29th Congress of the International Council of the Aeronautical Sciences
St. Petersburg, Russia; September 7-12, 2014

ENERGY APPROACH FOR AIRCRAFT VELOCITY OPTIMAL CONTROL DESIGN

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Keywords: *aircraft control design, energy approach, optimal control design*

Abstract

The report proposes the energy approach to optimal control design of the aircraft velocity vector based on the balance of energies and on considering such parameters of flight as the trajectory slope θ and roll γ , i.e. the coordinates of the state vector, which at a given speed of flight require the increase of thrust for the implementation of the allowable range of their values.

1 Introduction

Solution of the problem of automatic control of the aircraft velocity vector assumes that the parameters of movement do not extend beyond the operational constraints. Since the aircraft can perform the complex maneuvers the limits of the permissible range of velocity vector variation are complex functions of state coordinates and available energy resources.

The report proposes the energy approach to optimal control design of the aircraft velocity vector based on the balance of energies and on considering such parameters of flight as the trajectory slope θ and roll γ , i.e. the coordinates of the state vector, which at a given speed of flight require the increase of thrust for the implementation of the allowable range of their values.

For example, let us regard flight at a given altitude with the speed Vo. This mode corresponds to a certain value of thrust P_o . Suppose that the aircraft is turning through the change in the angle of roll. To maintain the desired altitude of flight, it is necessary to increase the angle of attack. The flight velocity is reduced by increasing drag. In this case, the thrust P_o is spent on maintenance of the new velocity V_1 and at a certain non-zero angle of roll $\gamma \neq 0$.

2 The triangle of power

For the geometric interpretation of the above it is proposed to use the equilateral triangle, which in the further will be called triangle of power. The height of the triangle will be interpreted as the thrust of the powerplant. On the sides of the triangle in a certain scale the values of velocity V, angle of roll γ and the trajectory slope θ are presented. Let us use the property of an equilateral triangle - the sum of the normals from arbitrary internal point of a triangle to its sides, is constant and equal to the height of the triangle. Then it can be argued that the lengths of the normals, hanging on the sides of the triangle, are proportional to the current values of the thrust needed to maintain the values V, γ and θ. As an example, a triangle of power for flight mode $V(t) = V_0$, $H(t)=H_0=const$, $|\gamma(t)|=|\gamma_0|$ is presented in Fig. 1.

Here is indicated:

OM - normal hanged from the point O to the side of AB of the power triangle, the length of which is proportional to the thrust necessary to create a speed of $V = V_0$;

DK - segment, which length is proportional to the magnitude of the velocity $V(t) = V_0$;

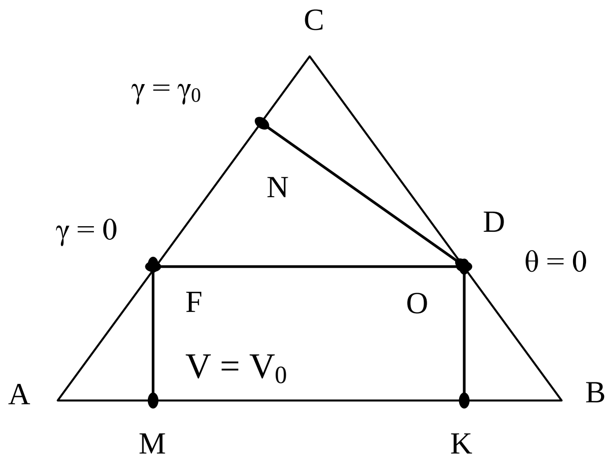

Fig. 1. Triangle of power

ON - normal hanged from the point O to the side of AC of the power triangle, which side represents the roll angle. The length of normal ON is proportional to the additional thrust required to create a roll, equal to γ_0 at $V = V_0$, H=const;

DN - segment, which length is proportional to the value of the roll. The start of the countdown of a roll is positioned at point D, defined as the point of intersection of the line DF with a side AC.

