

# CONTROL INPUT BLENDING LOGIC WITH AERODYNAMIC CONTROL AND REACTION JET CONTROL

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**Keywords:** *Dual-Control System, Control Allocation, Daisy-Chain, Pseudo-Inverse, Control Effectiveness Shaping*

## Abstract

*This paper deals with control input blending problem for vehicle system with tail-fin control surface and reaction jet. Two blending methods previously studied, Daisy-Chain Method and Pseudo-Inverse Method, have pros and cons contradicting with those of each other. With Daisy-Chain Method, reaction jet thrust is not required at steady-state, but its magnitude is immoderately large at transient state. In Pseudo-Inverse Method case, reaction jet thrust command is able to be designed small, but reaction jet thrust is non-zero at steady-state, resulting in permanent fuel consumption. Control Effectiveness Shaping Method is the blending logic proposed in this paper to solve the problems of those two methods. This method is designed from Pseudo-Inverse Method for small magnitude of reaction jet thrust at transient state, and modified by shaping the control effectiveness not to use this control input at steady-state. Simulations are conducted to verify the performance and the characteristics of proposed Control Effectiveness Shaping Method.*

## 1 Introduction

For tail-fin controlled vehicle, as angle-of-attack gets higher, larger normal force on the vehicle is induced. Thus, when high maneuverability is needed for vehicle, it is required to make desired high angle-of-attack in a short time. This is accomplished with huge tail-fin deflection angle command and large slew rate to follow that command rapidly. In the

case of vehicle at high altitude, since the control effectiveness of tail-fin deflection angle is small due to low air density, more immoderate control input command is demanded. However, tail-fin deflection angle has actuator limits, like deflection angle limit and slew rate limit, it cannot match the excessive input command derived by the autopilot system.

In order to compensate this degeneration of tail-fin deflection angle, reaction jet is introduced[1-4]. When reaction jet is equipped at the distant point from the center of mass, it exerts moment on the vehicle, reducing unacceptable burden on tail-fin. But, reaction jet thrust also has actuator limits, and it consumes much larger amount of fuel, which is finite, than tail-fin actuator. Thus, it is necessary to blend those two control inputs properly.

There are two ways of blending control inputs. One is designing autopilot system to define control input commands for both actuators, meaning that blending principle is integrated with autopilot system. In Di Zhou's method[5], two control inputs at steady state are defined to cancel the effect of them on the normal acceleration of vehicle. From this definition, the dynamics of the desired body angle rate is determined as the function of the normal acceleration and its derivative. The autopilot is designed by applying feedback linearization method to track this body angle rate command. Two control input commands are derived automatically by the autopilot system, but the reaction jet thrust is required at steady state, resulting in the fuel consumption over the total capacity.

The other way is designing autopilot system for one control input first, and dividing the control input command generated by the autopilot into control input commands of two actuators[6][7]. In this case, blending logics are separated from autopilot system, so it is able to apply any control law producing tail-fin deflection angle command. One blending logic is called Daisy-Chain Method, using the tail-fin deflection angle as the prior control input and the reaction jet input is activated only when the tail-fin actuator is saturated. This method has a merit in fuel saving because reaction jet thrust is not needed at steady-state, but its magnitude is overly large at transient state. Another one, Pseudo-Inverse Method, allocates the control input commands to minimize the cost function, which involves the priority of each control input. With this method, reaction jet thrust is small at transient state, but it is required at steady-state.

Control Effectiveness Shaping Method is suggested in this paper to solve the problems of Daisy-Chain Method and Pseudo-Inverse Method. The basic structure of this method is similar to that of Pseudo-Inverse Method. This enables to design the reaction jet input command at transient state to be small. To immoderate fuel consumption problem, the control effectiveness of reaction jet input is modified to converge to zero at steady-state by introducing shaping function. This modified control effectiveness is utilized for the control input allocation, and the resultant reaction jet thrust also goes to zero, showing the advantage in fuel consumption.

The contents of this paper are as follows. Section 2 deals with the vehicle angle-of-attack autopilot design. Three blending logics, Daisy-Chain Method, Pseudo-Inverse Method, and Control Effectiveness Shaping Method, are explained in Section 3. Numerical simulations are performed in Section 4. Section 5 addresses the overall conclusion.

