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MATHEMATICAL MODEL OF AIRCRAFT WAKE VORTICES DEVELOPMENT FOR TAKE-OFF/LANDING PHASES FOR WAKE VORTEX SAFETY SYSTEMS

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Abstract

One of the most important areas of research in aviation technology is an increase of air traffic control efficiency and flight safety provision in vicinity of airfields for take-off/landing flight phases, i.e. in conditions of possible entering of aircraft into wake vortices of other aircraft. Elaboration of wake vortex safety systems (WVSS) is rather effective solution of this problem. Math model of aircraft wake vortex is the main component of WVSS. The model should provide reliable information on intensity and spatial location of dangerous vortices areas. Calculation results should be available in real time scale. Presented in the paper is the detailed description of math model of aircraft wake vortices development. Examples of calculations of wake vortices characteristics for various aircraft, flight conditions and atmospheric states are presented. It is noted that the model elaborated could be utilized in WVSS both for take-off/landing flight phases and for cruise flight conditions during aircraft flight along flight routes.

1 Introduction

Aviation specialists, scientists and researchers of different countries are developed for a long time the methods of flight safety problem solution in conditions when aerodynamic wake vortex is the main factor influencing flight safety level [1] and are elaborated the systems enabling safe flights in the areas of intensive vortex flows generated by other aircraft air flows [2].

One of perspective direction of developments in this area is development of integrated system of detection and prognosis of vortex environment (so-called WVSS) with implemented wake vortex math model. This system should provide visualization of the vortex situation on the ATC staff screen and in the pilot cockpit also. WVSS should comprise both onboard and ground systems providing wake vortex safety for all flight phases including take-off and landing.

WVSS represents soft – hardware complex with digital communication lines, man-machine interface and corresponding meteorological support. WVSS is based on the technique of representation of visual information about vortex environment in the area of aircrew and ATC staff responsibilities. Aircraft trajectories parameters and data about dangerous vortex areas are provided onboard aircraft and ATC staff via digital data transmission lines. Special algorithms are developed to calculate dangerous vortex areas. Flight safety depends on the precision of these algorithms operation.

The experimental specimen of WVSS was developed in PJSC "Gromov Flight Research Institute" (GFRI) with participation of other Russian companies. Flight tests of this specimen were conducted in GFRI. For this purpose two flying test-beds (FTB) were specially instrumented: Tu-154M FTB – wake generator and L-39 FTB – wake sounder. The scheme of WVSS specimen flight tests is presented in Figure 1.

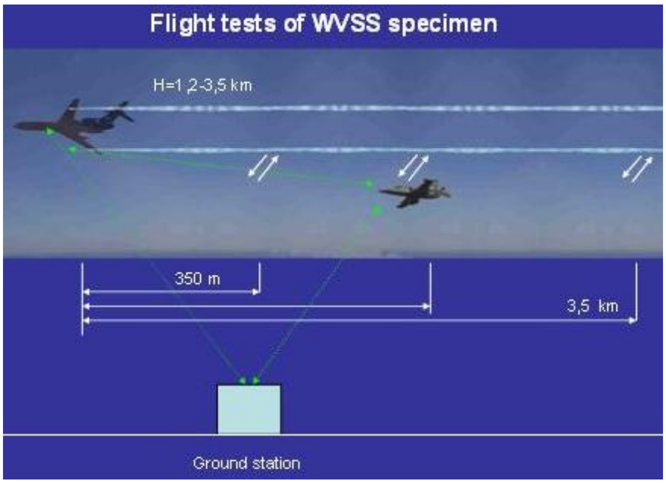

Fig. 1. Scheme of WVSS flight tests.

Fig. 2. View of wake-generator Tu-154M (left) via canopy of L-39. L-39 onboard LCD (right). H = 3170 m, Vias = 380 km/h, neutral atmosphere.

Tu-154M FTB was instrumented with onboard measurements system which records the necessary flight parameters, onboard weather station, satellite navigation system, onboard digital computer, WVSS computer, smoke generators “Smokewinder” with its control system. L-39 FTB was instrumented with the system of flight-navigation information representation in multifunctional LCD, onboard measurements system, satellite navigation system, WVSS computer and a system to record video image of airspace around the aircraft and pilot flight deck. Ground station – workplace of ATC staff includes computer, image representation system and base station.