The point O is called the equilibrium point, the position of which determines the possible values of the coordinates of V, γ and θ. In Fig. 1, point O coincides with the point F, which belongs to the side of BC. Note that side of BC represents trajectory slope θ and the point F corresponds $\theta = 0$. This means that in this case the flight with an increase of the altitude is impossible. For nonzero θ it is necessary to build another variant of the power triangle.

Note that the height h of the triangle meets the condition

$$\|h\| = \|OM\| + \|ON\|$$

and is determined by the characteristics of the engine. Scales of coordinates V, γ and θ are nonlinear and depend on the weight of the aircraft, the altitude and the Mach number of flight.

Fig. 2 presents the triangle of power, as on Fig.1, which additionally shows:

- D_1K_1 - minimum flight speed, equal $V_{пр\ огр}$;

- D_1N_1 - maximum roll angle for given power plant operating mode which enables to stabilize the flight altitude.

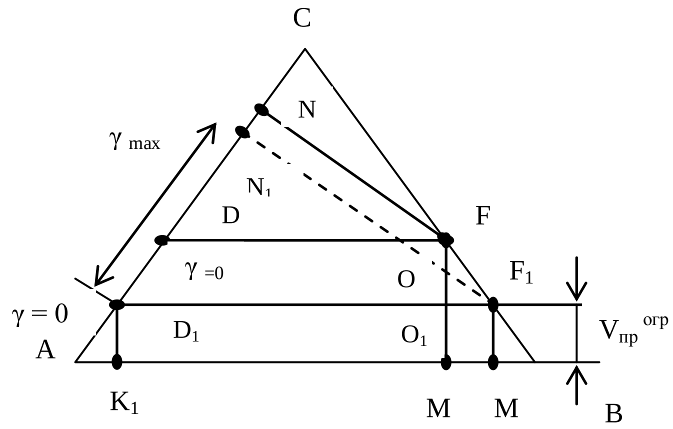

Fig. 2. Triangle of power with equilibrium point moved

As follows from the figure that in this case the equilibrium point has moved from point O to the point O_1. Further increase of the roll angle will lead to the breach of condition

$$V_{пр}(t) \geq V_{пр\ огр}, \qquad (2)$$

that means the aircraft velocity $V_{пр}(t)$ runs beyond the lower limit $V_{пр\ огр}$.

Therefore, it is necessary to form an adjustment signal that aimed to limit specified angle of roll, or for increasing thrust of the powerplant.

The case of equilibrium, or figurative, point belonging to the inner part of the power triangle ABC means that all managed coordinates are not equal to zero.

Figure 3 shows the current power triangle for medium-sized vessel for two situations: the climb at constant speed (more precisely, at constant Mach number) and the turn at constant flight altitude. Figure 3 shows that the flight was at the altitude $H = 2000$ m, flight speed corresponding Mach number M=0,6 and roll angle $\gamma = 0^0$, trajectory slope angle $\theta = 25^0$, if figurative point μ coincides with the point $F \in AB$. If the figurative point μ coincides with the point $E \in BC$ the parameters of flight are equal respectively $H = 2000$ м, $M = 0{,}6$, $\theta = 0$ и $\gamma = 72{,}3^0$.

The distribution of energy resources are as follows:

a) $\mu = F \in AB$

- $R_V = 2931$ kg – thrust of the power plant needed to create the flight speed $M = 0{,}6$ at altitude $H = 2000$ m;

- $R_\theta = 8169$ kg – thrust of the power plant required for the flight trajectory slope angle $\theta = 25^0$, Mach number $M = 0{,}6$ at altitude $H = 2000$ m;

- $R_\gamma = 0$ kg – thrust of the power plant required for the flight at $H = 2000$ m, $M = 0{,}6$, $\theta = 25^0$, $\gamma = 0$;

б) $\mu = E \in BC$ required thrust is distributed accordingly:

$R_V = 2931$ kg; $R_\theta = 0$ kg; $R_\gamma = 8169$ kg.

