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CONSTRAINED PREDICTIVE STRATEGIES FOR FLIGHT CONTROL SYSTEMS

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Abstract

The complexity of nowadays flight control systems arises from the engineering prescription to properly merge and integrate actuators, sensors and computing units within complex data network structures. The use of many different sensors and actuators, the presence of nonlinear phenomena, dynamic effects due to switching operating conditions as well as physical constraints due to input/state saturations are of paramount relevance when designing a modern flight control strategy, especially in the case of high performance aircrafts. In this paper we analyze the effectiveness of recent constrained predictive control strategies when they are used to deal with specific aircraft control problems. Numerical simulations are presented on a High Altitude Performance Demonstrator unmanned air vehicle adopting two different predictive strategies both dealing with redundant and saturating actuators.

1 INTRODUCTION

Flight Control problems are subject to input and state dependent constraints which can make the controller design a complex task. Saturating actuators, flight envelope limitations, restrictions due to comfort and safety requirements are examples of limitations affecting the aircraft. Anti-Windup (AW), Bumpless methods, AW/LQR, AW/H2, genetic algorithms and recovery guidance techniques, represent solutions discussed in the literature [1]. More recently, techniques based on set-invariance arguments and predictive control ideas [2]-[3] have gained in popularity due to their inherent capability to take directly into account prescribed constraints. In this paper two contrained control strategies for flgiht control purposes are discussed. An over-actuated unmanned aircraft developed at C.I.R.A. (Italian Aerospace Research Center), named High Altitude Performance Demonstrator (HAPD), is considered for simulation purposes. The paper is organized as follows. The mathematical model of the HAPD including aero-elastic modes is introduced in Section 2. In Section 3, the Model Predictive Control algorithm developed in [4] for norm-bounded uncertain systems, is discussed in view of its application to the HAPD. The proposed technique is oriented to the solution of constrained control allocation problems of over-actuated systems. In Section 4, the design and real-time implementation of a low-computational demanding predictive scheme known in literature as the command governor (CG) [5] is illustrated. This is oriented to the supervision of nonlinear dynamical systems, as aircrafts, subject to sudden switchings amongst operating conditions and set-points, and time-varying constraints. Numerical simulations are discussed in Section 5.

2 HAPD MATHEMATICAL MODEL

The High Altitude Performance Demonstrator is an over-actuated unmanned aircraft, see Fig. 1.

In particular, it has three pairs of elevators divided in inboard (IB), middle (MID) and outboard (OB), two pairs of ailerons divided in inboard and outboard, and two rudders, namely the upper (SUP) and lower (INF) rudder. Thrust is generated by eight independent electrically powered propellers. A mathematical model, which

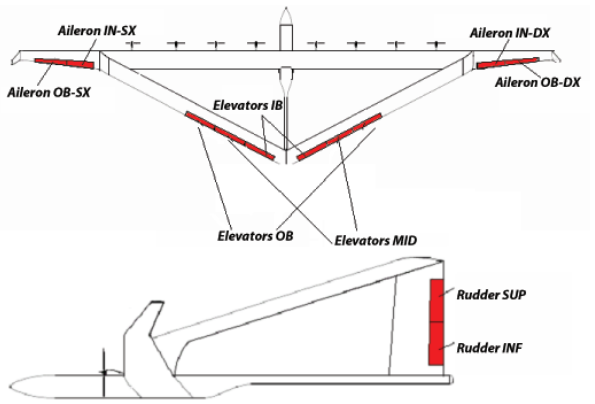

Fig. 1 HAPD model: twelve control surfaces and eight available propellers

takes into account flexibility, was developed by the Italian Aerospace Reserach Center (CIRA) under the following hypotheses: the inertia matrix I is independent from the aircraft elastic deformations; the linear elastic theory can be used to model the aero-elastic dynamics; aero-elastic modes are quasi-stationary. Under such assumptions, the *polar form* of the nonlinear equations of motion are (see e.g. [6])

$$M\dot{V} = T\cos\alpha\cos\beta - \bar{q}SC_D + Mg_1 \quad (1)$$

$$VM\dot{\beta} = -T\cos\alpha\sin\beta + \bar{q}SC_Y - MVr + Mg_2 \quad (2)$$

$$MV\cos\beta\dot{\alpha} = -T\sin\alpha - \bar{q}SC_L + MVq + Mg_3 \quad (3)$$

$$I_x\dot{p} - I_{xz}\dot{r} = \bar{q}S_bC_l + qr(I_y - I_z) + pqI_{xz} \quad (4)$$

