

A LOW ORDER METHOD FOR CO-AXIAL PROPELLER AND ROTOR PERFORMANCE PREDICTION

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Abstract

Rotary wings are widely used as propulsive or lifting devices in aerial vehicles. In the field of propulsion, propellers are very attractive because of their high efficiency due to high by-pass ratio. In the field of Vertical Take-Off and Landing (VTOL) vehicles, rotors are certainly the most efficient lifting devices, again because they can take advantage of a large amount of mass flow going through the rotors in hovering flight.

In the present study, dual rotor configurations are considered in order to better understand the reasons why they can bring significant advantages in terms of efficiency against the conventional single rotor. To do this, a low order method has been developed and is validated as a pre-design tool for the aerodynamic performance evaluation of rotors and propellers, single or dual co-axial configurations, used as propulsive and lifting devices. Despite the very low level of modeling of the method, interesting conclusions can be drawn, which show that co-axial lifting rotors and contra-rotating propellers are interesting for different reasons.

1 Introduction

Rotary wings are widely used as propulsive or lifting devices in aerial vehicles. Propellers are very attractive because of their high efficiency due to high by-pass ratio, especially when compared to turbofan engines. In the field of VTOL vehicles, rotors are certainly the most efficient lifting devices, again because they can take advantage of a large amount of mass flow going through the rotors in hovering flight.

It is interesting to note that in both fields, dual-rotor co-axial configurations have proven to bring significant benefits over single rotors or propellers, possibly because of the aerodynamic interactions between the rotors. Indeed, a renewed interest in Contra-Rotating Open Rotors (CROR) propulsion has emerged over the last 5 to 10 years, because co-axial contra-rotating propellers (Fig. 1, left) have a higher propulsive efficiency compared to single propellers. Similarly, it is well known since many years, that co-axial helicopter rotors have very good low speed performance compared to single rotors, one of the reasons for the development of the Kamov co-axial rotor helicopters in Russia (Fig. 1, right).



Fig. 1. Dual-rotor co-axial configurations. Left: Antonov-70 with contra-rotating propellers. Right: Kamov-52 with co-axial rotors

Nowadays, Computational Fluid Dynamics (CFD) methods are able to compute single or co-axial propellers and rotors with a very good accuracy [1]. However, low order methods are always needed in pre-design phases because of their negligible computational time (typically a few seconds) and the limited number of inputs. Such methods are most of the time based on the lifting-line theory and a more or less complex model for the wakes emitted by the blades. Still, these methods often suffer from

the fact that they require accurate 2D look-up tables to be used.

The objective of this paper is to propose a detailed analysis of the coaxial contra-rotating configurations, in order to highlight the reasons of the advantage of dual rotor compared to the single rotor configurations. In a first part, a newly developed low order method is described, with emphasis on its specificities compared to other computational methods. Then, the method is validated on both single and dual configurations, against experimental results when available, or against reference calculation results such as CFD. A discussion on the benefits brought by co-axial contra-rotating configurations is then done, before drawing some conclusions.

2 Method description

The low order method detailed below has been developed in the context of the development of a pre-design platform dedicated to VTOL vehicles called CREATION [2][3]. CREATION aims at being able to evaluate rotorcrafts by using models of different complexity levels depending on the data available. The reason why both rotor and propeller models are required is due to the renewed interest of compound helicopters, where an additional propulsive device (a propeller) is added to the traditional helicopter main rotor in order to fly at high speed. The X³ aircraft developed by Airbus Helicopters is an example of such an aircraft (Fig. 2).



Fig. 2. The X³ compound developed by Airbus Helicopters

The low order method, called PHB below, belongs to the Blade-Element Methods (BEM), characterized by the modeling of the blades as lifting-lines, discretized into spanwise segments. For the sake of simplicity, the method has been developed for axial configurations only, which means that it is valid only for:

- Helicopter rotors in hover (or vertical climb/descent),
- Propellers in axial flight (no incidence).

As a consequence there is no need for any time integration technique (or time loop) in PHB, and the kinematics of the blade segments is extremely simple and reduced to two velocity components: rotation (angular velocity Ω around the rotor axis of rotation \mathbf{e}_x) and translation (free stream velocity \mathbf{V}_0).

One of the main issue with such BEM methods is that they require a specific model in order to compute the so-called induced velocities, ie. the fluid velocity induced by the rotor or propeller wake. In the following, the induced velocities will be split into:

- Self-induced velocities \mathbf{V}_i , representing the velocities induced by one rotor on the blade segments of this rotor,
- Mutual-induced velocities $\mathbf{V}m_i$, representing the velocities induced by one rotor on the other rotor, in the case of dual-rotor configurations.

Furthermore, we will focus on the two components of each of these velocities:

- The axial component, noted by the subscript x, which is the component along the axis of rotation \mathbf{e}_x ,
- The tangential component, noted by the subscript θ , which is the component in the plane of rotation (along orthoradial vector \mathbf{e}_θ).