## 2 Vehicle Autopilot Design

The blending logics dealt with in this paper are separated from the autopilot. They divide the tail-fin deflection angle command into proper tail-fin deflection angle command and

reaction jet command. Thus, the autopilot is designed from the dynamic system only with tail-fin deflection angle and integrated with the blending logic in order to be applied to the vehicle system with tail-fin and reaction jet. The autopilot system is designed by applying backstepping control law. The longitudinal dynamic equations of vehicle with tail-fins and reaction jets are given as,

$$\begin{aligned}\dot{\alpha} &= \frac{QS}{mV} \left( C_{Z_\phi} \cos \alpha + C_{A_0} \sin \alpha \right) - \frac{1}{mV} T_x \sin \alpha + q \\ &\quad + \frac{QS}{mV} C_{Z_{\delta_q}} \cos \alpha \delta + \frac{1}{mV} \cos \alpha T_z \\ \dot{q} &= \frac{QSl}{I_{zz}} \left[ C_{m_\phi} + C_{m_q} \left( \frac{l}{2V} \right) q \right] + \frac{QSl}{I_{zz}} C_{m_{\delta_q}} \delta - \frac{l_T}{I_{zz}} T_z\end{aligned}\quad (1)$$

where the meanings of state variables and parameters in Eq.(1) are defined as below.

$\alpha$	: angle-of-attack
$q$	: pitch angle rate
$Q$	: dynamic pressure
$S$	: reference area
$l$	: reference length
$m$	: mass
$V$	: velocity
$I_{zz}$	: moment of inertia in z-axis
$l_T$	: distance between center of mass and reaction jet
$T_x$	: thrust in axial direction
$\delta$	: tail-fin deflection angle
$T_z$	: reaction jet thrust
$C_{(\cdot)}$	: aerodynamic coefficient

Since the tail-fin is considered as the only control input for autopilot design, the effects of the reaction jets are neglected as follows.

$$\begin{aligned}\dot{\alpha} &= \frac{QS}{mV} \left( C_{Z_\phi} \cos \alpha + C_{A_0} \sin \alpha \right) - \frac{1}{mV} T_x \sin \alpha + q \\ &\quad + \frac{QS}{mV} C_{Z_{\delta_q}} \cos \alpha \delta \\ \dot{q} &= \frac{QSl}{I_{zz}} \left[ C_{m_\phi} + C_{m_q} \left( \frac{l}{2V} \right) q \right] + \frac{QSl}{I_{zz}} C_{m_{\delta_q}} \delta\end{aligned}\quad (2)$$

Eq.(2) is expressed in simpler form by redefining variables.

$$\begin{aligned}\dot{x}_1 &= f_1 + x_2 + h_1 u \\ \dot{x}_2 &= f_2 + h_2 u\end{aligned}\quad (3)$$

where

$$\begin{aligned}x_1 &= \alpha \quad x_2 = q \quad u = \delta \\ f_1 &= \frac{QS}{mV} \left( C_{Z_\phi} \cos \alpha + C_{A_0} \sin \alpha \right) - \frac{1}{mV} T_x \sin \alpha \\ f_2 &= \frac{QSl}{I_{zz}} \left[ C_{m_\phi} + C_{m_q} \left( \frac{l}{2V} \right) q \right] \\ h_1 &= \frac{QS}{mV} C_{Z_{\delta_q}} \cos \alpha \quad h_2 = \frac{QSl}{I_{zz}} C_{m_{\delta_q}}\end{aligned}\quad (4)$$

In Eq.(3), since the effect of the term related to the control input on the angle-of-attack is much smaller than that of the pitch angle rate, meaning that  $|h_1 u| \ll |x_2|$ ,  $h_1 u$  can be ignored. Thus, the longitudinal dynamic equations of vehicle can be expressed in a strictly feedback form.

$$\begin{aligned}\dot{x}_1 &= f_1 + x_2 \\ \dot{x}_2 &= f_2 + h_2 u\end{aligned}\quad (5)$$

New state variables are defined to apply backstepping control law.