Flight tests revealed that system principally works, but it has some serious disadvantages mainly due to calculation models of vortex flows and dangerous areas determination. For example, during flight tests there were detected some cases of displaying of erroneous information about dangerous vortex flows. Figure 2 represents the case when visually Tu-154M FTB aircraft and dangerous vortex area are located at the left from sounder aircraft L-39 FTB, but onboard LCD displays this area at the right hand side from the L-39 FTB. These results point out on necessity of serious refinement of calculation models and this was done at the work presented.

2 Algorithm of aircraft wake vortex calculation

Block diagram of aircraft wake vortices calculation algorithm is presented in Figure 3.

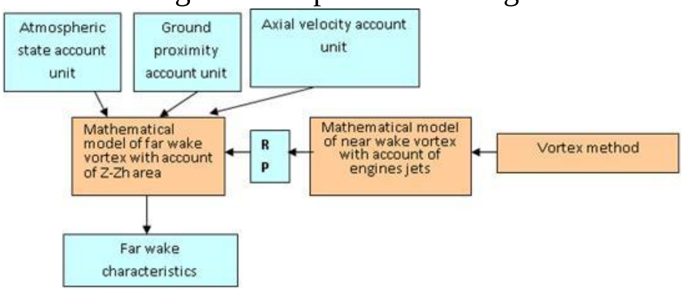

Fig. 3. Block diagram of aircraft wake vortices calculation algorithm.

Math model of near wake is constructed on the base of vortex method. Far wake math model is filled with initial data from near wake model through reference plane (RP). RP is located behind the aircraft perpendicular to flight velocity vector at the distance 0.5L, where L – aircraft length. All vortices falling in this plane are substituted by the vortices of infinite length, coming through the same points and having the same circulations as initial vortices. It allows proceeding to solution of a plane-parallel task, to which mathematical model of far wake is reduced. Math model of near wake was constructed basing on considerations of high accuracy provision during real time scale operation. Analogous models applied before [3, 4] utilized method of discrete vortices. This approach could provide good accuracy but it works very slowly, one should create appropriate data bases in advance to speed up the process. But data base discretization deteriorates the accuracy. The investigations of calculation techniques made it possible to choose sufficiently accurate model on the base of ***analytical - experimental approach*** formed by application of Zhukovsky theorem for lift force. In this case the aircraft is replaced by П-type vortex and circulation of wing vortices could be determined by equaling of aircraft lift force to weight. Next the circulation calculated is distributed between wing and horizontal tail proportionally to it squares. Thus four vortices are obtained with coordinates corresponding to the ends of wing ($\Gamma w1$, $\Gamma w2$) and horizontal tail ($\Gamma s1$, $\Gamma s2$). Development of six vortices is simulated if aircraft is in take-off/landing configuration (two from wing ends and two pairs from the ends of each flap ($\Gamma f1$, $\Gamma f2$ and $\Gamma f3$, $\Gamma f4$). Eight vortices are deployed from the aircraft itself, see Figure 4.

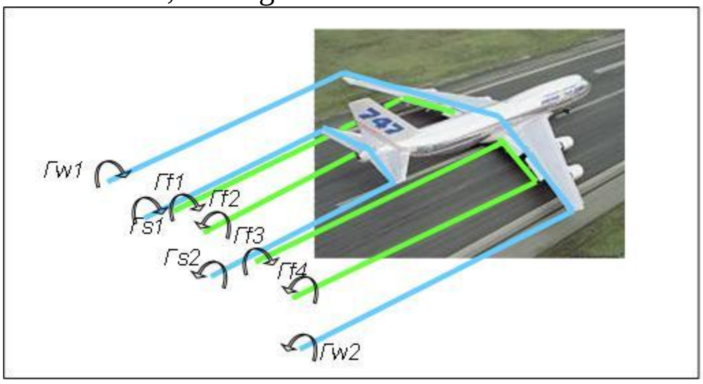

Fig. 4. Eight vortices from the aircraft.

Wing vortices circulation is distributed between all six vortices accordingly to the flap span, it area and deflection angle.

State of the atmosphere is accounted via Richardson number.

Account of the influence of axial velocity in the vortices nuclei on far wake characteristics in real flight is reduced to appearance of additional sources distributed along nuclei axis. More detail description of account of atmospheric state and axial velocity influence on the vortices development is presented in next sections.

Ground proximity is accounted via implementation of vortex system specularly reflected relatively to the earth surface.