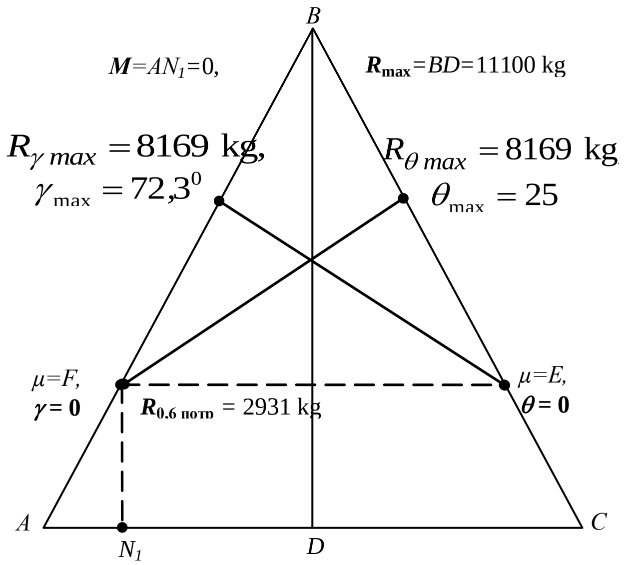

Fig. 3. Triangle of power for flight at $H = 2000$ m, $M = 0{,}6$

The maximum thrust of a power plant in this mode is R=BD=11100 kg note that for both cases the condition (2) holds. The sum of thrust, distributed to $R_V + R_\theta + R_\gamma$, must be equal to the maximum thrust of R generated by the individual channels power plant. Substituting specific thrust values for these parameters of flight will get

$2931\,кг + 8169\,кг + 0\,кг = 11100$ kg

or $\sum_{j=1}^{3} R_j \le R_{max}$.

Figure 4 shows two situations when equilibrium, or figurative, point is inside the triangle capacity ABC. The thrust distribution between the individual channels when flying at an altitude $H = 2000$ m, and with speed $M = 0{,}6$ and angles $\theta = 20^0$, $\gamma = 56^0$ shown in figure 4(a), and with angles $\theta = 10^0$, $\gamma = 69^0$ on figure 4(b). Here, as before, the basis is the fulfillment of the condition (2). Based on figure 4 really have:

а)

$$R_V + R_\theta + R_\gamma = 2931\,kg + 6609\,kg + 1560\,kg =$$
$$= 11100\,kg = R$$

б)

$$R_V + R_\theta + R_\gamma = 2931\,kg + 3377\,kg + 4792\,kg =$$
$$= 11100\,kg = R$$

Consider the build rule and basic properties of the limiting triangle of power. Let the flight performed at the specified altitude $H_з$ with a speed V_1. For definiteness, we believe that $\gamma = 0$ and $\theta = 0$. The current value of power plant thrust R_V is determined from the solution of the balance equations taking into account the integrated model "airplane - power plant". The value R_V forms the base for current triangle of power. Further, according to the model of a power plant the maximum permissible thrust R, corresponding to specified power plant mode, is determined. Then on the value of R is built limiting triangle of power AB_1C_1. The figurative point μ (figure 5) may be located either at the point B, or at the point E. If the point μ coincides with the point B, the height B_1C_1 omitted from this point will conform to the thrust value Rθ required to implement the mode of climb with the trajectory slope angle θ = 16,7°. Flight with a angle of trajectory greater than θ = 16,7° is impossible at altitude H=5000 m and M = 0,5, since the required value thrust R_θ^T exceeds the disposable thrust value Rθ. Therefore the possible range for slope angle θ = $\left[0^o \div 16{,}7^o\right]$.

The range of possible changes of roll angle is similarly defined. The figurative point μ must be moved to the point E. The resulting range of roll angle is $\gamma_з(t) \in \left[-63{,}8^o \div 63{,}8^o\right]$.

Consider the process of transition from the initial current power triangle to another possible power triangle when performing at the same time the turn and augmentation of altitude (turn with climb).Let aircraft performs flight at speed $M = 0{,}5$, and angles of roll $\gamma = 0$ and trajectory slope $\theta = 0$. It is assumed that the maneuver should be performed without afterburning mode of power plant with roll angle $\gamma_з(t) = 50^o$. The current power triangle ABC is shown in fig. 6.