$$\begin{aligned}z_1 &\triangleq x_1 - x_{1d} \\ z_2 &\triangleq x_2 - x_{2d}\end{aligned}\quad (6)$$

where  $x_{1d}$  and  $x_{2d}$  are the desired values of  $x_1$  and  $x_2$ . In order to obtain  $x_{2d}$ , which makes  $z_1$  converge to zero in finite time, a Lyapunov candidate function of  $z_1$  is defined.

$$V_1 = \frac{1}{2} z_1^2 \quad (7)$$

By taking a time derivative of Eq.(7), a Lyapunov stability condition is constructed.

$$\dot{V}_1 = z_1 (f_1 + x_2 - \dot{x}_{1d}) < 0 \quad (8)$$

From Eq.(8),  $x_{2d}$  is derived as follows.

$$x_{2d} = -f_1 + \dot{x}_{1d} - K_1 z_1 \quad \text{for } K_1 > 0 \quad (9)$$

The Lyapunov candidate function of  $z_1$  and  $z_2$  is defined in order to get proper  $u$ .

$$V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \quad (10)$$

A Lyapunov stability condition is derived from the time derivative of Eq.(10).

$$\dot{V}_2 = z_1 (z_2 - K_1 z_1) + z_2 (f_2 + h_2 u - \dot{x}_{2d}) < 0 \quad (11)$$

Thus, the tail-fin deflection angle command  $u$  is defined from Eq.(11) as below.

$$u = \frac{1}{h_2} (-f_2 + \dot{x}_{2d} - z_1 - K_2 z_2) \quad \text{for } K_2 > 0 \quad (12)$$

### 3 Blending Logic

Since the autopilot control input command  $\delta_{tot}$  defined in Section 2 is tail-fin deflection angle only, it is required to apply a blending logic which divide this control input into two control inputs, tail-fin deflection angle and reaction jet thrust, properly. But, those two control inputs has different dimensions, where one is angle and the other is force, so it is difficult to apply blending logic to a tail-fin control input signal derived from the autopilot. To overcome this problem, the autopilot control input command is normalized using control effectiveness first, and then blending logic is applied to this normalized input command with the consideration on control effectivenesses of two control inputs. In Section 2, the autopilot control input command is designed for tracking desired pitch angle rate, generated to follow the angle-of-attack command. Thus, the control effectiveness of control inputs are defined from the pitch angle rate dynamics in Eq.(1).

$$B_\delta = \frac{QSl}{I_{zz}} C_{m_{\delta_q}} \quad B_{T_z} = -\frac{l_T}{I_{zz}} \quad (13)$$

where  $B_\delta$  and  $B_{T_z}$  are control effectiveness of tail-fin deflection angle and reaction jet thrust.

From Eq.(13), a normalized input command is defined as

$$\mathbf{v} = B_{\delta} \delta_{tot} \quad (14)$$

Control input vector is defined from two control inputs.

$$\mathbf{u} = \begin{bmatrix} \delta \\ T_z \end{bmatrix} \quad (15)$$

The relationship between the normalized input command and the control input vector is set up from Eq.(14) and Eq.(15) as

$$\begin{bmatrix} B_{\delta} & B_{T_z} \end{bmatrix} \mathbf{u} = \mathbf{v} \quad (16)$$

Thus, the control input blending problem is defined as finding the proper control input commands which satisfies Eq.(16).

Each control input has its own actuator limits, and those actuator limits should be considered when defining the control input command to avoid actuator saturations and get better performance. The actuator limits of the vehicle system, deflection angle limits and slew rate limits of tail-fin and thrust limits of reaction jet, are described as follows.

$$\begin{aligned} \delta_{\min} < \delta < \delta_{\max} \\ \dot{\delta}_{\min} < \dot{\delta} < \dot{\delta}_{\max} \\ T_{z,\min} < T_z < T_{z,\max} \end{aligned} \quad (17)$$