Far wake characteristics are obtained as the results of calculations according to the algorithm described.

3 Account of the atmospheric state

Richardson number was utilized to account atmospheric state in far wake math model:

$$Ri = \frac{g}{\overline{T}} \frac{\Delta\overline{T} / \Delta z}{(\Delta\overline{N} / \Delta z)^2} \qquad (1)$$

Where g – gravity acceleration, $\overline{T}$ - mean temperature of the layer with thickness Δz. $\Delta\overline{T} / \Delta z$ and $\Delta\overline{N} / \Delta z$ - mean gradients of the temperature and a horizontal velocity of the wind in the layer with thickness Δz correspondingly.

Richardson number characterizes the relation of buoyancy force (numerator) and dynamic factor (denominator), i.e. relation of contributions of free and forced convections in atmosphere turbulence formation. The increase of the temperature gradient module corresponds to the state when buoyancy forces dominate. The increase of the velocity gradient corresponds to dynamic factor increase and characterizes the atmosphere as unstable. Atmosphere state is considered to be neutral at $-0.01 \le Ri \ge 0.01$ at that the thermal influence is minimum and only forced convection could exist in this case. At diminishing of the number $Ri < -0.01$ the buoyancy forces start to appear in more extent, mixed convection arises and at $Ri < -1.0$ the free convection mode is established. And vice versa, at the increase of $Ri > 0.01$ the buoyancy forces start to prevent the turbulence development. At $Ri > 0.25$ the flow starts to be practically laminar, turbulent mixing is almost absent. Thus, all considerations presented above could be reduced in the following Table 1.

Table 1.

Richardson number Ri	State of the atmosphere	Estimation in points
$Ri < -1.0$	Strongly unstable	5
$-0.01 > Ri \geq -1.0$	Unstable	4
$0.01 \geq Ri \geq -0.01$	Neutral	3
$2.5 \geq Ri > 0.01$	Stable	2
$Ri > 0.25$	Strongly stable	1

Estimations of the atmosphere state in points are introduced for convenience of this table utilizing. Strongly unstable atmosphere is assessed in 5 points and strongly stable – in 1 point. Points for other states of the atmosphere could be seen from the Table 1. As it was shown during flight tests the wake vortices development behind aircraft strongly depends on the atmosphere state. In the quiet atmosphere the wake vortices exist for a very long time while in unstable atmosphere they are destroyed quickly. Far wake math model utilizes the relation between atmospheric state points and reduced Reynolds number, obtained on the base of experimental data processing and thus it takes into account the dependence of air velocities field from atmospheric state.

4 Accounting of an influence of the axial velocity on the vortex propagation

Accounting of an influence of the axial velocity in the vortex nucleus on far wake characteristics in real flight is reduced to appearance of additional depression in the nucleus and additional velocities to the vortices center.

Let's consider that axial velocity (*Vx*) spatial distribution is known and also that this distribution is axisymmetric in the vortex nucleus.

$$V_x = V_x(x, r'), \quad r' = |\mathbf{r} - \mathbf{r}_0| \qquad (2)$$

where r – arbitrary point coordinate, r_0 – vortex coordinate. divV=0 because incompressible fluid model is considered. Thus, if axial velocity is not constant along X-axis:

$$\frac{\partial}{\partial y}V_y + \frac{\partial}{\partial z}V_z = -\frac{\partial}{\partial x}V_x \neq 0 \qquad (3)$$

In two-dimensional treatment in the YOZ plane this corresponds to the presence of two sources (sinks) with the density $q = -\frac{\partial}{\partial x}V_x$. These sources induce the velocity:

$$\mathbf{V}_q(\mathbf{r}) = \frac{1}{2\pi}\int \frac{\mathbf{r} - \mathbf{r}_q}{|\mathbf{r} - \mathbf{r}_q|^2} q(y_q, z_q)ds \qquad (4)$$

and the following takes place for it axisymmetric location:

$$\mathbf{V}_q(\mathbf{r}) = \frac{\mathbf{r} - \mathbf{r}_0}{|\mathbf{r} - \mathbf{r}_0|^2} Q(\mathbf{r}), \quad Q(\mathbf{r}) = \int_0^r r' \frac{\partial}{\partial x} V_x(r')dr' \qquad (5)$$

Thus intensity of the source (sink) Q(r) is determined via axial velocity value in the vortex nucleus and additional velocity in the arbitrary point is determined via Q(r).

