ANALYSIS OF INTER-LAMINAR STRESSES IN COMPOSITE THIN SHELL

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Abstract

In this study the interlaminar stresses in composite shell panels subjected to mechanical loading is studied. For solution of the problem, from the three-dimensional elasticity formulation and applying the boundary and displacement conditions a reduced displacement field is determined for laminated shell panels subjected to extension. Then the layerwise theory is used for solution of the problem. This model is then utilized to study the free-edge effect and interlaminar stresses in laminated panels subjected to an axial extension. Numerical results are then presented for the interlaminar stresses within the shell panels.

1 Introduction

In composite structures interlaminar stresses arise in regions near the edges due to material discontinuity and mismatch in the elastic properties of adjacent layers. These stresses can lead to delamination and failure of laminated composites at loads that are much lower than the failure loads predicted by the classical lamination theories. Accurate determination of three-dimensional stress state in the boundary-layer regions of laminated plates and shells is therefore crucial in order to correctly describe the laminates behavior and to prevent their early failure. The interlaminar stress distribution in composite laminates has been investigated for some years. The objective of present work is to determine the stress distribution in the interior and boundary-layer regions of cross-ply circular cylindrical shell panels under axial extension. Starting from the most general form of displacement field within the elasticity theory, for a long shell panel a reduced displacement field is obtained by making reasonable assumptions in conjunction with the physical behavior of cross-ply shell panels. Using this simplified displacement field a layerwise theory is then used to analyze the extensional problem of cross-ply shell panels with free edges. Also for cross-ply panels with a special set of edge supports an analytical elasticity solution is developed to demonstrate the accuracy of LWT solution.

The interlaminar stress distribution in composite laminates has been investigated for some years. The relatively recent survey paper of Kant and Swaminathan [1] reviews the appropriate papers on the interlaminar stresses in laminated composite plates. For completeness, however, the pertinent references on the subject are discussed here. In 1970 Pipes and Pagano [1, 2] employed the finite difference method to generate numerical results for interlaminar stresses in symmetric balanced laminates using the reduced form of the elasticity equations. In 1971 Pipe and Daniel [3] verified that in laminated plates the free-edge effect is confined to a boundary-layer region approximately equal to the laminate thickness. Tang and Levy [4] used the boundary-layer theory to study the interlaminar stresses in symmetric laminated composite plates in extension. Later Hsu and Herakovich [5] studied edge effects in symmetric angle-ply composite laminates by using a perturbation technique. Wang and Choi [6, 7] studied the boundary-layer effects in symmetric balanced laminate by means of Lekhnistskii's stress potentials. Wang and Crossman [8, 9] studied the edge-effect problem of a symmetric balanced composite laminate under uniaxial extension and thermal effects by finite element method. Whitcomb et al. [10] investigated the discrepancies in the results obtained by various authors and the reliability of the finite element approach. Murthy and Chamis [11] determined interlaminar stresses in composite laminates under various loadings such as in-plane and out-of-plane shear/bending by using a three dimensional finite element method. In 2000, Cho and Kim [12] proposed an iterative method and determined the stresses in composite laminates subjected to extension, bending, twisting and thermal loads by using the complementary virtual work and the extended Kantorovich method. Shau and Soldatos [13] determined stress distributions in angle-ply laminated plates subjected to the cylindrical bending with various sets of edge conditions. Recently Tahani and Nosier [14] studied the free-edge stresses in general cross-ply composite laminates under extensional and thermal loadings by using a layerwise laminated plate theory (LWT). They also used LWT to study the interlaminar stresses in the bending problems of cross-ply laminated plates [15].

The objective of present work is to determine the stress distribution in the interior and boundary-layer regions of symmetric and unsymmetric cross-ply circular cylindrical shell panels under axial extension. First a reduced displacement field is obtained by making reasonable assumptions in conjunction with the physical behavior of cross-ply shell panels. A layerwise theory is then used to analyze the extensional problem of cross-ply shell panels with free edges using this simplified displacement field. The numerical results is presented for the distribution of the interlaminar normal and shear stresses in the shell panel subjected to extension.

