29th Congress of the International Council
of the Aeronautical Sciences
St. Petersburg, Russia; September 7–12, 2014

GENERATION OF INITIAL GUESSES FOR OPTIMAL CONTROL PROBLEMS WITH MIXED INTEGER DEPENDENT CONSTRAINTS

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Abstract

Aircraft trajectory optimization problems often include discrete decisions such as flaps settings. In order to use discrete controls in optimal control problems (so-called Mixed Integer Nonlinear Programs, MINLP) reformulation methods are necessary to create a continuously differentiable problem. One of the most promising methods is the Variable Time Transformation (VTT) which is able to find the optimal switching sequence through optimization. However, the optimal result highly depends on the initial guess provided, if it even converges. This is especially true if discrete controls are involved. The aim of this paper is to produce more suitable initial guesses, which help to solve the MINLP fast and numerically stable.

1 Introduction

In dynamic systems often discrete controls arise which can only take values from a fixed set (e.g. $v = \{v_1, v_2, \ldots, v_N\}$). Discrete controls on an aircraft are the flaps, the landing gear and the spoilers. If discrete controls are used in an Optimal Control Problem (OCP), the problem is called Mixed Integer Nonlinear Program (MINLP). Taking into account discrete controls in an optimal control problem cannot be done directly, since the feasible set is disjoint and the discrete controls are not differentiable. A continuous differentiable reformulation is mandatory, since only gradient based optimization algorithms [5, 18] can manage the large number of optimization variables that appear in OCPs.

For the reformulation different methods are available, for instance dividing the problem into multiple phases [10, 11, 12, 17, 19], using the approach of inner convexification [11, 14, 16] or exploiting the shape of the hyperbolic tangent function [3, 13]. However, these methods either cannot find the optimal switching structure through optimization or are numerically not very stable. Currently, there are two approaches that can optimize discrete controls: the Variable Time Transformation (VTT) [4, 13] and the outer convexification [8, 15, 16]. Both are very similar in the formulation and only differ in the integration of the optimized dynamic system. In this paper the variable time transformation is used.

Additionally, to the changes of the discrete control, constraints that depend on their choice change instantly as well. On an aircraft, the speed constraints are dependent on the flap setting. Since in any optimization algorithm constraint bounds cannot be changed during an optimization run, a reformulation is necessary as well. Here, the vanishing constraint approach [6] is used which enables turning constraints "on" or "off" with a control function.

If the discrete control switching structure is subject to the optimization, some sort of switching cost function needs to be included to ensure that the optimal solution does not involve a switch at every discretization time step. However, this poses a problem, since the cost function introduces many minima into to OCP. Therefore, the solution depends on the initial guess provided. In this paper different approach to generate an initial guess to solve these type of problems are described and compared. The methods used are grid refinement, fixed bound optmization, B-Spline interpolation and the stitching of motion primitives. Additionally, the optimization process is divided into stages to ensure that the optimal switching structure is found.

The outline of the paper is as follows. Section 2 introduces the aircaft model used for the optimization, which is the BADA family 4 model from EUROCONTROL [2]. Special focus is given to the constraints that depend on the choice of the discrete controls. Afterwards, section 3 explains the variable time transformation, the switching costs and the vanishing constraints. The different initial guess approaches with the resulting optimization processes are introduced in section 4. In section 5 these approaches are applied on an example optimization problem. Section 6 concludes the paper.

2 Aircraft Model

The aircraft model used is the Base of Aircraft Data (BADA) from EUROCONTROL [2]. It offers a three degree of freedom model (3DOF) including equation for kinematics, aerodynamics, propulsion and is available for all common commercial aircrafts. The BADA family 4 takes into account different flap configuration with aerodynamic coefficients and dependent limits such as maximum lift $C_{L,max}$, maximum calibrated airspeed $V_{CAS,max}$ and the minimum and maximum vertical load factor $n_{z,min}, n_{z,max}$.

