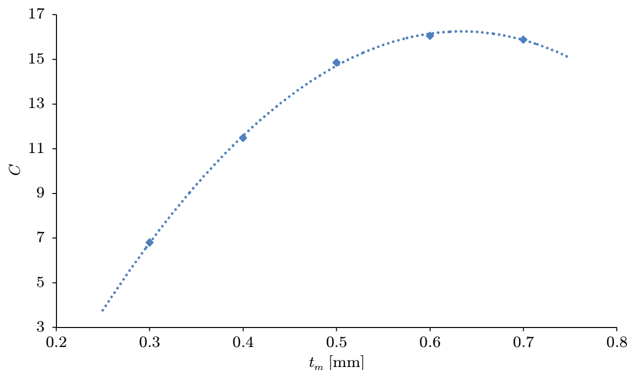

Fig. 7 C-coefficient plotted versus t_m

and t_m by combining all the equations. The approximate function in full-form is given by the following equation:

$$\log_{10}(N) = \frac{1}{n_m} \cdot \Big[\ln(\sigma_{lam}) \Big(a_1 + (n_m - 1)\left(b_1 + b_2 t_m + b_3 t_m^2 + b_4 t_m^3\right) \Big) + a_2 + (n_m - 1)\left(c_1 + c_2 t_m + c_3 t_m^2 + c_4 t_m^3\right) \Big] \quad (6)$$

For this particular case, the coefficients of the approximate functions are given in table 5.

Coefficients			
a_1	-4.253	a_2	32.426
b_1	4.304	c_1	-17.262
b_2	-17.644	c_2	103.598
b_3	8.694	c_3	-76.576
b_4	5.083	c_4	5.450

Table 5 Function coefficients for Glare2A

The approximate function is valid for $n_m \geq 2$. This function is also valid for lay-ups with out-of-the-range layer numbers, since the relationship between A_1/A_2 coefficients and n_m is described by a linear curve. It is expected that lay-ups with higher n_m will follow the same trend. However, in case of layer thickness, only values within the range should be considered, since the influence of t_m is not predictable outside the range due to the third order polynomial function, unless it is accounted for. In conclusion, the most accurate results are expected when the full lay-up and stress range is considered in the database for the regression analysis.

5 Results

The Alderliesten model is simplified to equation 6. Validity of curves are only per grade and the function alone has no physical meaning and cannot replace the model, but this fitness approximation can ensure that the drawbacks of the prediction model in the optimisation routine can be eliminated.

Constrained variables		
Initial crack length [mm]	a_0	26
Final crack length [mm]	a_f	120
Stress ratio [-]	R	0.1
Number fibre layers [-]	n_f	$n_m - 1$
Grade		Glare4B
Orientation fibre plies [°]	θ	$< 90/0/90 >$
Grade		Glare2A
Orientation fibre plies [°]	θ	$< 0/0 >$

Table 6 The constrained variables of the approximate function

Free variables		
Laminate stress [MPa]	S_{lam}	70 – 280 (step: 30)
Thickness metal layer [mm]	t_m	0.2 – 1.0 (step: 0.1)
Number metal layers [-]	n_m	2 – 20 (step: 1)

Table 7 The free variables of the approximate function

A larger design range is considered for verification. The process described in section 4 is repeated for Glare2A and Glare4B for the given parameters in table 6 and 7. The coefficients for both approximate functions are given in table 8. The data points obtained from the Alderliesten model are compared to the approximate function for Glare4B-9/8-0.6 and Glare2A-5/4-0.5, see figure 8. A perfect fit is obtained with a coefficient of determination of $R^2 = 0.9998$ and $R^2 = 0.9983$, respectively. In general, the approximate function replaces the prediction model with a coefficient of determination between $R^2 = 0.9500$ and $R^2 = 0.9999$, which means an overall good reproduction of the predicted data is obtained.

5.1 Optimisation

Initial optimisation results showed a decrease of computation time from average 5,500 to 5 seconds, while the same optimal lay-ups were obtained in both approaches. The scope of this paper will not include the implementation of the optimisation procedure or the discussion of the optimisation result.

5.2 Verification

Verification of the design optimisation methodology has been performed in two ways. First, the lay-ups predicted with the design optimisation approach using the simplified relation were implemented in the Alderliesten prediction model, to verify whether the resultant number of cycles comply with the initial design requirement. Secondly, the design optimisation approach using the approximate function has been verified against the design optimisation approach using the Alderliesten model. The latter is an alternative verification of the simplified relationship illustrated in figure and 8.

Glare4B				Glare2A			
a_1	-1.5693	a_2	11.9921	a_1	-1.644	a_2	12.352
b_1	-1.1439	c_1	11.8853	b_1	-0.399	c_1	8.725
b_2	-3.6207	c_2	15.4718	b_2	-5.404	c_2	24.200
b_3	5.2067	c_3	-24.4945	b_3	6.461	c_3	-31.011
b_4	-2.2579	c_4	10.9410	b_4	-2.474	c_4	12.182

Table 8 Function coefficients for Glare4B and Glare2A

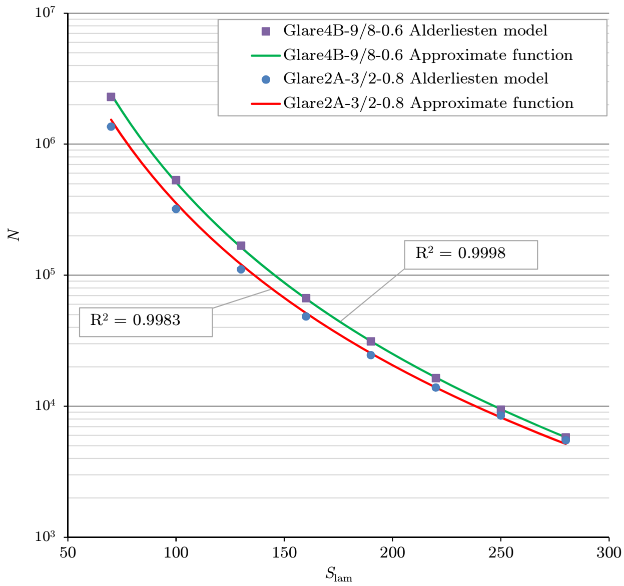

Fig. 8 Verification for Glare4B-9/8-0.6 and Glare2A-3/2-0.8

6 Conclusion

An fitness approximation for the crack propagation prediction model has been developed that can be used in the design optimisation approach for FML lay-ups. In this method, the Alderliesten crack growth model was replaced by an approximate function while genetic algorithm was implemented as optimisation routine. This approximate function was verified against data points obtained from the Alderliesten model and it achieved accurate results while the implementation in the optimisation caused the computation time to decrease immensely. However, the fitness approximation is limited to a single FML grade. The use of multiple approximate functions are needed when multiple grades are selected in the design procedure. In this case, the fitness of the lay-ups is calculated by the corresponding approximate function.

7 Future Work

Further research will focus on developing a design tool for an FML structure with the aim of developing a methodology to design and optimize FML lay-ups for fatigue & damage tolerance criteria. The optimisation procedure is implemented in genetic algorithm [11], which is used to find suitable lay-ups for given load en design requirements. Based on these requirements a range of satisfying lay-ups are obtained and a selection is made according to the lowest weight objective. In [4], the implementation of genetic algorithm is performed for FCI. The same procedure is also applied for FCP with the difference that the prediction model is replaced by a fitness approximation, as described in this paper. Further research will also focus on multi-constraint optimisation and compatibility between lay-ups on the segments in order to assure that the target model can be directly applicable as an optimisation tool for a complete structure.

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