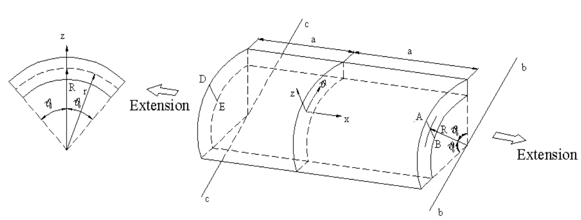

Fig. 1. Laminated circular cylindrical shell panel in extension; geometry and coordinate

2 Theoretical Formulation

A laminated circular cylindrical shell panel with perfectly bonded laminae of arbitrary fiber orientations is shown in Fig. 1. The panel is assumed to be long and extended in the x-direction. The displacement components in the *x*-, θ-, and *r*- directions of a material point initially located at (x,θ, r) in the kth ply of the shell are denoted by $u_1^{(k)}(x,\theta,r)$, $u_2^{(k)}(x,\theta,r)$, and $u_3^{(k)}(x,\theta,r)$ respectively. In Fig. 1, $r=R+z$ where R is the mean radius of the laminated shell panel. If the ends of the shell are assumed to be gripped in such a way so that these rotations are restrained while the ends are being extended, it can be shown that the displacement field of the panel could be reduced to;

$$\begin{aligned} u_1^{(k)}(x,\theta,r) &= C_6 x \\ u_2^{(k)}(x,\theta,r) &= v^{(k)}(\theta,r) \\ u_3^{(k)}(x,\theta,r) &= w^{(k)}(\theta,r) \end{aligned} \quad (2)$$

It mused be noted that in the layerwise theory each actual physical layer in a laminate is assumed that is made of imagined layers with same fiber direction as the actual layer. These imagined layers are often referred to as numerical (or mathematical) layers. So in the layerwise theory the panel is divided to N mathematical layers with N+1 mathematical surface. For this problem the displacements field of the k^{th} mathematical layer in the panel is shown by V_k and W_k. Now by applying the layerwise theory of Reddy the displacement field in the panel could be written based of the displacements of the mathematical surfaces as

$$\begin{aligned} u_1(x,\theta,z) &= C_6 x \\ u_2(x,\theta,z) &= V_k(\theta)\Phi_k(z) \\ u_3(x,\theta,z) &= W_k(\theta)\Phi_k(z) \end{aligned} \quad (3)$$

In which $\Phi_K(z)$ is the Lagrangian interpolation function. Now applying the infinitesimal theory of elasticity the strain field in the panel based on the displacement field in (3) could be obtained as

$$\varepsilon_x = C_6\,,\ \varepsilon_\theta = \frac{1}{R}(V_k' + W_k)\Phi_k\,,\ \varepsilon_z = W_k\Phi_k'$$
$$\gamma_{\theta z} = \frac{1}{R}(W_k' - V_k)\Phi_k + V_k\Phi_k'\,,\ \gamma_{\theta x} = \gamma_{xz} = 0 \qquad (4)$$

The equilibrium equations of the panel could be obtained using the principle of minimum total potential energy. The equilibrium equations of the layers are obtained as

$$\delta V_k:\ \frac{1}{R}M_\theta^k{,}_\theta - Q_\theta^k + \frac{1}{R}R_\theta^k = 0$$
$$\delta W_k:\ \frac{1}{R}R_\theta^k{,}_\theta - N_z^k - \frac{1}{R}M_\theta^k = 0 \qquad (5)$$

In which the generalized stress resultants are defined as;

$$(M_\theta^k, R_\theta^k) = \int_{-h/2}^{h/2}(\sigma_\theta, \sigma_{\theta z})\Phi_k dz\,,$$
$$(N_z^k, Q_\theta^k) = \int_{-h/2}^{h/2}(\sigma_z, \sigma_{\theta z})\Phi_k' dz \qquad (6)$$

In which the stresses could be obtained as

$$\begin{Bmatrix}\sigma_x\\ \sigma_\theta\\ \sigma_z\\ \sigma_{\theta z}\end{Bmatrix}^{(k)} = \begin{bmatrix}\overline{C}_{11} & \overline{C}_{12} & \overline{C}_{13} & 0\\ \overline{C}_{12} & \overline{C}_{22} & \overline{C}_{23} & 0\\ \overline{C}_{13} & \overline{C}_{23} & \overline{C}_{33} & 0\\ 0 & 0 & 0 & \overline{C}_{44}\end{bmatrix}^{(k)} \begin{Bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \varepsilon_z\\ \gamma_{\theta z}\end{Bmatrix}^{(k)} \qquad (7)$$

By substitution of (4) in to (7) and the subsequent results into (6) and (5) the equilibrium equations could be written in the form of displacement components. These equations could be solved numerically. Then the strains in each layers could be obtained and the stresses in the layers could be obtain by the stress-strain relation in (7).