This leads to the following relations:

$$\mathbf{V}_i = V_{ix} \mathbf{e}_x + V_{i\theta} \mathbf{e}_\theta \text{ (self)} \quad (1)$$

$$\mathbf{V}m_i = Vm_{ix} \mathbf{e}_x + Vm_{i\theta} \mathbf{e}_\theta \text{ (mutual)} \quad (2)$$

Note that the radial component of induced velocities is neglected in the present approach.

Self-induced velocities

In order to derive a simple model with as few unknowns as possible, the choice has been made to assume uniform axial induced velocity (not radius-dependent). Under this assumption, and following 1D momentum theory (also called Froude theory in helicopter text books), there is a very simple relation between V_{ix} and the rotor (or propeller) thrust T:

$$T = 2\rho S \cdot (V_0 + V_{ix}) \cdot V_{ix} \quad (3)$$

Where:

- ρ =air density, constant (hypothesis of incompressibility)
- S =surface of the rotor= $\pi(R^2-r_0^2)$, with R =rotor radius, and r_0 =rotor hub radius.

As far as the tangential component is concerned, it has been chosen to use a simple linear model:

$$V_{i\theta}(r) = r \cdot \omega_{i\theta} \quad (4)$$

Where $\omega_{i\theta}$ is constant (not radius dependent) and will be called “swirl” in the following. With this model and by integration of the Euler theorem well known in the field of turbomachinery (based on 1D assumption too), one can find a simple relation between the swirl $\omega_{i\theta}$ and the power P consumed by the rotor:

$$P = \rho \Omega \cdot \pi (R^4 - r_0^4) \cdot (V_0 + V_{ix}) \cdot \omega_{i\theta} \quad (5)$$

Where Ω is the rotor angular velocity.

Mutual-induced velocities

In the present work, for dual rotors, mutual induced velocities (axial and tangential components) are assumed to be proportional to the self-induced velocities. Let us note by index 1 the upper rotor for a coaxial helicopter rotor and the front propeller for a CROR), and index 2 the other rotor (the lower rotor or the aft rotor). It is assumed that the mutual axial induced velocity on rotor 2 is proportional to the self-induced axial velocity of rotor 1 (and same for rotor 2 to 1 interactions, and same for tangential components). This can be summarized by the following relations:

$$Vm_{ix2} = k_{x12} \cdot V_{ix1} \quad (6)$$

$$Vm_{i\theta2} = k_{\theta12} \cdot V_{i\theta1}$$

$$Vm_{ix1} = k_{x21} \cdot V_{ix2}$$

$$Vm_{i\theta1} = k_{\theta21} \cdot V_{i\theta2}$$

The four coefficients k_{x12} , $k_{\theta12}$, k_{x21} , $k_{\theta21}$ are constant and fixed once for all in the method. Based on simple physical considerations, we can anticipate that:

- k_{x12} should be between 1 and 2 since, based on 1D theory, the fluid is accelerated by the rotor and the induced velocity far downstream is twice its value on the rotor disk,
- k_{x21} should be between 0 and 1, since upstream the rotor the induced velocity is lower than its value on the rotor disk, and equals 0 far upstream,
- $k_{\theta12}$ should be close to 2, since the swirl behind a propeller very quickly reaches twice its induced value on the propeller disk,
- $k_{\theta21}$ should be close to zero, since the swirl component does not propagate upstream the rotor.

Note that the model used here has already been applied in other studies such as [4].

Because of wake contraction, rotor 2 may not be completely inside the wake of rotor 1, so that one part of rotor 2 may not be influenced by rotor 1 (the outer part of the disk: green arrows in Fig. 3). This is especially true for co-axial helicopter rotors in hover. For CROR applications, the effect of wake contraction can be less important, because the aft rotor often has a rotor diameter which is smaller than the one of the front rotor. In order to account for this effect, an option is introduced in the BEM method which allows the user to specify on which radius extension a rotor is under the influence of the other rotor (parameter r_{int} , meaning that only the part of the rotor between r_0 and r_{int} is influenced by the other rotor, with $r_{int} \leq R$).

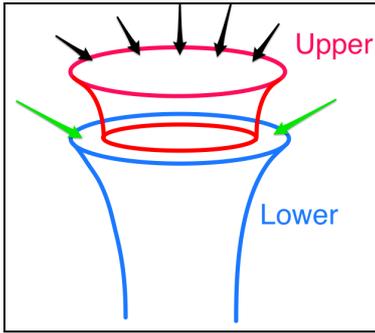


Fig. 3. Wake contraction on co-axial rotors in hover

Aerodynamic sectional coefficients

Once the self and induced velocities are known, since it is very easy to compute the blade kinematics, one has access to the fluid velocity with respect to the blade segment, hence the sectional Mach number M and aerodynamic incidence α , as in any other BEM (Fig. 4).