### 3.1 Daisy-Chain Method

In Daisy-Chain Method, the tail-fin deflection angle is used as primary control input. However, when tail-fin actuator is saturated by the actuator limits defined in Eq.(17), the actual tail-fin deflection angle cannot track the autopilot control input command. Thus, the tail-fin deflection angle command should be designed with considering the actuator limits. The actual limit of the tail-fin deflection angle which can be accomplished during the sampling time  $\Delta t$  is determined not only by the tail-fin deflection angle limit but also by the slew rate limit. So, new tail-fin deflection angle limits considering the slew rate limits are defined as

$$\begin{aligned} \bar{\delta}_{\max} &= \min(\delta_{\max}, \delta + \Delta t \dot{\delta}_{\max}) \\ \bar{\delta}_{\min} &= \max(\delta_{\min}, \delta + \Delta t \dot{\delta}_{\min}) \end{aligned} \quad (18)$$

From Eq.(18), in Daisy-Chain Method case, tail-fin deflection angle command  $\delta_c$  is bounded as below.

$$\bar{\delta}_{\min} < \delta < \bar{\delta}_{\max} \quad (19)$$

The difference between the autopilot control input command  $\delta_{tot}$  and the tail-fin deflection angle command  $\delta_c$  is calculated as follows.

$$\Delta\delta = \delta_{tot} - \delta_c \quad (20)$$

This control input command loss  $\Delta\delta$  is compensated by the reaction jet thrust in this method. From Eq.(15), Eq.(16) and Eq.(20), the reaction jet thrust command is defined as

$$T_{z,c} = \left( \frac{B_{\delta}}{B_{T_z}} \right) \Delta\delta \quad (21)$$

The overall Daisy-Chain Method is expressed as the block diagram in Fig. 1.

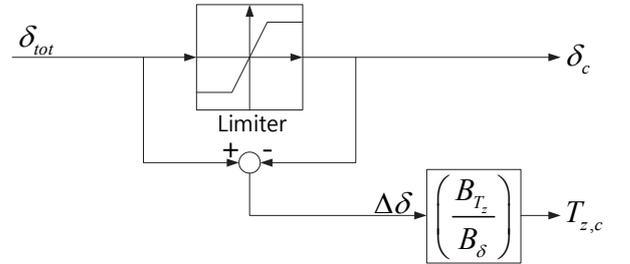


Fig. 1. Daisy-Chain Method

### 3.2 Pseudo-Inverse Method

In Pseudo-Inverse Method, a cost function considered for the blending logic is defined as follows.

$$\min_{\delta, T_z} J = \begin{bmatrix} \delta & T_z \end{bmatrix} \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} \begin{bmatrix} \delta \\ T_z \end{bmatrix} \quad (22)$$

Two weighting functions,  $W_1$  and  $W_2$ , are chosen to define the priority of each control input. Also, in order to consider the actuator limits, which show the capability of each

control input, the pseudo-control effectivenesses are defined.

$$\hat{B}_\delta = B_\delta \delta_{\max} \quad \hat{B}_{T_z} = B_{T_z} T_{z,\max} \quad (23)$$

To reconstruct a control input blending problem using the pseudo-control effectivenesses in Eq.(23), the pseudo-control input variable vector  $\hat{\mathbf{u}}$  is defined as follows.

$$\hat{\mathbf{u}} = \begin{bmatrix} \frac{\delta}{\delta_{\max}} \\ \frac{T_z}{T_{z,\max}} \end{bmatrix} \quad (24)$$

With Eq.(23) and Eq.(24), Eq.(16) is rewritten as

$$\begin{bmatrix} \hat{B}_\delta & \hat{B}_{T_z} \end{bmatrix} \hat{\mathbf{u}} = \nu \quad (25)$$

Thus, in this method, the control input blending problem is redefined as finding the pseudo-control input command  $\hat{\mathbf{u}}_c$  which satisfies Eq.(25) and minimizes Eq.(22). The solution of this problem is given as below.

$$\hat{\mathbf{u}}_c = \mathbf{W}^{-1} \hat{\mathbf{B}}^T (\hat{\mathbf{B}} \mathbf{W}^{-1} \hat{\mathbf{B}}^T)^{-1} \nu \quad (26)$$

where

$$\mathbf{W} = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} \quad \hat{\mathbf{B}} = \begin{bmatrix} \hat{B}_\delta & \hat{B}_{T_z} \end{bmatrix} \quad (27)$$