There are 8 states and 4 inputs for the aircraft model. Tables 1 and 2 state the names and symbols. Using a parameter vector for the aerodynamic coefficients, different flaps settings can be simulated. The flaps are represented through an integer variable $k = 1,2,3$; the corresponding flap settings are CRUISE, APPROACH and LANDING. Atmospheric influences are taken into account using the International Standard Atmosphere (ISA).

Table 1 Aircraft States

Name	Description	Unit
x	x-position in NED frame	m
y	y-position in NED frame	m
z	z-position in NED frame	m
V	kinematic velocity	m/s
χ	kinematic course angle	rad
γ	kinematic climb angle	rad
m	aircraft mass	kg
μ	kinematic bank angle	rad

Table 2 Aircraft Controls

Name	Description	Unit
C_L	lift coefficient	-
$\dot{\mu}$	time derivative of bank angle	rad/s
δ_T	thrust lever position	-
k	flap configuration setting	-

2.1 Equations of Motion

Aircraft dynamics are considered in the kinematic frame K, the aircraft position is given relative to a local north east down frame (NED). Thus the position ODEs are

$$\begin{aligned} \dot{x} &= V \cdot \cos\chi \cdot \cos\gamma \\ \dot{y} &= V \cdot \sin\chi \cdot \cos\gamma \\ \dot{z} &= -V \cdot \sin\gamma. \end{aligned} \tag{1}$$

The translational dynamics are defined as

$$\begin{aligned} \dot{V} &= \frac{T - D(k)}{m} - g \cdot \sin\gamma \\ \dot{\chi} &= \frac{L \cdot \sin\mu}{m \cdot V \cdot \cos\gamma} \\ \dot{\gamma} &= \frac{L \cdot \cos\mu}{m \cdot V} - \frac{g \cdot \cos\gamma}{V}. \end{aligned} \tag{2}$$

BADA version 4 has generic models for the thrust $T(\delta_T, M, \ldots)$, the drag $D(k, C_L, M, \ldots)$, and the lift $L(C_L, M, \ldots)$ where the drag is dependent on the current flap setting k. Here, M represents the Mach number, R the universal gas constant and T the current air temperature:

$$M = \frac{V}{a}, \qquad a = \sqrt{\kappa \cdot R \cdot T}. \tag{3}$$

2.2 Limitations

Additionally to the aircraft dynamics, constraints have to be taken into account to ensure a realistic flight path. Along normal continuous constraints such as bank angle limitations there are constraints that depend on the choice of the discrete control.

The discrete change in the flap settings influences aircraft limitations instantaneously. If the flaps are deployed, the maximum lift coefficient

$$C_L \leq C_{L,max}(k) \tag{4}$$

is increased, while the upper bound for the Calibrated Air peed (CAS)

$$V_{CAS} \leq V_{CAS,max}(k) \tag{5}$$

is reduced. Additionally the vertical load factor

$$n_{z,min}(k) \leq n_z \leq n_{z,max}(k) \tag{6}$$

box constraint changes. Please note, that for the vertical load factor, the model differs between clean (CRUISE) and nonclean (APPROACH, LANDING) configurations. These flap dependent constraints are taken into account with vanishing constraints described in section 3.

3 Discrete Control and Discrete Constraints

If discrete controls are used in an optimal control problem with gradient based optimization algorithms, a continuous problem reformulation is necessary. There have been many approaches in the past, some of them discussed in [13], but only a few offer the ability, that the switching structure is subject to the optimization. One of these approaches is the variable time transformation [4] used in this paper. At the same time constraints that depend on the discrete control choice have to be taken into account. In this paper, vanishing constraints [6] are used.