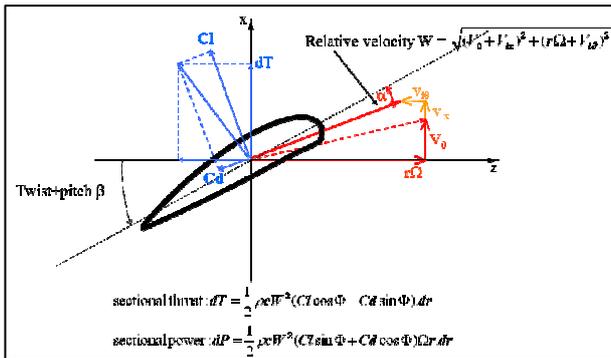


Fig. 4. Lifting-line theory in a BEM method

Given M and α , the use of 2D look-up tables provides the sectional aerodynamic lift C_l and drag C_d coefficients. Here again, for the sake of simplicity, it has been chosen to use an unique look-up table, whatever the section airfoil, whatever the Mach number, so that C_l and C_d are only a function of α . After a spanwise integration, and knowing the number of blades, the rotor thrust T_{integ} and power P_{integ} can be deduced.

System resolution for single rotor

In the case of single rotor configurations, the two unknowns of the problem are:

- The axial self-induced velocity V_{ix}
- The tangential self-induced velocity $V_{i\theta}$ (or more precisely $\omega_{i\theta}$, see Eq. (4)).

And we have two equations:

- Rotor thrust: $T_{integ} = T$ as in (3)

- Rotor power: $P_{integ} = P$ as in (5)

This leads to a nonlinear system of two equations with two unknowns, which is solved iteratively using a gradient method.

System resolution for dual rotor configurations

In the case of dual rotor configurations, the four unknowns of the problem are:

- The axial self-induced velocities of each rotor V_{ix1} and V_{ix2} ,
- The tangential self-induced velocities of each rotor $V_{i\theta1}$ and $V_{i\theta2}$ (or more precisely $\omega_{i\theta1}$ and $\omega_{i\theta2}$).

Indeed, mutual induced velocities are no longer unknowns thanks to Eq. (6). And we have the four equations derived from 1D momentum theory on single rotor that can be extended to dual rotors:

$$T_{integ1} = T_1 = 2\rho S_1 \cdot (V_0 + V_{ix1} + V_{m_{ix1}}) \cdot V_{ix1} \quad (7)$$

$$T_{integ2} = T_2 = 2\rho S_2 \cdot (V_0 + V_{ix2} + V_{m_{ix2}}) \cdot V_{ix2} \quad (8)$$

$$P_{integ1} = P_1 = \rho \Omega \cdot \pi \cdot (R_1^4 - r_{01}^4) \cdot (V_0 + V_{ix1} + V_{m_{ix1}}) \cdot \omega_{i\theta1} \quad (9)$$

$$P_{integ2} = P_2 = \rho \Omega \cdot \pi \cdot (R_2^4 - r_{02}^4) \cdot (V_0 + V_{ix2} + V_{m_{ix2}}) \cdot \omega_{i\theta2} \quad (10)$$

This leads to a nonlinear system of four equations with four unknowns, which is solved iteratively using a gradient method.

3 Validation of the method

3.1 Single rotor configurations

Compound propeller

The propeller of a compound such as the one illustrated in Fig. 2 has to operate in quite different aerodynamic conditions depending on the flight condition:

- In cruise flight, both propellers operate as conventional propulsive devices,
- In hover flight ($V_0=0$), since the propellers ensure the anti-torque function, one of the two propellers operates like a helicopter in hover generating positive thrust, but the other

one has to generate negative thrust (reverse mode with zero wind).

Current BEM PHB has been validated on a generic 4-bladed compound propeller for which reference numerical solutions existed prior to the present study.

In cruise condition (advancing Mach number $M_0=0.35$ in the present case), a comparison of PHB results with results obtained by another BEM code named LPC2 [6] is done in Fig. 5. Current low-order method PHB successfully reproduces the power vs. lift curve (top of Fig. 5), and predicts with a quite good accuracy the propulsive efficiency (bottom of Fig. 5). However, these good results should not hinder the fact that the spanwise lift distribution predicted by PHB is different from the one of the reference BEM LPC2 (Fig. 6), due to the assumption of uniform axial velocity: in LPC2, a more realistic spanwise lift distribution is obtained, with a lift decrease near the tip, because the wake model is more sophisticated (helical wake, see [6]).

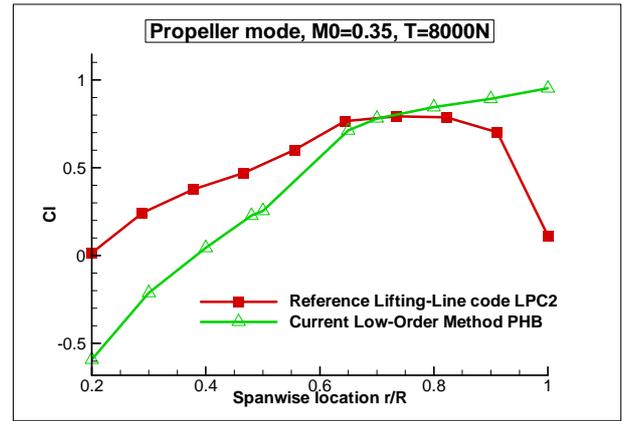


Fig. 6. Sectional lift distribution at $M_0=0.35$.