Once  $\hat{\mathbf{u}}_c$  is obtained as Eq.(26), the actual control input commands are given from Eq.(24).

$$\begin{bmatrix} \delta_c \\ T_{z,c} \end{bmatrix} = \begin{bmatrix} \delta_{\max} & T_{z,\max} \end{bmatrix} \hat{\mathbf{u}}_c \quad (28)$$

### 3.3 Control Effectiveness Shaping Method

In Daisy-Chain Method, the reaction jet thrust command is designed to compensate the lack of tail-fin control effect due to its actuator limits. Thus, the reaction jet thrust is almost not required during steady-state, when the tail-fin deflection angle and slew rates are small. However, since large fin deflection angle and slew rate are needed during transient state, the reaction jet thrust command can be excessively

huge level. On the other hand, in Pseudo-Inverse Method, since the control input commands are designed based on a cost function, deriving immoderately large reaction jet command during transient state is avoided by defining proper weighting functions in Eq.(22). But, as shown in Eq.(26), both control inputs are used simultaneously whenever the autopilot control input command is non-zero. Since non-zero autopilot control input command is required to track the desired angle-of-attack value during steady-state, the reaction jet thrust should be used in this stage. This means the reaction jet thrust, which consumes limited fuel, should operate all the time in this method. Control Effectiveness Shaping Method is designed to overcome weaknesses and have strengths of previous explained two methods. In other words, reaction jet thrust command is designed not to have a large value during transient state and not to be used during steady-state.

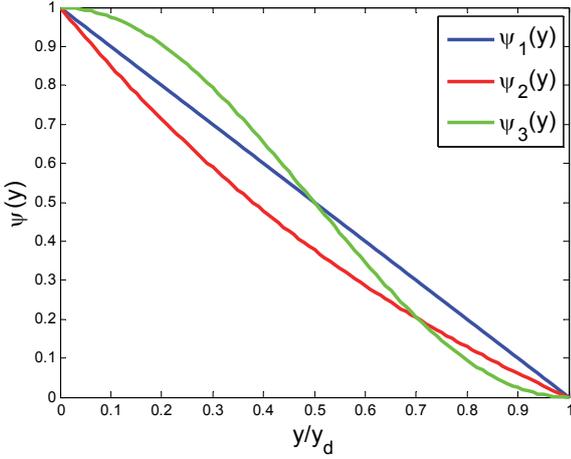
In Control Effectiveness Shaping Method, control input commands are determined by taking account of the cost function given in Eq.(22) to avoid the excessively large reaction jet thrust command. Moreover, in order not to use reaction jet thrust in steady-state, pseudo-control effectiveness of the reaction jet, given in Eq.(23), is modified as below.

$$\bar{B}_{T_z}(y) = \hat{B}_{T_z} \psi(y) \quad (29)$$

The independent variable  $y$  in Eq.(29) is the state variable which is controlled by the autopilot system and it is angle-of-attack for the autopilot designed in Section 2. The shaping function  $\psi(y)$  is defined to have a value of one at the initial state and converges to zero as  $y$  reaches to its desired value,  $y_d$ . In the case that the initial value of  $y$  is zero, functions in Eq.(30) can be defined as shaping function  $\psi(y)$ .

$$\begin{aligned}\psi_1(y) &= 1 - \frac{y}{y_d} \\ \psi_2(y) &= \frac{1}{1 - e^{-1}} \left( e^{-y/y_d} - e^{-1} \right) \\ \psi_3(y) &= \frac{1}{2} \left[ 1 + \cos \left( \pi \frac{y}{y_d} \right) \right]\end{aligned} \quad (30)$$

The graphs of shaping functions in Eq.(30) are shown in Fig.2.