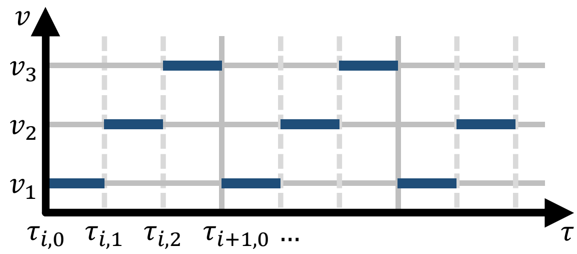

Fig. 1 Variable Time Transformation - Minor Grid [4]

3.1 Variable Time Transformation

The variable time transformation distorts the time grid to find the correct switching sequence through optimization. Let $N_TS \in \mathbb{N}$ be the number of discretization points. Then, assuming an equally distributed discretization, the step size is

$$h = \frac{t_f - t_0}{N_{TS} - 1} = \tau_{i+1,0} - \tau_{i,0} \tag{7}$$

with the resulting discretization grid (called major grid, see Fig. 1) of

$$t_i = t_0 + i \cdot h, \quad i = 0, \ldots, N_{TS} - 1. \tag{8}$$

For the variable time transformation this grid needs to be divided further. If $N \in \mathbb{N}$ is the number of discrete choices of the discrete controls, each major grid interval h is divided into N minor grid steps.

$$\tau_{i,j} = t_i + j \cdot \frac{h}{N}, \quad j = 0, \ldots, N-1 \tag{9}$$

The integration is carried out along the minor grid. Each of the minor grid steps represents a discrete control choice v_k (see Fig. 1).

For every minor grid step an optimizable variable $w_{k,i} \in [0,1]$ is defined which can stretch or shrink it, thus transforming the time. A value of $w_{k,i} = 0$ shrinks the minor grid to a singularity, rendering the corresponding discrete control inactive. On the other hand, a value of $w_{k,i} = 1$ stretches the discrete control over the major grid interval. The other discrete choices have been eliminated in this case. At each discretization step i the constraint

$$\sum_{k=1}^{N} w_{k,i} = 1 \tag{10}$$

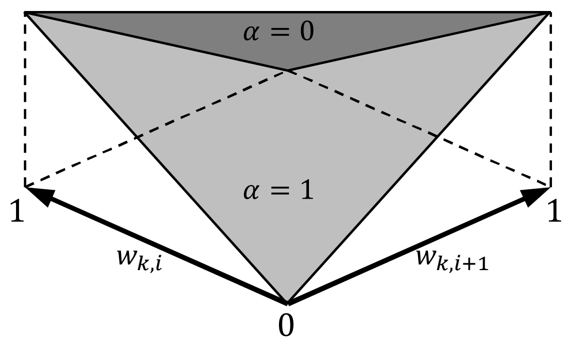

Fig. 2 Switching Cost Model [9]

has to be fulfilled. Thus, the major grid and minor grid always coincide at the major grid nodes. For the implementation, the integration algorithm has to be adapted. The state dynamics $f_k(x,u)$ is multiplied by the optimization parameter

$$\dot{x} = w_{k,i} \cdot f_x(x,u). \tag{11}$$

The variable time transformation enables the optimization algorithm to switch at every discretization step. However, especially for many application (e.g. flaps) slow switch dynamics are desired. To limit the number of switches a penalty is added to the cost function.

3.2 Switching Costs

Since flaps or the landing gear on an aircraft shall not be switch with high frequency, the number of switches needs to be limited. Additionally, it is desired that only one discrete control is chosen for a discretization step, which means the $w_{k,i}$ shall either be 0 or 1. Therefore, a switching cost approach is used [9]. A penalty function

$$J_p = \left(2 \cdot \alpha_{k,i} - 1\right) \cdot \left(w_{k,i} + w_{k,i+1} - 1\right) + 1 \tag{12}$$

is added to the cost function. Additional optimization variables $\alpha_{k,i} \in [0,1]$ need to be added to the OCP for every $w_{k,i}$. As before k represents the choice of the discrete control (minor grid step) and i the current index of the time discretization (major grid step). The variable $\alpha_{k,i}$ influences the gradient of the "two-dimensional plane" (see Fig. 2). Dependent on the sum of $w_{k,i} + w_{k,i+1}$ the optimal value for $\alpha_{k,i}$ is either 0 or 1.