When the propeller acts as a helicopter rotor in hover (no wind, $V_0=0$) generating positive thrust, Fig. 7 shows that the evolution of the rotor Figure of Merit FM which measures classically the rotor efficiency is pretty well predicted by PHB, although FM is underestimated by 3 cts in average (1 ct = 0.01 FM). This deviation is all the most acceptable considering that here the reference solution is a CFD Navier-Stokes calculation, known to provide results with very good accuracy for this kind of flight condition. Furthermore, it is interesting to note that PHB predicts the strong loss of rotor efficiency when the rotor provides negative thrust (reverse mode), although the PHB FM values are too low in reverse mode compared to the ones predicted by CFD (green lines in Fig. 8).

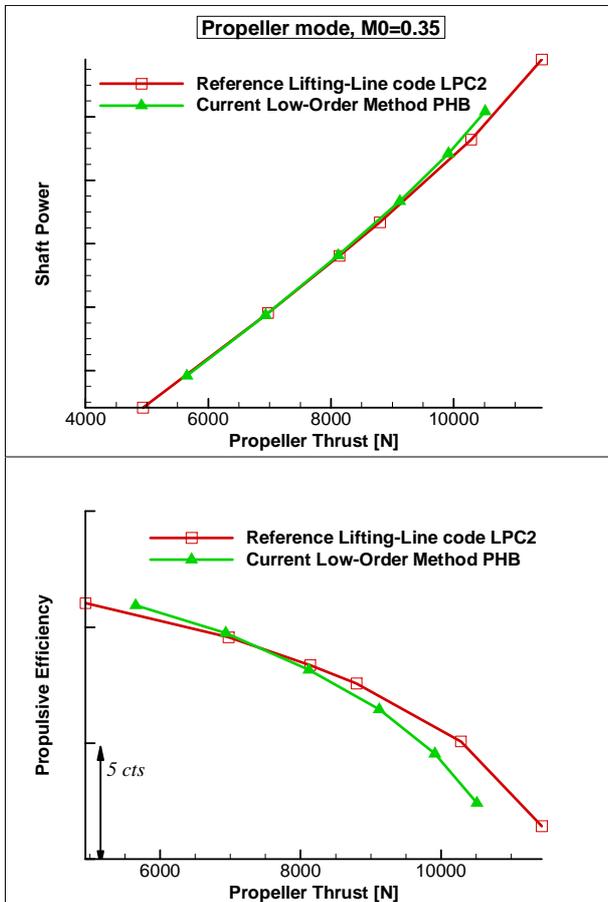


Fig. 5. Global performance of a single propeller at $M_0=0.35$. Validation of PHB method