**Fig. 2. Shaping Functions**

The next step is redefining the control input blending problem shown in Eq.(25) by exchanging the pseudo-control effectiveness of the reaction jet  $\hat{B}_{T_z}$  with the modified pseudo-control effectiveness  $\bar{B}_{T_z}$  defined in Eq.(29) as below.

$$\begin{bmatrix} \hat{B}_\delta & \bar{B}_{T_z} \end{bmatrix} \bar{\mathbf{u}} = \nu \quad (31)$$

The modified pseudo-control input variable vector  $\bar{\mathbf{u}}$  is defined from Eq.(16), Eq.(23) and Eq.(29) as

$$\bar{\mathbf{u}} = \begin{bmatrix} \frac{\delta}{\delta_{\max}} \\ T_z \\ \frac{T_z}{T_{z,\max} \psi(y)} \end{bmatrix} \quad (32)$$

The control input blending problem is defined to obtain the modified pseudo-control input command  $\bar{\mathbf{u}}_c$  that satisfies Eq.(31) and

minimizes cost function given as Eq.(22). The solution of this blending problem is given as follows.

$$\bar{\mathbf{u}}_c = \mathbf{W}^{-1} \bar{\mathbf{B}}^T (\bar{\mathbf{B}} \mathbf{W}^{-1} \bar{\mathbf{B}}^T)^{-1} \nu \quad (33)$$

where

$$\mathbf{W} = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} \quad \bar{\mathbf{B}} = \begin{bmatrix} \hat{B}_\delta & \bar{B}_{T_z} \end{bmatrix} \quad (34)$$

In Eq.(29), due to the property of  $\psi(y)$ ,  $\bar{B}_{T_z}(y)$  is equal to  $\hat{B}_{T_z}$  at the initial state and it goes to zero as  $y$  approaches to  $y_d$ . This means, from Eq.(33) and Eq.(34), that the portion of the reaction jet thrust reduces gradually as  $y$  approaches to  $y_d$ , and it converges to zero when the autopilot system is in steady-state. Thus, the reaction jet thrust is not needed for angle-of-attack control during steady-state. Since the modified pseudo-control input command is obtained as Eq.(33), the control input commands are determined from Eq.(32) as below.

$$\begin{bmatrix} \delta_c \\ T_{z,c} \end{bmatrix} = \begin{bmatrix} \delta_{\max} & T_{z,\max} \psi(y) \end{bmatrix} \bar{\mathbf{u}}_c \quad (35)$$

## 4 Simulations

Numerical simulations are performed in this Section to analyze performances and characteristics of Control Effectiveness Shaping Method and compare this algorithm with other methods, Daisy-Chain Method and Pseudo-Inverse Method. The motion of vehicle is simulated with six degree-of-freedom dynamic model. The autopilot gains are designed to be

$$K_1 = 20 \quad K_2 = 20 \quad (36)$$

The actuator limits are defined as follows.

$$\begin{aligned} \delta_{\min} &= -30^\circ & \delta_{\max} &= 30^\circ \\ \dot{\delta}_{\min} &= -450^\circ/\text{sec} & \dot{\delta}_{\max} &= 450^\circ/\text{sec} \\ T_{z,\min} &= -20kN & T_{z,\max} &= 20kN \end{aligned} \quad (37)$$

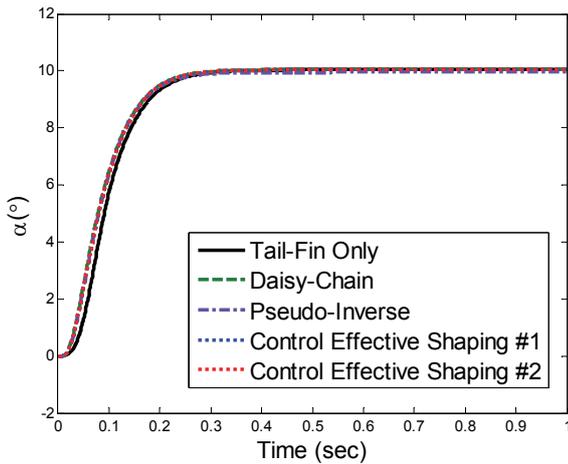
The sampling time is set to be  $\Delta t = 0.0002$  sec. The weighting function matrix in cost function for Pseudo-Inverse Method and Control Effectiveness Shaping Method is