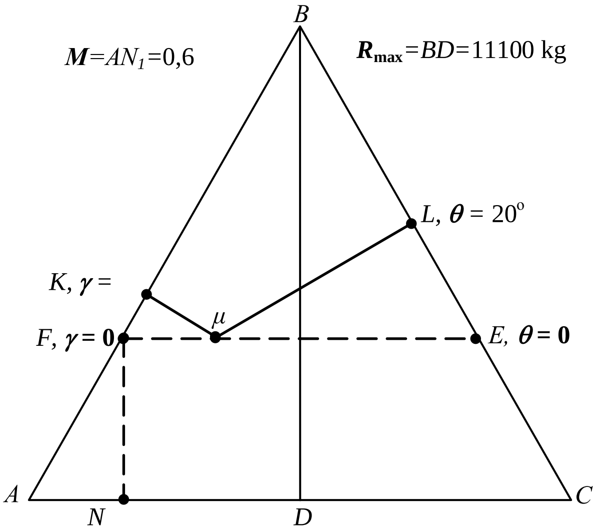

Fig. 4(a) - The power triangle, the figurative point μ is located within the inner part of the power triangle ABC

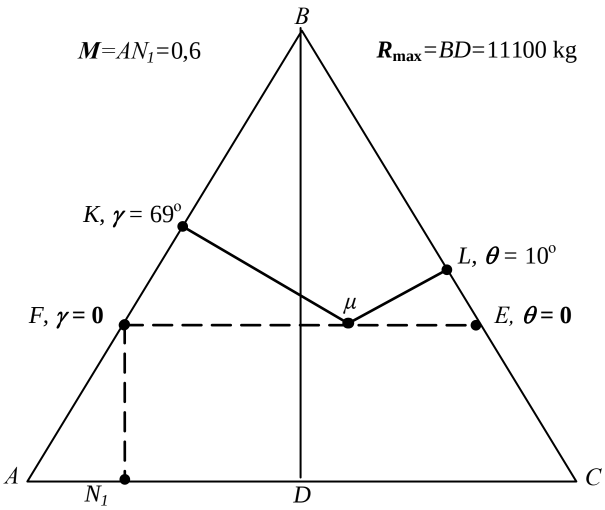

Fig. 4(b) - The power triangle, the figurative point μ is located within the inner part of the power triangle ABC