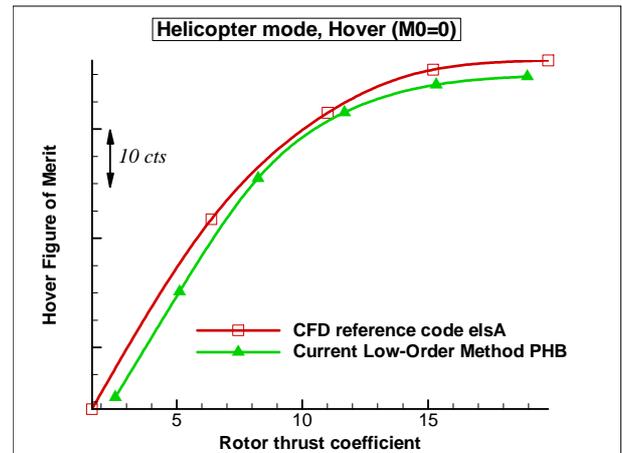


Fig. 7. Prediction of Hover Efficiency. Validation of PHB method

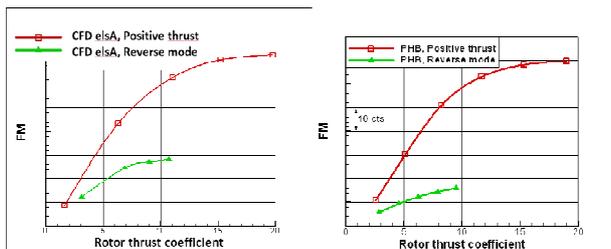


Fig. 8. Hover Efficiency in normal and reverse modes. Validation of PHB method

Tilt-rotor propeller in cruise flight

What is important in pre-design codes is not only their ability to compute realistic performance, but also to be able to reproduce the difference of performance between two designs. In order to evaluate the PHB method with respect to this specific capability, we have used the results of a tilt-rotor optimization study that was done in the framework of the ADYN European Project [7]. In this project, starting from a reference rotor called TILTAERO, ONERA designed an improved rotor called ADYN. Both rotors, illustrated in Fig. 9, were tested in the ONERA high speed wind-tunnel SIMA in 2006-2007 and their cruise performance were measured. For a given flight condition defined by $M_0=0.4$, tip rotational Mach number $M_{tip}=0.493$, Fig. 10 shows the difference of efficiency between the ADYN and the TILTAERO rotors as a function of propeller thrust. The positive values on the Y-axis indicate the better performance of the ADYN optimized rotor than the reference TILTAERO. Both current low-order method PHB and another BEM reference method reproduce this experimental trend, however with a lower slope when the rotor thrust increases. For low thrust values, BEM methods underestimate the improvement brought by the ADYN rotor; on the contrary at high thrust, they tend to slightly overestimate it. Given the simplicity of the method implemented in PHB, this result is believed to be satisfactory at this stage.



Fig. 9. TILTAERO (left) and ADYN (right) rotors in SIMA wind-tunnel

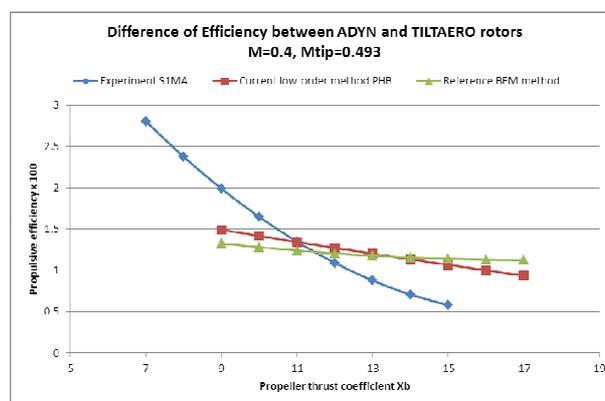


Fig. 10. Difference of efficiency between two rotors in propeller mode.

3.2 Dual rotor configurations

In this section, the PHB method is evaluated for dual rotor configurations, first on a CROR then on co-axial helicopter rotor in hover.

CROR

An 11-9 bladed CROR configuration is considered here, for which reference computational results existed prior to this study. These reference results were obtained from CFD simulations with the ONERA elsA software [8], in which a quasi-steady approximation of the CROR behavior was used, through a mixing-plane boundary condition at the interface between the two propellers. More details about this averaging technique can be found in [6]. The reference computational point was obtained for an advancing Mach number $M=0.23$ (typical of Take-Off condition). For this Mach number, the low-order method PHB was run and the pitch angles of each propeller were adjusted in order to match each propeller

thrust of the CFD result. Once these pitch angles were obtained, a sweep in upstream Mach number was done both in PHB and in the CFD, with unchanged pitch angles. The thrust of each propeller is plotted vs. the advancing Mach number in Fig. 11. The agreement between PHB prediction and the reference CFD results is quite good for the considered range of Mach numbers, at least for $M \geq 0.1$. For very low Mach numbers, the thrust of each propeller is considerably reduced in the CFD results, whereas this trend is not reproduced in the BEM method PHB, especially on the aft propeller. The reason for these discrepancies certainly lie in the fact that for such low Mach numbers, the flow over a significant part of the blades is separated, and the simple PHB assumptions are no longer valid (use of a single airfoil table, uniform induced velocities). Furthermore, it is likely that for these low Mach number conditions, the interaction coefficients k^{**} (Eq. (6)) of the PHB interaction model are no longer valid. This shows a limitation of the method for off-design conditions, but also a nice validation at nominal conditions between $M=0.1$ and 0.3 .

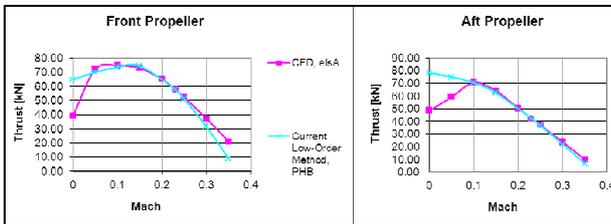


Fig. 11. Propeller Thrust-Mach number curves at prescribed pitch angles.

Co-axial helicopter rotor

Co-axial helicopter rotor performances in hover computed by the BEM PHB are validated by using recent experimental results obtained by US Army [5] on a scaled rotor. The power coefficient C_p required by a rotor system, as defined by momentum theory, can be written as a function of rotor thrust coefficient C_t by:

$$C_p = C_{p0} + K/2^{1/2} \cdot C_t^{3/2} \quad (11)$$

Where C_{p0} is the profile power coefficient and K the induced power factor (representing the induced power losses of the rotor). Note that the following definitions of power and thrust coefficients are used:

$$\begin{aligned} C_t &= T / (\rho S (R\Omega)^2) \\ C_p &= P / (\rho S (R\Omega)^3) \end{aligned} \quad (12)$$

The performance of a 3x3 coaxial rotor is compared to that of a single 6-bladed rotor (with same solidity) in Fig. 12. We can see here the clear interest of the coaxial rotor which requires less power than the 6-bladed rotor. One way to quantify the corresponding induced power gains is to compute the K_{sep} coefficient introduced in [5] defined as follows:

$$\text{6-bladed rotor: } C_p = 2C_{p0} + K_6/2^{1/2} \cdot C_t^{3/2} \quad (13)$$

$$\text{Coaxial: } C_p = 2C_{p0} + K_{sep} \cdot K_6/2^{1/2} \cdot C_t^{3/2}$$

K_{sep} represents the ratio of induced power of coaxial rotor to the induced power of the 6-bladed isolated rotor. Using best fit curves, its value computed by current PHB method is: $K_{sep} \sim 0,853$. This represents a gain of the coaxial configuration of 15%, which is a bit optimistic compared to the 10% value measured in [5].