As an example, for A320 aircraft there were obtained the results of wake vortices calculations for two cases – with and without account of axial velocity influence.

Figure 5 demonstrates A320 aircraft disturbed velocities fields at flight speed Vias = 850 km/h, flight altitude H = 6 000 m at the distances X = 0 km and X = 2,85 km behind the aircraft at the condition Vx = 0 (no axial velocity). Presented in Figure 6 are the analogous velocities fields at the condition Vx > 0 (axial velocity is available). Noticeable increase of disturbed velocities in case of axial flows accounting is observed.

Such influence of axial velocity on velocities field in a wake points out that axial flow in vortices nuclei couldn't be neglected. And experimental data indicate that axial flows in the nuclei are actually available and rather intensive. Character and parameters of these flows obtained via experimental data processing were used for accounting of axial velocity influence on the wake structure.

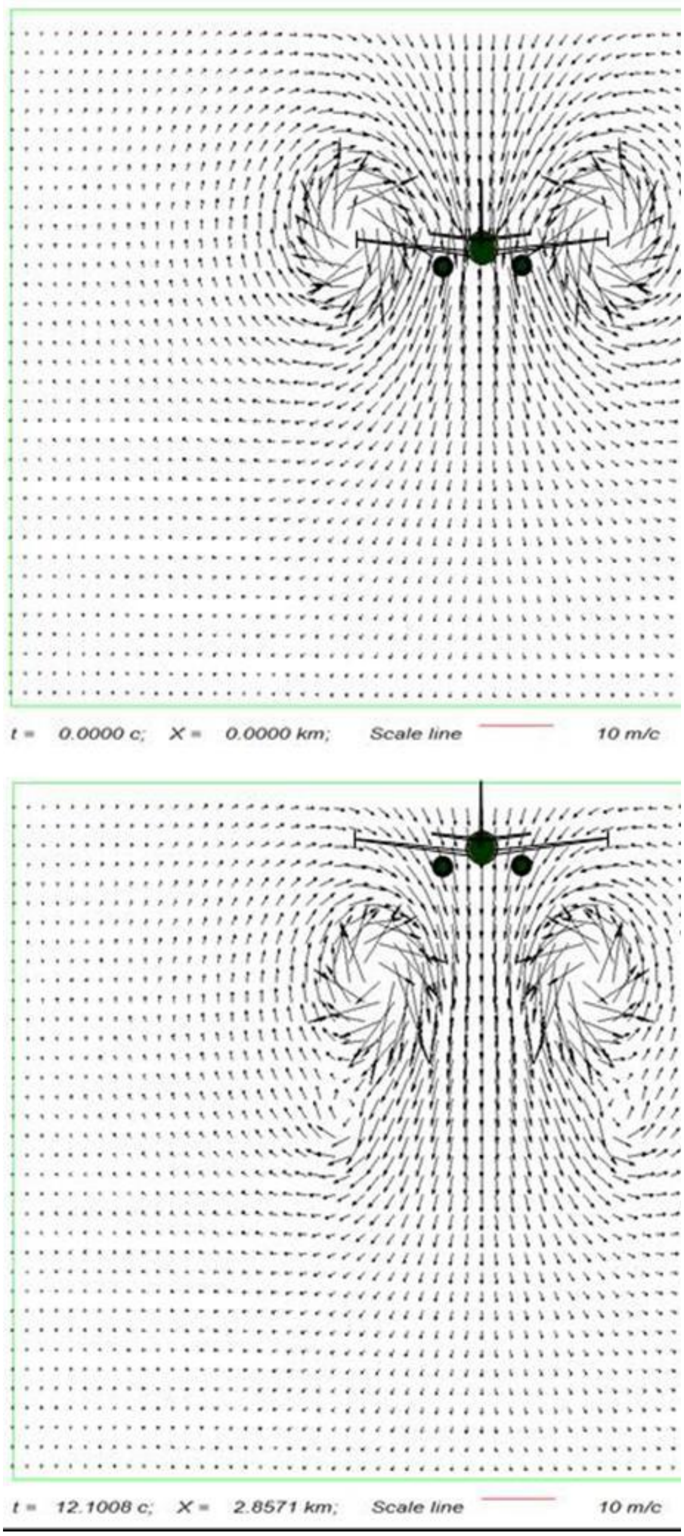

Fig. 5. A320 disturbed velocities field without Vx account.