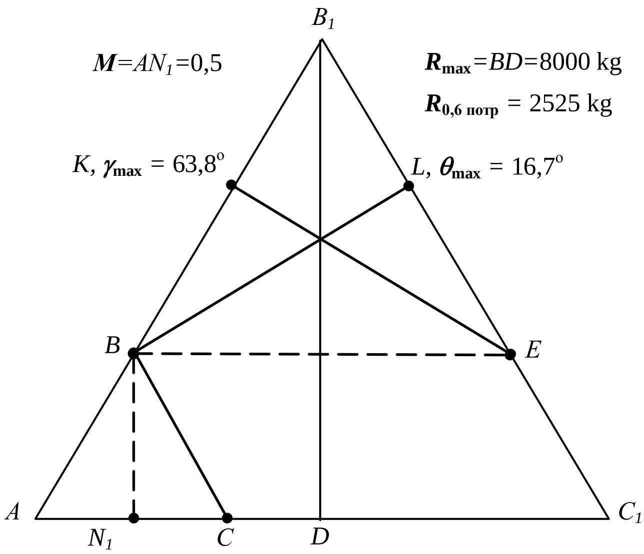

Fig. 5 - The power triangle at flight altitude H=5000 m and Mach number M = 0,5

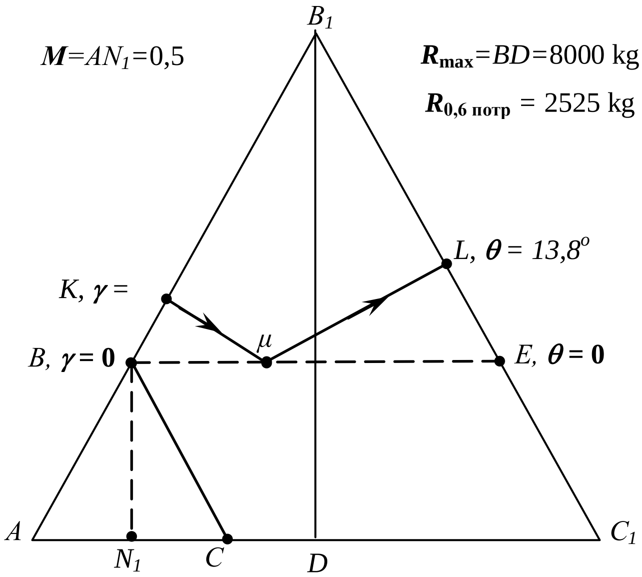

Fig. 6 - The power triangle corresponding to the turn with climb at flight altitude H=5000 m , Mach number M = 0,5, roll angle $\gamma_з(t) = 50^o$

Then on the side AB_1 the segment BK is delayed from point B in upward direction. The length of segment BK corresponds the roll angle $\gamma_з(t) = 50^o$ in the scale of roll. Then from point K a perpendicular to side AB_1 is drawn until it intersects with a segment BE. The point of intersection of the perpendicular and segment BE forms the required figurative point μ.

Segment μK is proportional to the value of the increment of power plant thrust required for flight at speed $M = 0{,}5$ and $\gamma(t) = 50^o$. Next, from the point μ drop a perpendicular to the side B_1C_1. The point L of intersection of that perpendicular and the side B_1C_1 determines the maximum possible angle of trajectory that can be implemented without afterburning mode of operation of the power plant. Indeed, the sum of segments BN_1, μK, μL is the height of the power triangle or, in other words, maximum thrust of the power plant on this mode of flight.

Consider the harmonization process for given values of controlled coordinates on the flight modes most frequently encountered in practice. Assume that aircraft performs the climb at given values of the velocity $V_з(t)$ and angle of trajectory slope $\theta_з(t)$.

The current power triangle ABC and the achievable power triangle AB_1C_1 are shown in figure 7.

The current thrust value R, as can be seen from figure 7, is slightly lower than maximum allowable. Therefore, this mode of flight can be carried out without restrictions on specified values. But already at the height of H = 4270 m the required thrust value will coincide with the maximum possible (figure 8).

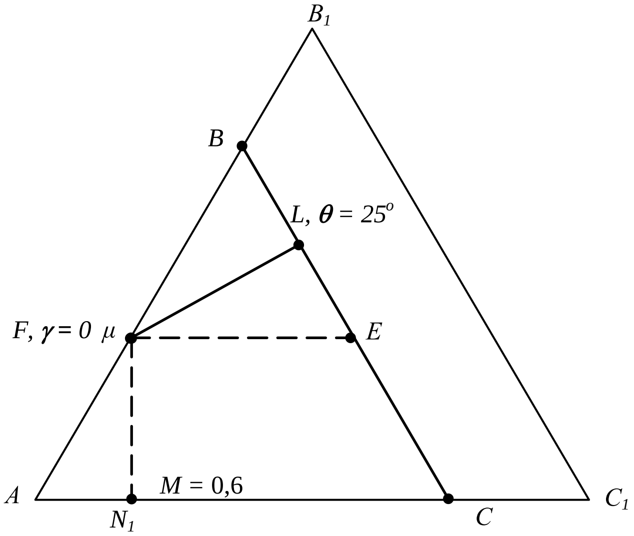

Fig. 7. The current power triangle ABC and the achievable power triangle AB_1C_1 for parameters of flight: H=1000 м, M = 0,6, $\theta = 25^o$