$$I_y\dot{q} = \bar{q}S_cC_m + rp(I_z - I_x) + (r^2 - p^2)I_{xz} \quad (5)$$

$$-I_{xz}\dot{p} + I_z\dot{r} = \bar{q}S_bC_n + pq(I_x - I_y) - qrI_{xz} \quad (6)$$

$$\dot{\phi} = p + q\tan\theta\sin\phi + r\tan\theta\cos\phi \quad (7)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \quad (8)$$

where $\bar{q} = 0.5\rho V_{TAS}^2$ is the dynamic pressure, T the thrust, $V_{TAS} = \|V_B - V_W\|$ the true air speed,$V_B = (u_B, v_B, w_B)$ the 6DoF linear velocity vector, $V_W = (u_W, v_W, w_W)$ the atmospheric wind velocity vector, $V = \|V_B\|$, $\omega_B = (p, q, r)$ the rotational velocity vector, ϕ the roll angle, θ the pitch angle, $\alpha = \arctan\left(\frac{w_B - w_w}{u_B - u_w}\right)$ the angle of attack, $\beta = \arcsin\left(\frac{v_B - v_w}{V}\right)$ the sideslip angle ρ the air density, I_x, I_y, I_z, I_{xz} the moments and products of inertia in body axes and

$$\begin{aligned} g_1 &= g(-\cos\alpha\cos\beta\sin\theta + \\ &\quad + \sin\beta\sin\phi\cos\theta + \sin\alpha\cos\beta\cos\phi\cos\theta) \\ g_2 &= g(\cos\alpha\sin\beta\sin\theta + \\ &\quad + \cos\beta\sin\phi\cos\theta - \sin\alpha\sin\beta\cos\phi\cos\theta) \\ g_3 &= g(\sin\alpha\sin\theta + \cos\alpha\cos\phi\cos\theta), \end{aligned}$$

with g the gravity acceleration. Moreover by resorting to the generalized state variables η_i and $\dot{\eta}_i$, aero-elastic modes are modelled by means of a second order linear state space description:

$$M_{\eta_i}\ddot{\eta}_i + \zeta_{\eta_i}\dot{\eta}_i + M_{\eta_i}\omega_{\eta_i}\eta_i = Q_{\eta_i}, \; i = 1, \ldots, n_a \quad (10)$$

where M_{η_i} is the generalized mass of the $i-th$ mode, ζ_{η_i} the generalized damping coefficient, ω_{η_i} the generalized natural frequency and Q_{η_i} the generalized force. Notice that due to aero-elastic dynamics the aerodynamic coefficients $(C_D, C_Y, C_L, C_l, C_m, C_n)$ and generalized forces Q_{η_i} depend on 6DoF variables $(V_{TAS}, \alpha, \beta, p, q, r)$, on surfaces control deflection and on generalized state variables η_i and $\dot{\eta}_i$. Finally the thrust T is assumed to be a known function of the throttle command δ_T.

Table 1 HAPD: main parameters

Parameters	Value	Units
Wing Area (S)	13.5	m^2
Wing Span (S_b)	16.55	m
Mean Chord (S_c)	0.557	m
Mass (M)	184.4	kg
Elevators Slew Rates	±200	deg/s
Ailerons Slew Rates	±200	deg/s
Rudders Slew Rates	±200	deg/s
Ailerons deflections	±25	deg
Elevators deflections	±25	deg
Rudders deflections	±25	deg

3 Method 1: Model Predictive Control allocation scheme

Consider the discrete time system, obtained with a zero order hold continous to discrete conversion of the aircraft model,

$$\begin{array}{rcl} x(t+1) & = & f(x(t),u(t)) \\ y(t) & = & h(x(t),u(t)) \end{array} \quad (11)$$

with $x \in \mathbb{R}^{n_x}$ denoting the state, $u \in \mathbb{R}^{n_u}$ the control input, $y \in \mathbb{R}^{n_y}$ the output. Suppose that for each x, u, and each t there exists a matrix function $\Gamma(\cdot,\cdot) : \mathrm{R}^{n_x} \times \mathrm{R}^{n_u} \to \Omega(\Delta)$, such that $f(x,u) = \Gamma(x,u)\begin{bmatrix} x \\ u \end{bmatrix}$ where $\Omega(\Delta)$ is a convex set of matrices of appropriate dimensions. Then, any property ensured for the uncertain linear system