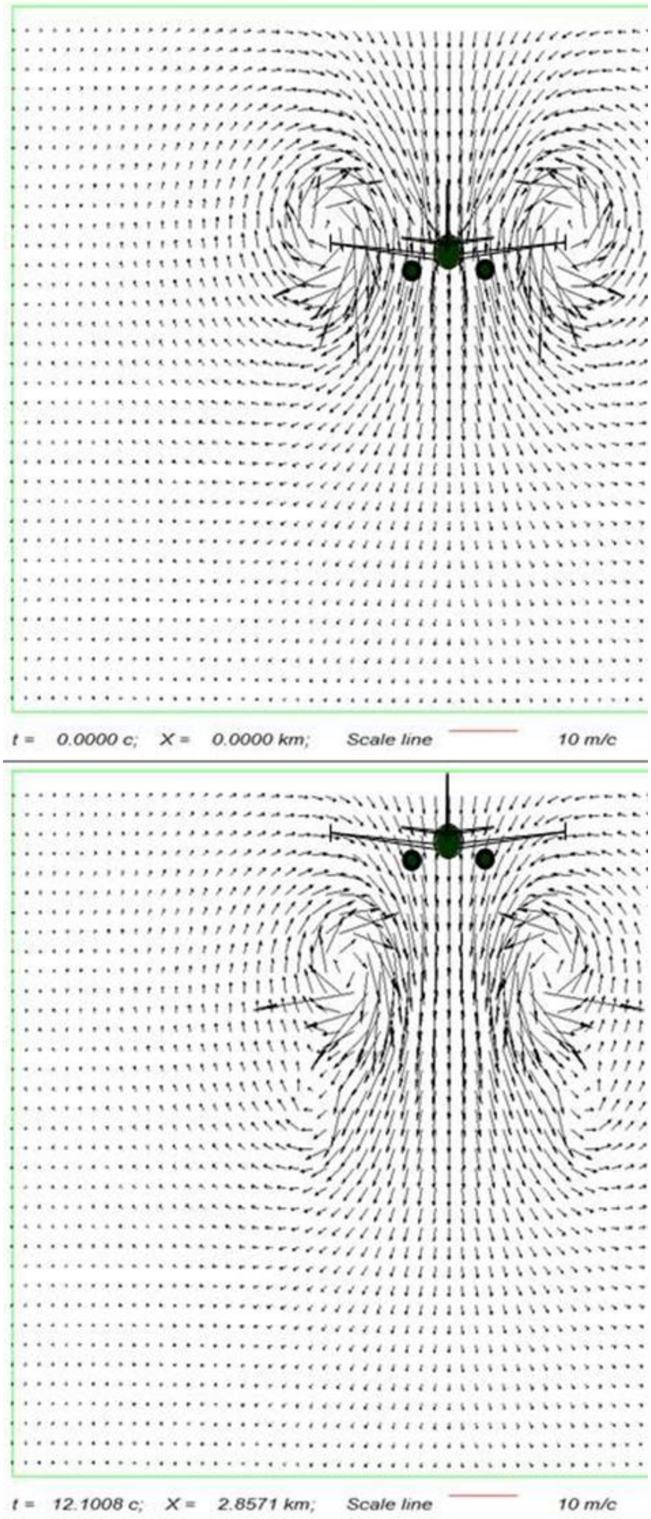

Fig. 6. A320 disturbed velocities field with Vx account.

5 Far wake mathematical model

At the basis of the far wake mathematical model there is exact solution of Helmholtz equation, which has the following form for plane-parallel flow:

$$\frac{\partial \Omega_x}{\partial t} + W_y \frac{\partial \Omega_x}{\partial y} + W_z \frac{\partial \Omega_x}{\partial z} = \nu \left(\frac{\partial^2 \Omega_x}{\partial y^2} + \frac{\partial^2 \Omega_x}{\partial z^2} \right) \quad (6)$$

where Ω_x - angular velocity vector ($\vec{\Omega}$) component and W_y and W_z - components of the fluid motion linear velocity vector ($\vec{W}$), ν - kinematic viscosity coefficient. Equation (1) describes the process of vortex diffusion in viscous incompressible fluid. Solution description is presented in [5].