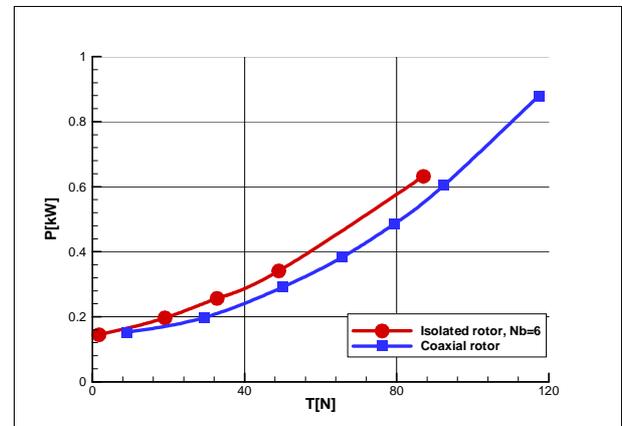


Fig. 12. Power consumption of a 3x3 coaxial rotor compared to that of an isolated 6-bladed rotor. Untwisted blades. RPM=1200

An interesting point is to try to quantify how each of the two rotors of the coaxial configuration behaves compared to that of the isolated 3-bladed rotor. To do this, following what is done in [5], two coefficients K_{upp} and K_{low} can be defined:

$$\text{3-bladed rotor: } C_p = C_{p0} + K_3/2^{1/2} \cdot C_t^{3/2} \quad (14)$$

$$\text{Coaxial, upper: } C_p = C_{p0} + K_{upp} \cdot K_3/2^{1/2} \cdot C_t^{3/2}$$

$$\text{Coaxial, lower: } C_p = C_{p0} + K_{low} \cdot K_3 / 2^{1/2} \cdot C_t^{3/2}$$

Quoting [5], they “represent the induced power lost by the upper rotor because of the influence of the lower rotor and vice versa, respectively”. Fig. 13 gives an illustration on how the power-thrust curves of these rotors look like, according to PHB results. We can see that the upper rotor is only very slightly penalized by the lower rotor interaction, whereas the lower rotor consumes significantly more power than in isolated condition, because of the interaction with the upper rotor.

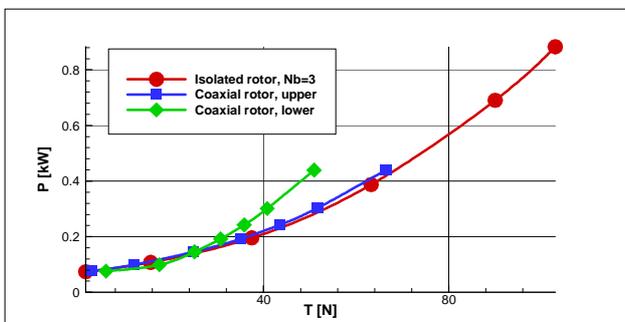


Fig. 13. Power consumption of the upper and lower rotors of the 3x3 coaxial configuration compared to that of an isolated 3-bladed rotor. Untwisted blades. RPM=1200

These remarks are summarized by the K_{upp} and K_{low} values indicated in Table 1. PHB prediction of the upper rotor performance is good according to the value of K_{upp} , which is very close to experiment. PHB predicts the significant power penalty of the lower rotor due to interactions with the upper rotor highlighted by the high value of the K_{low} coefficient, which is even larger than in experiment (1.61 instead of 1.41).

	K_{sep}	K_{upp}	K_{low}
Experiment [5]	0.90	1.10	1.41
Current low-order method PHB	0.85	1.07	1.61

Table 1: Interaction coefficients for a coaxial rotor in hover

Despite these differences, all experimental trends are reproduced by the low order method PHB, which is considered as validated in this hover configuration.

4 Discussion

In this part, a discussion is proposed in order to better understand the origin of the advantages of dual rotor configurations compared to single ones. Part of the study is done with the help of the low order method PHB.

Zeroing the swirl

In order to quantify the influence of the swirl, an interesting exercise is to modify temporally the interaction model in PHB by assuming no tangential mutual velocities. This is very easy to do by just modifying the two coefficients kt_{12} and kt_{21} and set them to zero. By doing this, aerodynamic interactions between the rotors are only axial interactions.