$$\begin{array}{rcl} x(t+1) & = & \Phi x(t) + Gu(t) + B_p p(t) \\ y(t) & = & Cx(t) \\ q(t) & = & C_q x(t) + D_q u(t) \\ p(t) & = & \Delta(t) q(t) \end{array} \quad (12)$$

with $p,q \in \mathbb{R}_{n_p}$ denoting additional variables which account for the uncertainty, holds true locally also for the nonlinear system (11), see [7]. In (12) the set

$$\Omega(\Delta) := \left\{ \tilde{A} + \tilde{B}\Delta\tilde{C} \,|\, \|\Delta\| \leq 1 \right\} \quad (13)$$

with $\tilde{A} = \begin{bmatrix} \Phi & G \\ C & 0 \end{bmatrix}, \tilde{B} = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, \tilde{C} = \left[C_q \; D_q\right]$ is the image of the matrix norm unit ball under a matrix linear-fractional mapping. In the sequel we shall assume that the plant is subject to the ellipsoidal input $u(t) \in \Omega_u$ and state evolution constraints $Cx(t) \in \Omega_x$ with

$$\Omega_u \triangleq \left\{ u \in \mathbb{R}^{n_u} : \|u\|_2^2 \leq \bar{u}^2, \;\; \bar{u} \in \mathbb{R}^+ \right\} \quad (14)$$

$$\Omega_x \triangleq \{ x \in \mathbb{R}^{n_x} : \|Cx\|_2^2 \leq \bar{x}^2, \;\; \bar{x} \in \mathbb{R}^+ \} \quad (15)$$

By resorting to the ideas developed in [8], we propose a two-stage optimization procedure. Specifically, in the first phase, hereafter denoted as *Reference Trajectory module*, a feasible reference trajectory is made available to the MPC controller. Then, the control allocation problem is formulated as a *receding horizon tracking* problem with the reference trajectory achieved at the previous step and attacked by means of the MPC framework discussed in [9]. The scheme of the proposed architecture is depicted in Fig. 2.

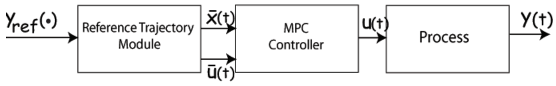

Fig. 2 Control allocation architecture

The control allocation problem can be stated as follows:

Control Allocation (CA) problem - *Given an initial time instant t_0 and a reference trajectory $y_{ref}(\cdot)$, determine at each time instant $t \geq t_0$ a command input $u(t)$ such that the output $y(t)$ of the plant model (12)-(13) subject to (14)-(15) tracks $y_{ref}(t)$, $\forall t \geq t_0$, as closely as possible in a 2-norm sense.* □

In the sequel, the reference to be tracked will be denoted as $r(\cdot) = y_{ref}(\cdot)$, $y_{ref}(\cdot) \in \mathcal{Y}$, where $\mathcal{Y}$ accounts for limitations in the angular acceleration provided by the actuators.

3.1 Reference Trajectory module

Consider a time-varying reference trajectories, the solution of the **CA** problem is subject to the computation over an horizon of a finite length $N \in \mathbb{N}$ of a feasible input sequence such that the sequence output $\{y_i(t) = Cx_i(t)\}_{i=0}^N$ corresponds to the given reference sequence $\{r_i(t)\}_{i=0}^N$. To this end it is mandatory that a nominal plant model is available: within the proposed norm-bounded framework this translates to consider the so-called *central* dynamics, i.e.

$$\begin{array}{rcl} x(t+1) & = & \Phi x(t) + Gu(t) \\ y(t) & = & Cx(t) \end{array} \quad (16)$$

Then, the following problem must be solved:

Reference Trajectory (RF) problem - *Given the reference sequence $\{r_k(t)\}_{k=0}^N \subset \mathcal{Y}$ and an initial state condition $x(t)$ such that $Cx(t) = r(t)$, determine an input sequence $\{u_k(t)\}_{k=0}^{N-1} \subset \Omega_u$ such that the corresponding solution of*

$$x_k(t+1) = \Phi x_k(t) + Gu_k(t), \;\; y_k(t) = Cx_k(t) \quad (17)$$

yields $y_k(t) = r_k(t)$ and satisfies $\{x_k(t)\}_{k=1}^N \subset \Omega_x$. □

In order to ensure the resolvability of the **RF** problem, we shall further assume that: **A1** the nominal system (16) is controllable and observable; **A2** at each time instant t the reference trajectory $r(t)$ is known over the horizon $I_t = [t, t+N]$, i.e. the sequence $\{r_k(t)\}_{k=0}^{N}$ is available to the controller for all $k \in \mathbb{N}$.