3 Numerical Results

The numerical results for distribution of the stresses are presented in this section. In what follows several numerical examples are presented for cross-ply panel subjected to uniform extension loading. The panel is assumed to made from high modulus Graphite/Epoxy lamina with mechanical properties as, E_1=137.9 Gpa, E_2=E_3=14.48Gpa, G_{12}=G_{13}=G_{23}=5.86Gpa, ν_{12}= ν_{13}= ν_{23}=0.21.

As said before in the layerwise theory each actual physical layer in a laminate is assumed that is made of imagined layers which are often referred to as mathematical layers. So for increasing the accuracy in the results, layers that are near the interfaces at which inter-laminar stresses are interested may be divided further with respect to the layers that are far from the desired interfaces. The number of subdivision through the thickness can be greater than, equal to, or less than the number of material layers. In what follows, several numerical examples are presented for cross-ply shell panels subjected to uniform extension. In this study each physical layer is divided into 6 mathematical layers.

Fig. 2 and Fig. 3 show the distribution of interlaminar normal and shear stresses in symmetric $[90^\circ/0]_s$ cross-ply panel in the interfaces of layers. It is seen from Fig. 2 that the interlaminar normal stress σ_z for this panel in 0˚/0˚, (z/h_k=0) interface is negative in the edge. It is to be noted that compressive interlaminar normal stress does not cause to delamination of the layers. Also it is seen that σ_z in z/h_k=-1 and z/h_k=1 which is the interface of 90/0 and 0/90 interface approximately is equal for this panel. From Fig. 3 it is seen that transverse shear stress $\sigma_{\theta z}$ in the middle surface of this panel is zero. It is seen that the transverse shear stress in the thickness of $[90^\circ/0]_s$ at z/h_k=-1 and z/h_k=1 approximately are equal in magnitude and is apposite in sign.

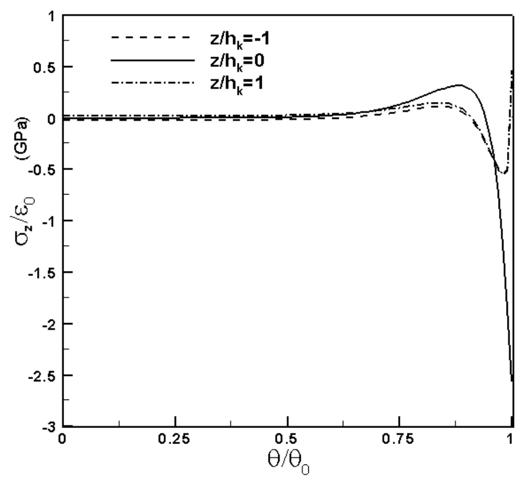

Fig. 2. Distribution of interlaminar normal stress in the interfaces of $[90^\circ/0]_s$ panel, (R/h=30, 2b/h=8)

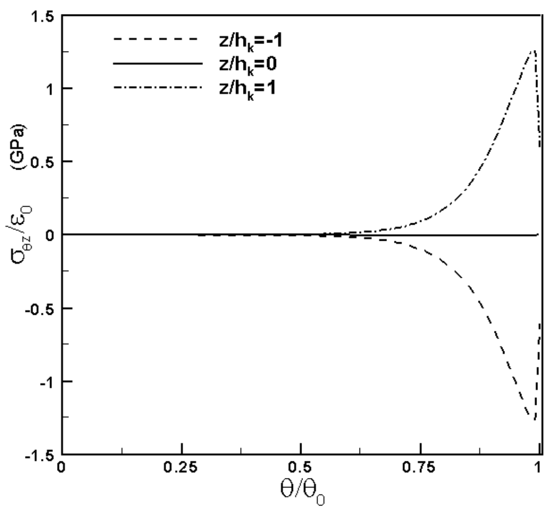

Fig. 3. Distribution of interlaminar shear stress in the interfaces of $[90^{\circ}/0]_s$ panel, (R/h=30, 2b/h=8)