3.3 Vanishing Constraints

The constraints (4-6) depent on the flap setting k. In any optimization algorithm the bounds for constraints cannot be changed during the optimization run. Therefore a formulation is necessary that accounts for changed constraints limits if the discrete control changes. Here, vanishing constraints are used to disable constraints if the corresponding discrete control is not active. A vanishing constraints has the form

$$w_k \cdot g_k \leq 0 \tag{13}$$

where constraint $g_k \leq 0$ is switched "on" or "off" dependent on a control function $w_k \geq 0$. In this case w_k coincides with the variable time transformation optimization parameter. If $w_k = 0$ the constraints $w_k \cdot g_k$ holds for any $g_k \in \mathbb{R}$. In case of $w_k > 0$ the constraints $g_k \leq 0$ has to be fulfilled. Thus the constraints (4-6) need to be adapted

$$\begin{aligned} w_k \cdot (C_L - C_{L,max}) &\leq 0 &(14)\\ w_k \cdot (V_{CAS} - V_{CAS,max}) &\leq 0 &(15)\\ w_k \cdot (n_z - n_{z,max}) &\leq 0 &(16)\\ w_k \cdot (n_{z,min} - n_z) &\leq 0 &(17) \end{aligned}$$

to the structure of the vanishing constraints. Please note that the box constraints (6) is divided into two separate constraints (16,17).

3.3.1 Vanishing Constraint Relaxation

Vanishing constraints in their original form violate the constraint qualification [7] at the point $w_k = g_k = 0$. To resolve this, the relaxation approach

$$w_k \cdot (t + g_k) - t \leq 0 \tag{18}$$

softenes the curve at the origin using a relaxation parameter t. The constraint qualification is no longer violated, but for $w_k = 1$ the constraint $q_k \leq 0$ is enforced.

4 Initial Guess Generation

In each optimal control problem a certain mission shall be optimized while minimizing a cost function. Here, the mission is defined by a series of waypoints. At each waypoint the states and state bounds are defined. The bounds are used in the optimization at the beginning and end of each phase. The state specified in the waypoint is used for the initial guess generation.

Additionally to the initial guess generation of the states and continuous controls, the discrete controls need to be initialized as well. Here the calculation of the switching structure follows a simple assumption. The extension of the flaps increases the drag of the aircraft. Since the cost function is the fuel consumption, it is assumed that the best choice for the flap setting is the lowest one that is still valid.

4.1 Optimization Process

If switching costs are present in an optimal control problem, changing the structure of the discrete controls becomes very difficult since every change initially increases the cost function. In other words, the switching cost model introduces many minima in the optimal control problem.

Therefore, the optimization needs to be divided into several stages. The switching cost penalty shall be used in the last optimization stage only. If the initial switching structure is not optimal, the lack of a switching penalty enables the optimization algorithm to find the local optimal one. The vanishing constraints remain active ensuring a feasible trajectory. In many cases, the resulting solution already has integer values for the $w_{k,i}$. In the last stage, the switching costs are activated and the optimization is restarted from the intermediate solution. During this stage, the number of switches is reduced to a suitable minimum and integer values are enforced.

4.2 Initial Guess Algorithms

In this subsection the different initial guess approaches are described. The grid refinement approach and fixed bound approach have poor initial guesses but follow two different strategies to generate an optimal solution. Contrary to this, the B-Spline approach and the motion primitive approach produce a good initial guess which is on the other hand more complex to generate.

4.2.1 Grid Refinement

The easiest way to generate an initial guess for an optimal control problem is by simply initializing the states and controls linearly. As mentioned before, the state data written into the mission waypoints is used to generate the initial guess. Here, the states and controls of the phases are initialized by linearly interpolating from one waypoint to the next. Assuming $\tau \in [0,1]$ is the normalized phase time, the interpolation formula is

$$\eta(\tau) = \eta_i + (\eta_{i+1} - \eta_i) \cdot \tau. \tag{19}$$

Please note that the η acts as a placeholder for x, y, z and V. In order to achieve a feasible trajectory between two waypoints, the kinematic course angle and the kinematic climb angle

$$\chi(\tau) = \arctan\left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right) \tag{20}$$