If in initial moment t = 0 the vortex with circulation Γ_{+i} is located in the point (y_i, z_i) in parallel to Ox axis, then in accordance with exact solution of equation (1) for any arbitrary point in the plane zOx there will be:

$$\Omega_{x(i)}(y,z,t) = \frac{\Gamma_{+i}}{8\pi\nu} \frac{1}{t} e^{-\frac{(y-y_i)^2 + (z-z_i)^2}{4\nu t}} \quad (7)$$

In accordance with the Stokes theorem and with account of the initial condition $\Gamma_{+i}(y,z,0) = const$, the expression for circulation to be defined in the point (x,z) in arbitrary time moment is written as:

$$\Gamma_{+i}(y,z,t) = \Gamma_{+i}(y,z,0) \left[1 - e^{-\frac{(y-y_i)^2 + (z-z_i)^2}{4\nu t}} \right] \quad (8)$$

Velocity vector components at this point in arbitrary time moment are defined by the following expressions:

$$W_{y(i)} = \frac{\Gamma_{+i}}{2\pi} \frac{z - z_i}{(y-y_i)^2 + (z-z_i)^2} \left[1 - e^{-\frac{(y-y_i)^2 + (z-z_i)^2}{4\nu t}} \right] \quad (9)$$

$$W_{z(i)} = \frac{\Gamma_{+i}}{2\pi} \frac{y - y_i}{(y-y_i)^2 + (z-z_i)^2} \left[1 - e^{-\frac{(y-y_i)^2 + (z-z_i)^2}{4\nu t}} \right] \quad (10)$$

or in non-dimensional form:

$$\overline{W}_{y(i)} = \frac{\overline{\Gamma}_i}{2\pi} \frac{\bar{z} - \bar{z}_i}{(\bar{y} - \bar{y}_i)^2 + (\bar{z} - \bar{z}_i)^2} \left[1 - e^{-\mathrm{Re}\frac{(\bar{y} - \bar{y}_i)^2 + (\bar{z} - \bar{z}_i)^2}{4\tau}} \right] \quad (11)$$

$$\overline{W}_{z(i)} = \frac{\overline{\Gamma}_i}{2\pi} \frac{\bar{y} - \bar{y}_i}{(\bar{y} - \bar{y}_i)^2 + (\bar{z} - \bar{z}_i)^2} \left[1 - e^{-\mathrm{Re}\frac{(\bar{y} - \bar{y}_i)^2 + (\bar{z} - \bar{z}_i)^2}{4\tau}} \right] \quad (12)$$

In (11)-(12) there are $\overline{\Gamma}_i = \Gamma_{+i} / W_\infty b$, $\bar{y} = y / b$, $\bar{z} = z / b$, $\tau = W_\infty t / b$, $\mathrm{Re} = W_\infty b / \nu$, W_∞ - velocity of non-disturbed flow.

Thus the basis of far wake mathematical model is constituted of expressions (11)-(12) where reduced value of Reynolds number Re^* is introduced instead of Reynolds number Re and instead of coefficient ν there is introduced the reduced coefficient ν^*. The value of ν^* coefficient is obtained by means of experimental data treatment and depends on atmospheric state. Account of vortices dissipation and diffusion is made via this coefficient ν^*.

6 Examples of aircraft wake vortices calculations for various flight conditions utilizing the model developed

Results of Tu-154M aircraft wake vortex flow calculations for flight conditions corresponding to those, depicted in Figure 2, i.e. H = 3170 m, Vias = 380 km/h, neutral atmosphere are presented in Figure 7. Below in the Figure 7 there is presented a picture, which will be displayed on L-39 aircraft onboard LCD at the distance of X = 420 m behind aircraft-generator (Tu-154M), as it was during flight tests. The picture demonstrated corresponds to the photo, illustrated in Figure 2.