In the case of coaxial helicopters in hover, we see that the corresponding PHB calculation results are not modified compared to the case with the correct model including tangential mutual velocities. This is illustrated in Fig. 14, which shows that the power of the coaxial configuration is insensitive to swirl effects. It has been checked that the lower efficiency is almost not affected by the swirl generated by the upper rotor. This result somehow confirms what is often found in the literature, that is to say, for coaxial helicopter rotors in hover, swirl effects are not significant.

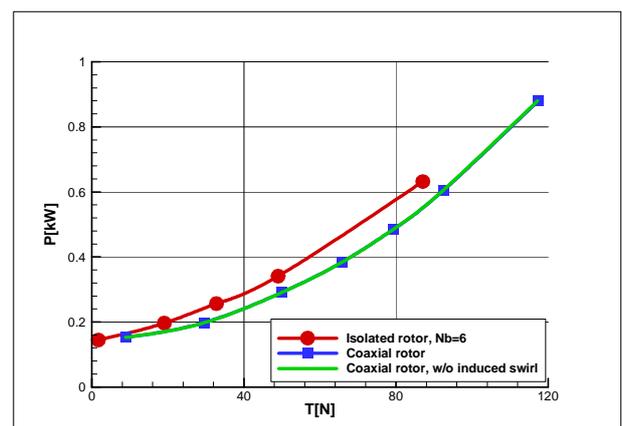


Fig. 14. Influence of swirl on the power consumption of a 3x3 coaxial rotor compared to that of an isolated 6-bladed rotor.

Doing the same exercise for CROR leads to opposite conclusions, as can be seen in Fig. 15. Calculation results without swirl in the mutual induced velocities (red bars) are

characterized by a large decrease of the thrust generated by the aft rotor (top of Fig. 15), with a corresponding 4 counts reduction of its efficiency (bottom of Fig. 15). This is because the aft rotor no longer benefits from the swirl of the front rotor. Here again, this is not surprising and very consistent with what is found in the literature.

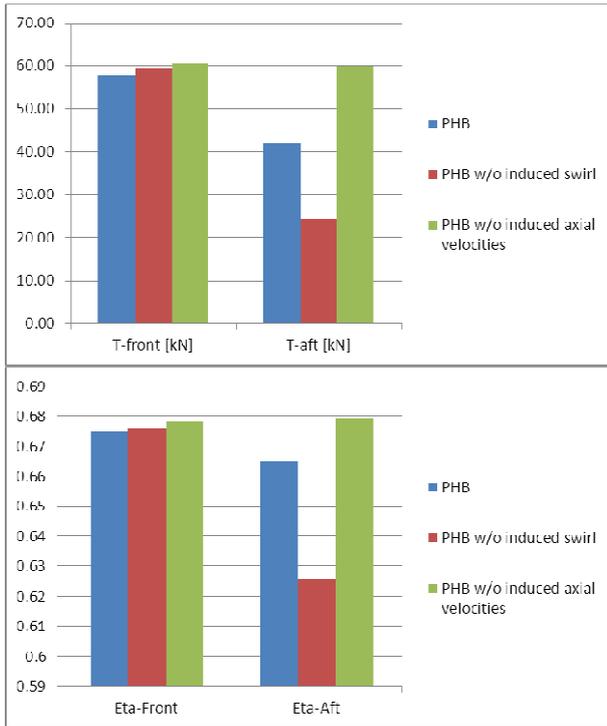


Fig. 15. Influence of interaction coefficients on CROR performance. Top: Thrust. Bottom: Efficiency

Coaxial helicopter rotors

Since the swirl is not at the origin of the interest of coaxial helicopter rotors in hover, we need to understand why such rotors are more interesting in terms of induced power consumption than a single rotor with equivalent solidity. The first thing to be reminded is that each of the two rotors in interaction behave worse than the same rotor in isolated conditions, as indicated by the K_{upp} and K_{low} coefficients which are always higher than 1 (Table 1). Since swirl has almost no effect, it has to be concluded that axial interactions are not favorable to the aerodynamic behavior of the rotors. But we should keep in mind that, due to wake contraction, only part of the lower rotor is in interaction with the upper rotor (Fig. 3). An interesting exercise is to run PHB method

assuming that the whole surface of the lower rotor is in interaction with the upper rotor (r_{int}/R coefficient equal to 1 instead of 0.85). Fig. 16 clearly shows that with this new calculation the lower rotor performance gets worse, and that the coaxial performance gets equivalent to that of the isolated rotor of equivalent solidity. It can be concluded that the reason why coaxial helicopter rotors are interesting lies in the fact that the external part of the lower rotor is not in interaction with the upper rotor wake, thus taking benefit of unperturbed air. In other terms it is as if the dual rotor system had an increased equivalent surface.