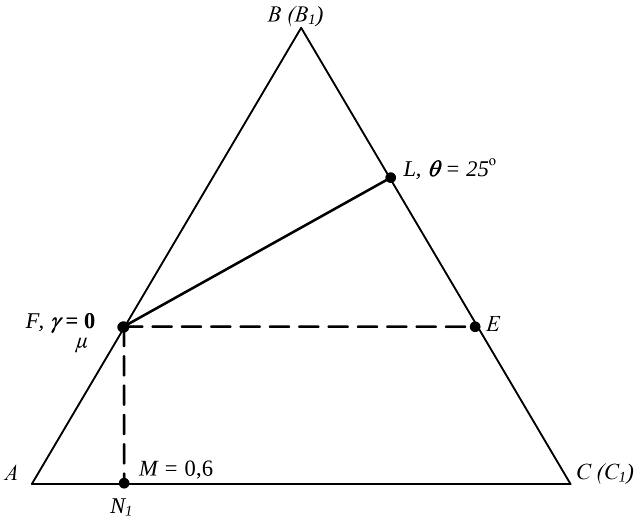

Fig. 8. The current power triangle ABC and the achievable power triangle AB_1C_1 for parameters of flight: H= 4270 м, M = 0,6 и $\theta = 25^o$

Further growth of the height H = 4500 m also accompanied by decrease power plant thrust. In this case the required thrust is less than the maximum possible. This means that the further climb trajectory slope angle $\theta = 25^o$ is impossible and the correction of the required trajectory slope angle value is necessary. Figure 9 shows this correction procedure .The correction process continues until figurative point μ is aligned with the point B_1.

Physically this means that the plane was transferred to level flight and a further climb in this operating mode of the power-plant is impossible.

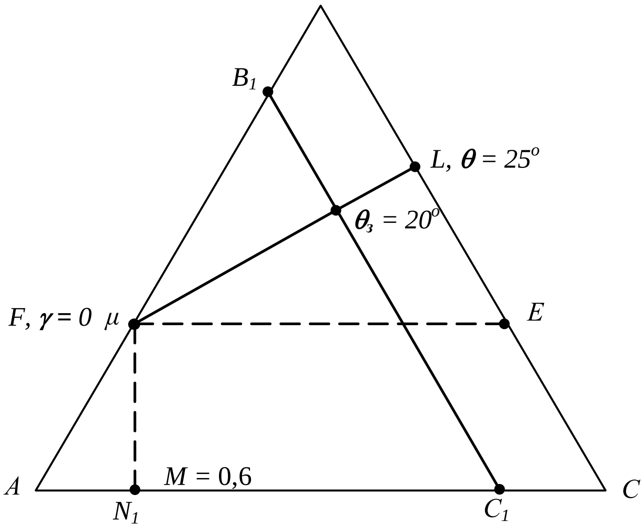

Fig. 9. The power triangle for the correction of the required trajectory slope angle $\theta_3(t)$

3 The approach to flight control design

The report also considers the application of the proposed energy approach to the problem of aircraft flight velocity vector optimal control design. The control algorithms are based on the concept of inverse problems of dynamics. This concept considers the problem of synthesis of control algorithms along with the condition of providing the prescribed dynamic characteristics of synthesized systems [1-3].

Let dynamics of the object on control time interval $[t_0, t_к]$ be described by a system of ordinary differential equations

$$\dot{x}(t) = f(x,u,t)$$

with $x(t)$ - n- dimensional state vector, $f(x,u,t)$ - n- dimensional vector function, $u(t)$ - m - dimensional control vector.

It is required to determine the control $u(t)$, which on control time interval $[t_0, t_к]$ provides the minimum for integral scalar control functional, that includes scalar non-negative function L(x,u,t) and terminal part $S_K[x(t_K)]$.

The control must also provide that the discrepancies between the state coordinates

$x(t)$ and the desired path $y_c(t)$ would meet the condition of the following type:

$$\psi_1\left[\lambda,\dot{F}(x,y_c),\ddot{F}(x,y_c),\ldots F^{(k)}(x,y_c)\right]$$

with λ,β - arbitrary matrixes;

ψ_1 and ψ_2 - m - dimensional non-linear vector functions,

The results of this paper have been achieved due to the support of Russian Fund for Fundamental Research, project 14-08-01109-a.

Conclusion

The paper proposes a new form for presenting the relation between principal parameters of flight - the triangle of power, based upon the general idea of energy equilibrium. The build rule and basic properties of this triangle are discussed. The paper also reveals the triangle of power connection to some typical aircraft flight modes. It is also shown that proposed approach is useful for the problem of aircraft optimal control design.

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