$$\gamma(\tau) = \arccos\sqrt{\frac{d}{l}} \tag{21}$$

$$d = (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 \tag{22}$$

$$l = (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2 \tag{23}$$

$$\mu(\tau) = 0 \tag{24}$$

are calculated using trigonometry. The bank angle is set to zero. At each discretization point, the lowest flap setting is chosen. The continuous controls can then be calculated using an inverse dynamics model. The resulting initial guess is feasible between the waypoints, but if more than two waypoints are involved for a mission, at intermediate waypoints the trajectory has defects. Therefore, this initial guess is poor and a strategy is needed such that the optimization algorithm is able to find an optimal solution.

In the grid refinement approach, the discretization density between two waypoints is reduced with the aim to reduce the number of optimization variables. The coarse discretization must be chosen in such a way, that it eases the optimization process, but at the same time is able to render the physical behaviour correctly. As mentioned before, the first optimization is carried out without switching costs. Afterwards, the

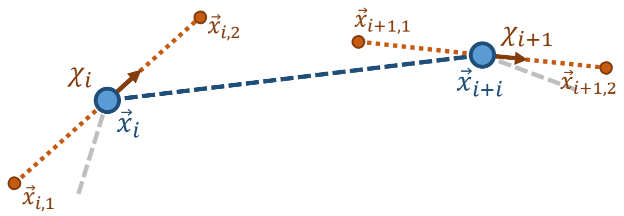

Fig. 3 Helper Waypoints

coarse discretized optimal solution is interpolated to generate a finer discretized initial guess. Then, the switching costs are activated and the second and final optimization is carried out.

4.2.2 Fixed Bound

The fixed bound initial guess is generated linearly as the grid refinement approach. However, instead of using a coarse discretization to make the optimization problem easier to solve, in the first optimization stage, the discrete controls are fixed and therefore not subject to the optimization. Thus the MINLP is converted to a continuous optimal control problem which is much easier to solve for the optimization algorithm. As before, the first stage is carried out without switching costs.

Once the first stage is sucessfull, a continuous feasible trajectory is found without defects at the mission waypoints. Then, in the second stage, the discrete controls can be taken into account in the optimization. In the third and last stage, the switching costs are activated.

4.2.3 B-Spline Interpolation

The aim of the B-Spline interpolation is to provide a connected smooth trajectory through all waypoints. The interpolation is carried out based on the information given in the waypoints that define the mission. Since the optimization is done in MATLAB, the build-in B-Spline interpolation algorithm is used. Interpolations are carried out for the position x, y, z and the speed V. However some information is lost throughout the process. For instance, the desired kinematic heading and climb angle is not taken into account. Therefore, intermediate waypoints are defined that influence the interpolation (see fig. 3). Using all these waypoints the interpolation is carried out.

In order to generate the full kinematic state of the aircraft, the derivatives of the splines need to be calculated as well. Therefore, for the position x, y, z the first derivative x', y', z' and second derivatives x'', y'', z'' are necessary. For the speed the first derivative V' is generated. Please note that the derivatives are with respect to the spline parameter $s \in [0,1]$ and not with respect to the real time t. To transform the derivatives into the time domain, the following calculations need to be carried out. The first time derivative is calculated using

$$\dot{x} = x' \cdot \frac{V}{p'} \qquad (25)$$

$$p' = \sqrt{x'^2 + y'^2 + z'^2} \qquad (26)$$

where the calculation of $\dot{y}, \dot{z}, \dot{V}$ are analog to $\dot{x}$. For the second derivative of the position values the equation

$$\ddot{x} = \frac{V}{p'^2} \cdot \left(x''V + x'V'\right) - \frac{x'V^2}{p'^4} \cdot \left(x'x'' + y'y'' + z'z''\right) \qquad (27)$$

is used. The kinematic course angle and kinematic climb angle are calcualted using the equations

$$\gamma = -\arctan\left(\frac{\dot{z}}{\sqrt{\dot{x}^2 + \dot{y}^2}}\right) \qquad (28)$$

$$\dot{\gamma} = -\frac{\ddot{z}\sqrt{\dot{x}^2 + \dot{y}^2} - \dot{z}\frac{\dot{x}\ddot{x} + \dot{y}\ddot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}}}{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \qquad (29)$$

$$\chi = \arctan\left(\frac{\dot{y}}{\dot{x}}\right) \qquad (30)$$

$$\dot{\chi} = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{\dot{x}^2 + \dot{y}^2}. \qquad (31)$$