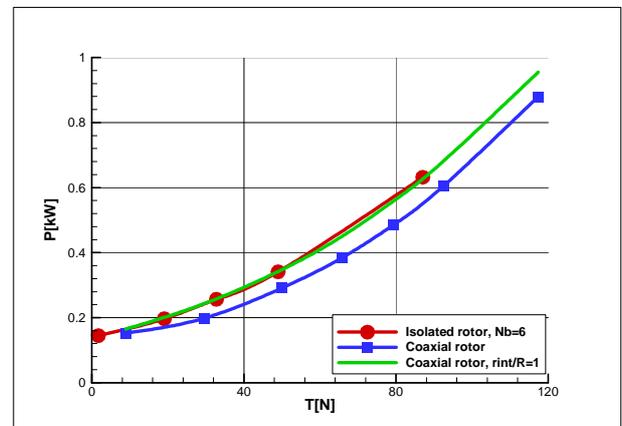


Fig. 16. Influence of upper rotor wake contraction on the power consumption of a 3x3 coaxial rotor compared to that of an isolated 6-bladed rotor.

CROR configurations

In the case of CROR, another interesting exercise is to temporarily zero axial mutual induced velocities just by setting the interaction coefficients $k_{x12}=k_{x21}=0$. Fig. 15 (bottom) shows that by doing this, performance of the aft rotor is increased (increase of efficiency by 1.5 counts), confirming the fact that axial interactions are not beneficial.

Before concluding, one should try to figure out why swirl has such an influence in CROR configurations and not in the case of coaxial helicopter rotors. Let us come back to equations (3) and (5) derived from 1D momentum theory and let us divide equation (5) by equation (3). Using an average value of tangential velocity $V_{i\theta}$ (given the linear model of eq. (4)) and after some easy algebraic operations, we obtain:

$$V_{i\theta}/V_{ix} = f(r_0, R) \cdot P / (T \cdot R \Omega) \quad (15)$$

Where $f()$ is a simple function of r_0 and R the value of which is close to 1.

The beauty of this simple relation is that it shows that the power to thrust ratio is proportional to the tangential to axial induced velocity ratio, for a single rotor or propeller. We can also express directly the following ratio which aims at comparing the induced swirl to the blade rotation velocity:

$$V_{i\theta}/(R\Omega) \sim V_{ix}/(R\Omega) \cdot P / (T \cdot R \Omega) \quad (16)$$

Using propeller non dimensional quantities (J =advancing coefficient and η =propulsive efficiency), we obtain the following equation:

$$V_{i\theta}/(R\Omega) \sim V_{ix}/(R\Omega) \cdot J / (\pi \eta) \quad (17)$$

Let us give typical values for the coefficients in the previous equation.

For CROR in low speed: $M=0.2$, $J=1$, $\eta=0.55$, for $Ct=0.5$ we can compute using 1D theory $V_{ix}/R\Omega=9.4\%$ so that $V_{i\theta}/(R\Omega) > 5\%$, which is significant.

For CROR in high speed: $M=0.78$, $J=3.3$, $\eta=0.75$, for $Ct=0.6$ we can compute using 1D theory $V_{ix}/R\Omega=3\%$ so that $V_{i\theta}/(R\Omega) > 4\%$. Here again, $V_{i\theta}$ represents something significant compared to the rotation speed of the propeller.

For coaxial helicopter rotor in hover, using typical helicopter notations (solidity σ , thrust coefficient Zb and hover figure of merit FM), we obtain:

$$V_{i\theta}/(R\Omega) \sim V_{ix}/(R\Omega) \cdot Zb^{3/2} / FM \cdot \sigma^{1/2} / 20 \quad (18)$$

With the following typical values: $Zb=20$, $FM=0.75$, $\sigma=0.085$, we can compute using 1D theory $V_{ix}/R\Omega=6.5\%$ so that $V_{i\theta}/(R\Omega) < 1\%$. In this case, the swirl $V_{i\theta}$ is very small compare to the rotation speed of the rotor, which confirms the fact that the effect of swirl is almost negligible in this case.

5 Conclusions and future work

In this study, co-axial contra-rotating rotors have been studied aiming at a better understanding of the reasons why such

configurations generally offer better performance than single-rotor configurations.

Making use of a simple BEM code based on 1D momentum theory, it has been shown that axial interactions between the two rotors are not favorable to aerodynamic performance. For co-axial helicopter rotors in hover, the external part of the lower rotor is not in interaction with the upper rotor, because of wake contraction, resulting in a higher equivalent rotor disk surface and improved performance. In this configuration, tangential velocities induced by the upper rotor are almost negligible compared to the rotational speed of the rotors.

On the contrary, for contra-rotating propellers, the swirl generated by the front rotor is important and allows the aft rotor to benefit from additional tangential velocity, which is the origin of performance improvement.

The BEM code PHB developed in this study will be improved in a near future. One of the main areas of improvement is the interaction model, which is currently based on four coefficients which values were fixed once for all. A model to relate these coefficients to physical parameters such as the distance between the two rotors or wake contraction or rotor loading would be very helpful in order to have a more predictive method. To achieve this, it is believed that specific good quality databases related to rotor-rotor interactions are needed, first in axial flight conditions, and then for any kind of flight conditions.

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