$$\mathbf{W} = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \quad (38)$$

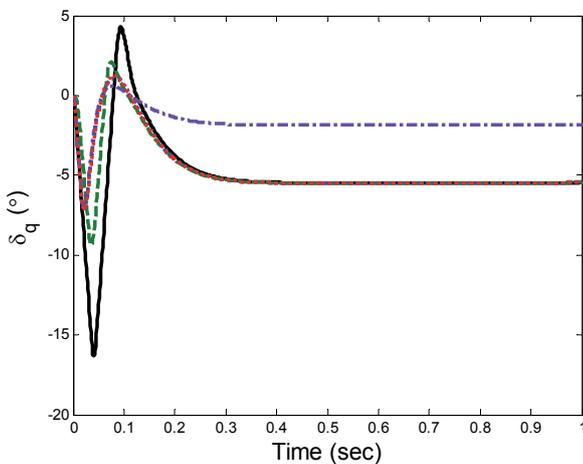
Two shaping functions for Control Effectiveness Shaping Method are chosen for this simulation.

$$\begin{aligned} \psi_1(y) &= 1 - \frac{y}{y_d} \\ \psi_2(y) &= \frac{1}{1 - e^{-1}} (e^{-y/y_d} - e^{-1}) \end{aligned} \quad (39)$$

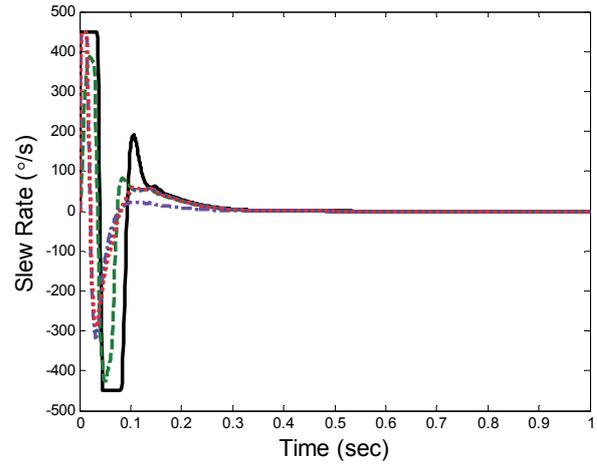
Simulation results for the angle of attack command  $\alpha_{cmd} = 10^\circ$  are presented below.



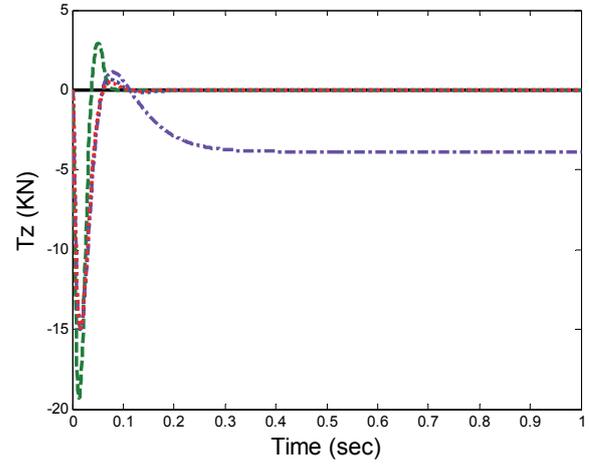
a) Angle-of-attack



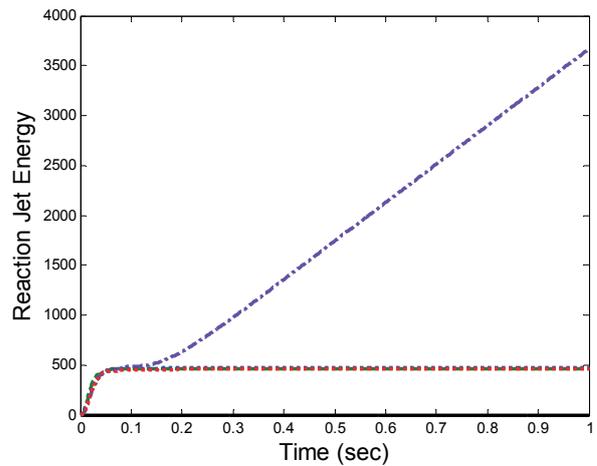
b) Tail-fin Deflection Angle



c) Tail-fin Slew Rate



d) Reaction Jet Thrust



e) Reaction Jet Thrust Energy

**Fig. 3. Simulation Results**

Reaction jet thrust energy means the total amount of reaction jet thrust used and it is calculated by integrating the absolute value of reaction jet thrust through the simulation time as described below.

$$\int_0^t |T_z(\tau)| d\tau \quad (40)$$

From Fig. 3 - a), it is shown that the angle-of-attack converges to the desired value in finite time only with tail-fin deflection angle. However, due to the slew rate limit, the tail-fin actuator is saturated as observed in Fig. 3 - c), meaning that the slew rate of the tail-fin actuator is smaller than that required to follow the control input command. This results in the degeneration of the autopilot system, and reaction jet is introduced to solve this problem.