Calculation of the bank angle is achieved through the loads in y and z direction.

$$n_y = \frac{V\dot{\chi}\cos\gamma}{g} \qquad (32)$$

$$n_z = \frac{V\dot{\gamma}}{g} + \cos\gamma \qquad (33)$$

$$\mu = \arctan\left(\frac{n_y}{n_z}\right) \qquad (34)$$

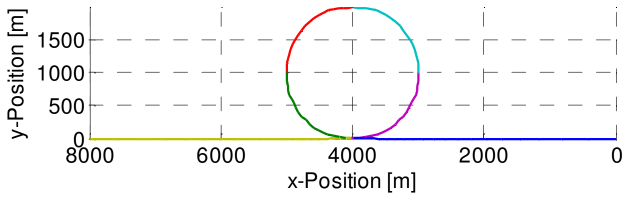

Fig. 4 Initial Guess generated using B-Spline interpolation

Once the discrete controls are chosen, the lift coefficient C_L and thrust lever position δ_T are calculated using an inverse model of the aircraft. The time derivative of the bank angle $\dot{\mu}$ is generated using finite differences.

Using this initial guess, the optimization is started without the switching costs. Once the optimal solution has been found, the switching costs are activated and the second optimization stage is carried out.

4.2.4 Stitching of Motion Primitives

The key idea behind the stitching of motion primitives approach is that every trajectory can be divided into reoccuring segments. This means, every trajectory can be created by stitching together motion primitives from a library. These motion primitives consist of trimmed flight conditions and transitions from one trimmed condition to another. If these transitions and trimmed conditions are carried out in an optimal way, the resultion flight path is optimal, or at least very close to the optimal solution [1].

For this approach a library needs to be generated. Then, the initial guess is stitched together using the library trajectories. Afterwards the optimization is started with disabled switching costs. After the correct switching sequence is found, the second and last optimization stage is carried out with switching costs. For simplicity, in this paper, the trajectory is stitched together with optimal trajectories that have been generated previously.

5 Results

The method is now applied to a multiple shooting optimization problem in which the fuel consumption is minimized. An Airbus A320-200 [2] performs a 360 deg turn while maintaining an approximate radius of $1000m$. Fig. 4 displays the initial guess for the B-Spline approach. Before and after the turn a straight segment of $4000m$ is attached. At the beginning and end of the flight path the aircrafts true air speed shall be between $100m/s$ and $120m/s$. Maximum allowed deviation from the reference altitude of $2000m$ is $\pm 10m$. The target major grid step size for the phase is $\tau = 0.05$, the coarse step size for the grid refinement approach is $\tau = 0.1$.

Fig. 5 shows the optimal solution generated with the B-Spline initial guess. The resulting switching structure can be seen in the top left subplot alongside with the calibrated air speed, the lift coefficient as well as the vertical load factor. In red the discrete control dependent bounds are shown, which are kept at every discretization step.

The switching structure of the discrete control starts and finishes with the clean configuration (CL) and switches to the approach configuration (APR) for the turn. The landing configuration (LDG) does not appear to be optimal in this case and was not chosen by the optimization.

Fig. 6 display the optimization results for the different initial guess approaches. All approaches produce approximately the same result, although the grid refinement and motion primitive approach result in a slightly different trajectory.

The discrete control switching for the grid refinement and the fixed bound approach start with the approach configuration, whereas the B-Spline and motion primitives approach start from the cruise approach. Otherwise the switching structure is the same. The discrete control switching structure has been stacked vertically to show the differences more cleary. Discrete control dependent constraints are not shown to keep the plots more tidy.