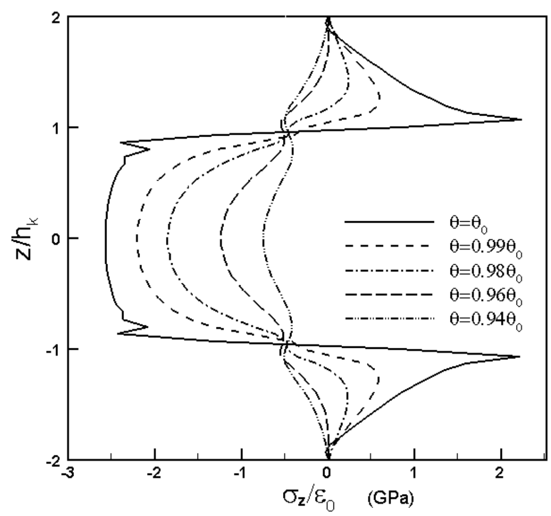

Fig. 4. Distribution of interlaminar normal stress σ_z through the thickness in $[90^{\circ}/0]_s$ panel under uniform axial strain ε_0, (R/h=30, 2b/h=8)

The variation of σ_z through the thickness of the symmetric cross-ply $[90^{\circ}/0]_s$ panel under uniform axial strain ε_0 is in the vicinity of edge shown in Fig. 4. It is noted that the value of transverse normal stresses σ_z diminishes away from the free edge and rise suddenly near the edge. Also it is seen that the maximum positive value of σ_z in this panel occur in the 90 layer and the maximum negative value of σ_z is occur within 0 layers both near the 0/90 interfaces at the free edge and there are sharp jumps in interlaminar stresses in the interfaces of layers with different orientation. The distribution of interlaminar normal stress is symmetric to middle plan of panel (z=0). Also the variation of transverse shear stresses $\sigma_{\theta z}$ through the thickness of this symmetric cross-ply panel is shown in Fig. 5. The distribution of interlaminar shear stress is anti symmetric to middle plan of panel ($z = 0$). From Fig. 5 it is noted that exactly at the free edge $\theta=\theta_0$ the shear stress $\sigma_{\theta z}$ satisfy the traction free boundary condition expect near the interfaces of layers with different orientation. Also it is seen that in Fig. 6 and Fig. 5 the stresses at $z/h_k=\pm 1$ are equal to zero so it is concluded that the LWT solution satisfies traction free boundary conditions in top and bottom surfaces of the panel.

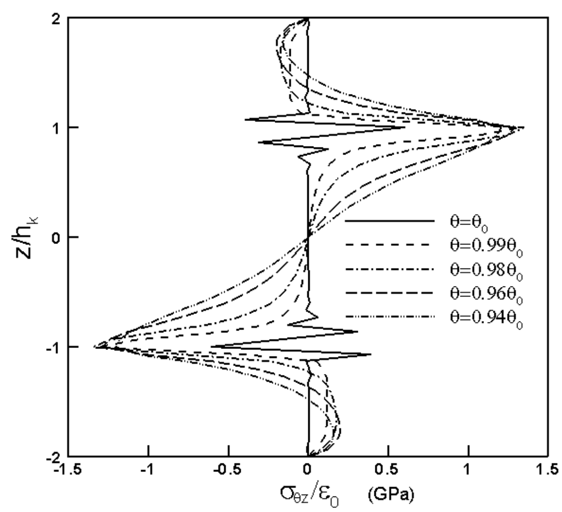

Fig. 5. Distribution of interlaminar shear stress $\sigma_{\theta z}$ through the thickness in $[90^{\circ}/0]_s$ panel under uniform axial strain ε_0, (R/h=30, 2b/h=8)

4 Conclusion

The interlaminar stresses in composite shell panels subjected to mechanical loading are studied. For solution of the problem, from the three-dimensional elasticity formulation and applying the boundary and displacement conditions a reduced displacement field is determined for laminated shell panels subjected to extension. Then the layerwise theory is used for solution of the problem. This model is then utilized to study the free-edge effect and interlaminar stresses in laminated panels subjected to an axial extension. In the numerical results the Graphite/Epoxy laminated composite shell panel is studied. The numerical results are then presented for the distribution of stresses and interlaminar stresses within symmetric and unsymmetric cross-ply shell panels.

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