In Daisy-Chain Method case, the tail-fin slew rate is not saturated as shown in Fig. 3 - c). Also, from Fig. 3 - d) and e), it is noticed that the reaction jet thrust is zero and the reaction jet thrust energy does not grow larger at the steady-state. But, among all the blending methods used, the largest magnitude of reaction jet thrust is required at the transient state, and its maximum value is  $19.29kN$  as appeared in Fig. 3 - d).

The tail-fin actuator saturation is relieved by Pseudo-Inverse Method, as shown in Fig. 3 - c). From Fig. 3 - d), the maximum magnitude of the reaction jet thrust is  $14.98kN$ , meaning that smaller magnitude of thrust is required at the transient state than Daisy-Chain Method case. However, as shown in Fig. 3 - d) and e), the reaction jet thrust is not zero at steady-state, resulting in the gradual increase of the reaction jet thrust energy.

In this simulation, two blending logics based on Control Effectiveness Shaping Method are used, and they are differed by their own shaping function,  $\psi_1(y)$  and  $\psi_2(y)$  given in Eq.(39). Both blending logics relieved the tail-fin actuator saturation as observed in Fig. 3 - c). Also, from Fig. 3 - d) and e), Control Effectiveness Shaping Method is considered to have the strengths of both blending logics, Daisy-Chain Method and Pseudo-Inverse Method. The largest magnitude of the reaction jet thrust for the blending logic with  $\psi_1(y)$  is

$14.96kN$  and that for the blending logic with  $\psi_2(y)$  is  $14.95kN$ , which are almost the same with that of Pseudo-Inverse Method. This shows that, compared with Daisy-Chain Method case, reaction jet thrust with smaller magnitude is used at the transient state. At the steady-state, the reaction jet thrust converges to zero and the reaction jet thrust energy almost does not grow in both blending logic cases. Moreover, since the time histories of the reaction jet thrust and the final values are differed by the shaping functions used, it is able to shaping the reaction jet thrust command by designing a proper shaping function.

## 5 Conclusion

Control input blending problem for vehicle system with two control inputs, tail-fin deflection angle and reaction jet thrust, is handled in this paper. The autopilot is designed by applying backstepping control law to the vehicle dynamics only with tail-fin, and it generates the tail-fin deflection angle command to track the desired angle-of-attack value. Blending logic is designed to divide this autopilot control input command into two control input commands properly. Two previously studied blending logics, Daisy-Chain Method and Pseudo-Inverse Method, are introduced. In Daisy-Chain Method, the tail-fin deflection angle is the prior control input and the reaction jet input is used when the tail-fin deflection angle command exceeds the actuator limit. This method makes the reaction jet thrust converge to zero at steady-state, but it utilizes large magnitude of reaction jet thrust at transient state. Pseudo-Inverse Method defines the control input commands that minimize the cost function, whose weighting function matrix shows the priority of each control input. In this method, smaller reaction jet thrust is required at transient state. However, the weakness of this method is that the reaction jet thrust has non-zero value at steady-state, resulting in the continuous energy consumption. Control Effectiveness Shaping Method is designed to have the contrasting strengths of those two blending logics. This method is modified

Pseudo-Inverse Method with pseudo-control effectiveness shaped to converge to zero at steady-state. As a result, with Control Effectiveness Shaping Method, the reaction jet thrust used at transient state is small and it is not required at steady-state. Numerical simulation results implies those features of Control Effectiveness Shaping Method, and the capability of shaping reaction jet thrust history with proper shaping function is also shown.

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### Acknowledgements

This research was supported by Agency for Defense Development under the contract UD110031CD.

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