In Fig. 7 the different initial guess approaches are compared with respect to the required optimization time as well as the resulting cost function. The motion primitive approach is the fastest, since the initial guess was stitched together with optimal trajectories. The grid re-

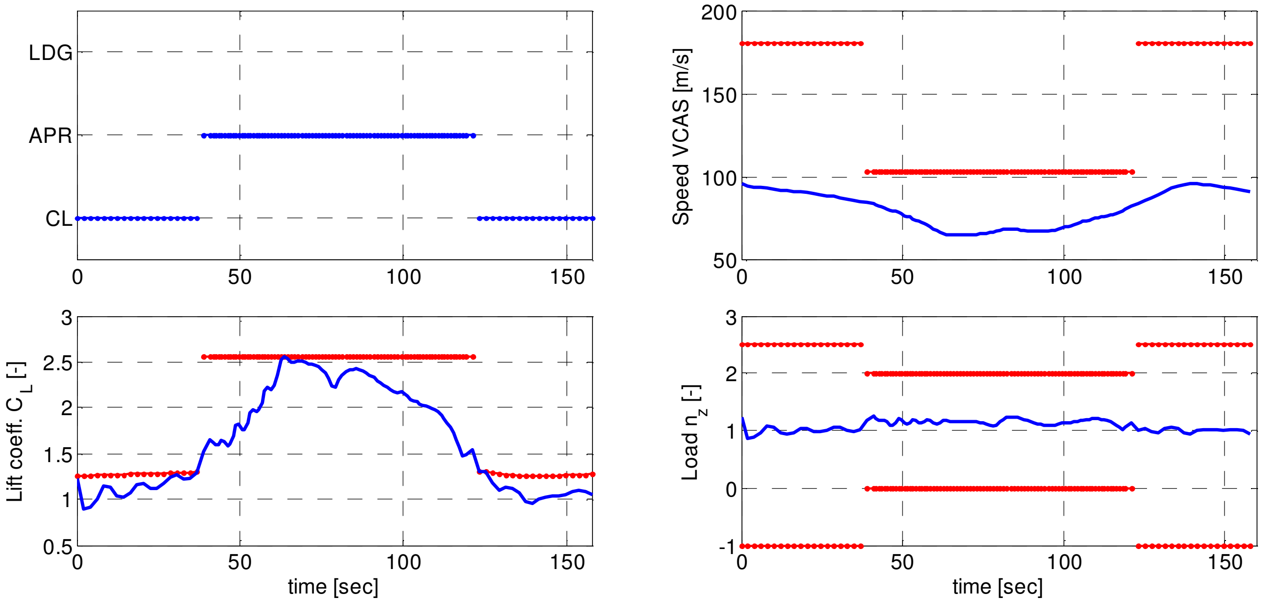

Fig. 5 Optimal solution from B-Spline initial guess

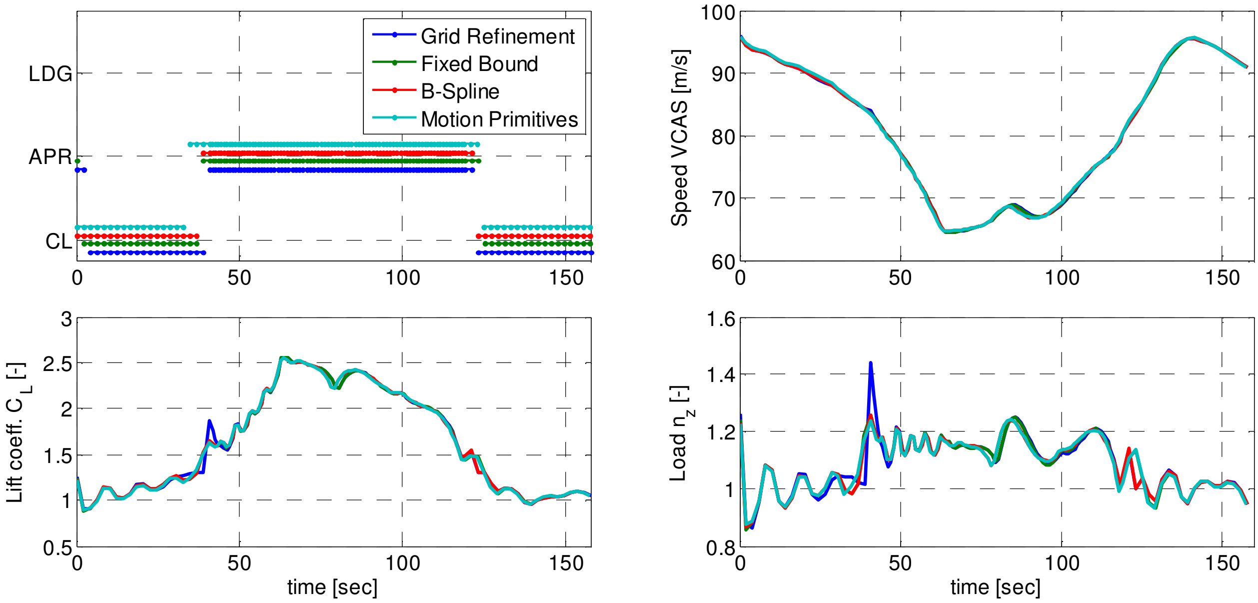

Fig. 6 Optimal solution from different initial guess approaches

finement approach is very fast as well due to the fact, that in the first pass the problem size is much smaller. The B-Spline approach takes around 100 seconds longer to solve. The fixed bound approach has, due to the three optimization stages, the longest optimization time. Considering the cost functions the grid refinement approach has the best fuel consumption but, due to an additional switch, a higher switching cost (SC). The B-Spline approach has the lowest overall cost function, the fixed bound approach the highest. Finally, the motion primitive approach has the highest fuel consumption.

Comparing the different approaches it is clear, that the fixed bound approach is not recommended. The motion primitive approach produces good results, however, it is necessary to create a library of optimal trajectories. There-

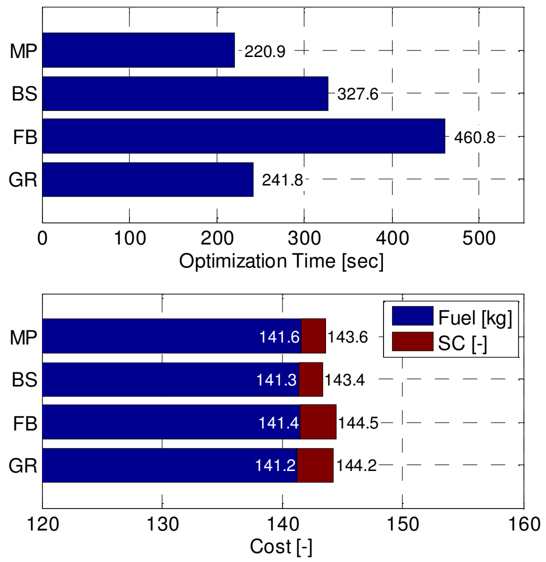

Fig. 7 Comparison of different initial guess approaches (MP = Motion Primitives, BS = B-Spline, FB = Fixed Bound, GR = Grid Refinement)

fore this approach is very complex to implement. Simple to implement is the grid refinement approach and it produces a good fuel consumption. However, there may be OCPs in which the initial guess is too poor for the optimization algorithm. To make this approach more robust an optimization stage can be added between the two current stages, in wich the switching costs remain switched off and the optimization is carried out with the higher discretization density. However, this increases the optimization time significantly. Overall, for complex optimization problems, the B-Spline approach is favoured, since it produces a smooth trajectory without phase defects.

6 Conclusion

In this paper a method has been shown to taken into account discrete controls and discrete control dependent constraints in an aircraft optimization problems. Different approaches for an initial guess and according solution processes have been compared with respect to optimization time and cost function. All approaches generate comparable results, however, the B-Spline approach is favoured. This approach produces high quality smooth tajectories without phase defect with relative little computational effort.

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