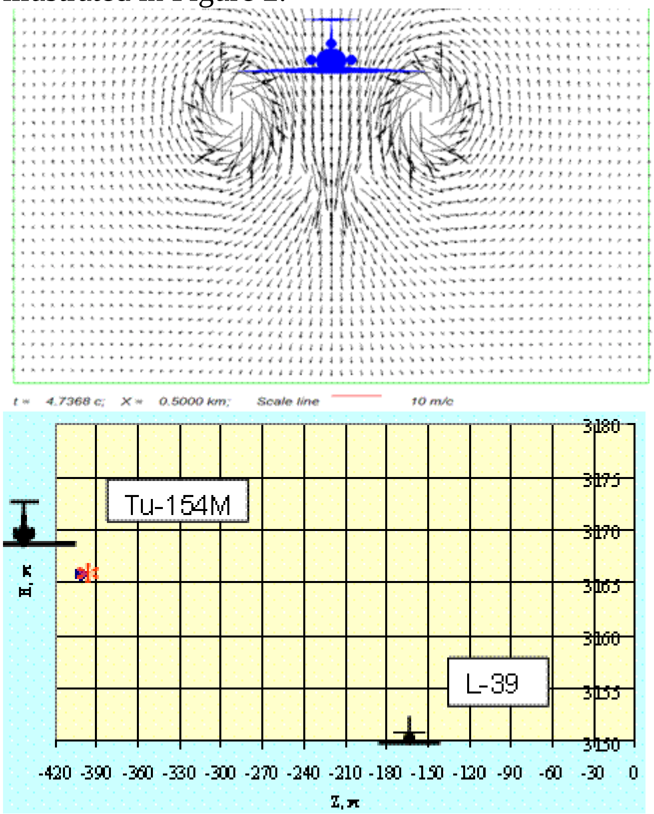

Fig. 7. Example of wake vortex calculations.

The math model presented was developed for implementation in WVSS at take-off/landing flight phases, but it also could be applied for cruse flight conditions. Depicted in Figures 8 and 9 are results of B747 wake vortices development calculations for flight conditions H=6000 m, Vias = 800 km/h, stable atmosphere: Figure 8 – velocities field at the distance X=4047,6 m behind the aircraft and Figure 9 – vortices axes locations in vertical plane. Stabilizer vortex, being less intensive than the wing vortex, exists for a long time in a wake for aircraft with aerodynamic layout analogous to B747.

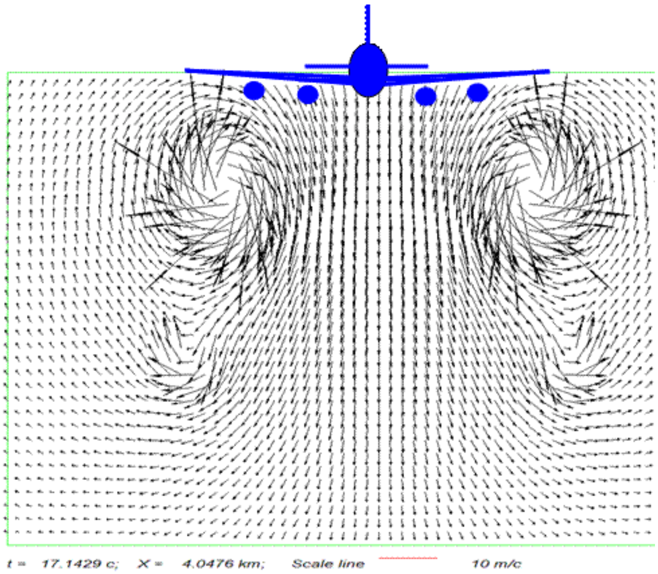

Fig. 8. B747 velocities field at the distance X=4047.6m, Vias=800km/h, stable atmosphere.

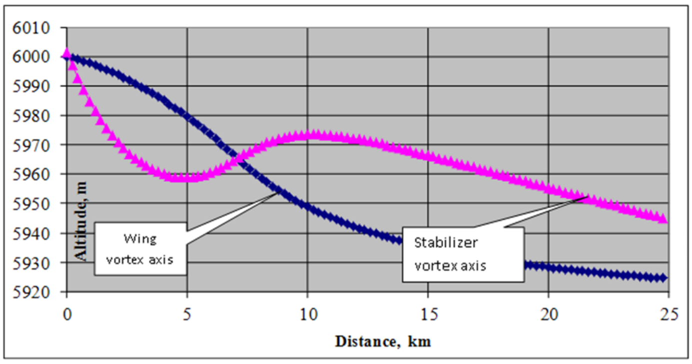

Fig. 9. B747 wake vortices axes location, H=6000m, Vias=800km/h, stable atmosphere.

7 Conclusions

It was elaborated the math model of aircraft wake vortex development, providing calculations of intensity and special location of dangerous vortex areas in real time scale.

Math model on the basis of experimental data made it possible to account the influence of atmosphere state, ground proximity and availability of axial flows in vortices nuclei on the structure and development of aircraft wake.

Math model is intended for implementation in WVSS both for take-off/landing and cruse flight conditions.

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