A convenient solution of **RF** problem involving the solution of a quadratic constrained optimization problem is discussed in [9].

3.2 MPC tracking scheme

The tracking problem will be formulated as a regulation problem by using the reference sequences $\{\bar{x}_k(t)\}_{k=1}^{N}$ and $\{\bar{u}_k(t)\}_{k=0}^{N-1}$. By means of standard coordinate transformations $\tilde{x}_k(t) = x_k(t) - \bar{x}_k(t)$ and $\tilde{u}_k(t) = u_k(t) - \bar{u}_k(t)$ the following modifications of the basic MPC algorithm result:
1. Family of virtual commands:

$$\tilde{u}(\cdot|t) := \begin{cases} K\hat{x}_k(t) + c_k(t) & -\bar{u}_k(t), \\ & k = 0, \ldots, N-1 \\ K\hat{x}_k(t), & k \geq N \end{cases} \tag{18}$$

2. Quadratic index:

$$\begin{aligned} &V(\tilde{x}(t), P, \tilde{c}_k(t)) := \|\tilde{x}(t)\|^2_{R_x} + \\ &\sum_{k=1}^{N-1} \left(\max_{\hat{\tilde{x}}_k(t)} \|\hat{\tilde{x}}_k(t)\|^2_{R_x} + \|\tilde{c}_{k-1}(t)\|^2_{R_u} \right) \\ &+ \max_{\hat{\tilde{x}}_N(t)} \|\hat{\tilde{x}}_N(t)\|^2_P + \|\tilde{c}_{N-1}(t)\|^2_{R_u} \end{aligned} \tag{19}$$

where $\hat{\tilde{x}}_k(t) = \Phi_K x(t) + \sum_{i=0}^{k-1} \Phi_K^{k-1-i}(G(c_i(t) - \bar{u}_i(t)) + B_p p_i(t)) - \bar{x}_k(t)$ and $\tilde{c}_k(t) = c_k(t) - \bar{u}_k(t)$, $k = 0, \ldots, N-1$.
3. Upper bound to (19):

$$\max_{p_0 \in \tilde{S}_0} \quad \hat{\tilde{x}}_1^T R_x \hat{\tilde{x}}_1 + \tilde{c}_0^T R_u \tilde{c}_0 \leq J_0 \tag{20}$$

$$\max_{\substack{p_i \in \tilde{S}_i \\ i=0,\ldots,k \\ k=1,\ldots,N-2}} \quad \hat{\tilde{x}}_{k+1}^T R_x \hat{\tilde{x}}_{k+1} + \tilde{c}_k^T R_u \tilde{c}_k \leq J_k \tag{21}$$

$$\max_{\substack{p_i \in \tilde{S}_i \\ i=0,\ldots,N-1}} \quad \hat{\tilde{x}}_N^T P \hat{\tilde{x}}_N + \tilde{c}_{N-1}^T R_u \tilde{c}_{N-1} \leq J_{N-1} \tag{22}$$

where

$$\begin{aligned} &\tilde{S}_i(t) := \{p \,|\, \|p\|_2^2 \leq \max_{\hat{\tilde{x}}_i(t)} \|C_K \hat{\tilde{x}}_i(t) + \\ &+ D_q \tilde{c}_i(t)\|_2^2\}, i = 0, 1, \ldots, k-1. \end{aligned} \tag{23}$$

Moreover, the $i-th$ input constraint with $i = 1, \ldots, n_u$ is recast into

$$|e_i^T \left(K\hat{\tilde{x}}_k(t) + \tilde{c}_k(t)\right)|^2 \leq \bar{u}_i^2 \tag{24}$$

where e_i is the $i-th$ vector of the canonical basis and $\bar{u}_i$ the input constraint. Then, the tracking MPC scheme, hereinafter named **NB-MPC**, is as follows:

Off-line -

1. Given the initial state $\tilde{x}(0)$, compute the triplet (K, Q, ρ) by solving the optimization problem subject:

$$\min_{Q,Y,X,\rho,\lambda,t} \rho \quad s.t. \tag{25}$$

$$\begin{bmatrix} 1 & \tilde{x}(t)^T \\ \tilde{x}(t) & Q \end{bmatrix} \geq 0, \tag{26}$$

$$\begin{bmatrix} Q & Y^T R_u^{1/2} & Q R_x^{1/2} & QC_q^T + Y^T D_q^T & Q\Phi^T + Y^T G \\ R_u^{1/2} Y & \rho I_{n_u} & 0 & 0 & 0 \\ R_x^{1/2} Q & 0 & \rho I_{n_x} & 0 & 0 \\ C_q Q + D_q Y & 0 & 0 & \lambda I_{n_x} & 0 \\ \Phi Q + GY & 0 & 0 & 0 & Q - \lambda B_p B_p^T \end{bmatrix} \geq 0 \tag{27}$$

$$\begin{bmatrix} X & Y \\ Y^T & Q \end{bmatrix} \geq 0,\ X_{ii} \leq \bar{u}_i^2, i = 1, \ldots, n_u, \tag{28}$$

$$\begin{bmatrix} \bar{x}^2 Q & (C_q Q + D_{qu} Y)^T & (\Phi Q + GY)^T C^T \\ C_q Q + D_{qu} Y & t^{-1} I_{n_x} & 0 \\ C(\Phi Q + GY) & 0 & I - t^{-1} G B_p B_p^T C^T \end{bmatrix} \geq 0 \tag{29}$$

with $P = \rho Q^{-1}$, $K = YQ^{-1}$, $\lambda > 0$ and $t > 0$.

On-line -

1. At each time instant t, given $\tilde{x}(t)$, solve the following optimization problem

$$[J_k^*(t), \tilde{c}_k^*(t)] \triangleq \arg\min_{J_k, c_k} \sum_{k=0}^{N-1} J_k$$

subject to (20)-(22) and

$$\max_{\substack{p_i \in \tilde{S}_i \\ i=0,\dots,k-1}} \left\| e_i^T \left(K\hat{\tilde{x}}_k(t) + \tilde{c}_k \right) \right\|_2^2 \leq \bar{u}_i^2,$$
$$k = 0,1,\dots,N-1,\, i = 1,\dots,n_u$$
$$\max_{\substack{p_i \in \tilde{S}_i \\ i=1,\dots,k-1}} \left\| C\hat{\tilde{x}}_k(t) \right\|_2^2 \leq \bar{x}^2,\, k = 1,2,\dots,N$$

2. Feed the plant with $\tilde{u}(t) = K\tilde{x}(t) + \tilde{c}_0^*(t)$;
3. Update $t = t+1$ and go to step 1.1

Finally, it's worth to note that it could be of some interest try to solve the MPC tracking problem by considering a partial knowledge of full state. In this case the MPC tracking problem has to be recast into a **Constrained Output Feedback Stabilisation (COFS) Problem** for which some preliminary results are collected in [10].

4 Method 2: Hybrid Command Governor (HCG) approach

Consider the following linear system

$$\begin{cases} x(t+1) &= \Phi x(t) + Gg(t) + G_d d(t) \\ y(t) &= H_y x(t) \\ c(t) &= H_c x(t) + Lg(t) + L_d d(t) \end{cases} \tag{30}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $g(t) \in \mathbb{R}^m$ is the input vector, hereinafter called CG action, $d(t) \in \mathcal{D} \subset \mathbb{R}^{n_d}$ $\forall t \in \mathbb{Z}_+$ is the exogenous disturbance vector with $\mathcal{D}$ a specified convex and compact set such that $0_{n_d} \in \mathcal{D}$; $y(t) \in \mathbb{R}^m$ is the plant output vector which is required to track $r(t)$; $c(t) \in \mathcal{C} \subset \mathbb{R}^{n_c}$ $\forall t \in \mathbb{Z}_+$ is the constrained output vector, with $\mathcal{C}$ a specified convex and compact set. Assume that all the eigenvalues of matrix Φ are in the open unit disk and system (30) is offset-free $H_y(I_n - \Phi)^{-1}G = I_m$. The CG design deals with the problem of generating, at each time instant t, the set-point $g(t)$ as a function of the current state $x(t)$ and reference $r(t)$ such that constraints are always fulfilled along the system trajectories and possibly $y(t) \approx r(t)$. In view of linearity of (30), it is possible to separate the effects of the initial conditions and input from those of disturbances so that the disturbance-free solutions of (30) to a constant command $g(t) = w$ are: $\bar{x}_w := (I_n - \Phi)^{-1}Gw$, $\bar{y}_w := H_y(I_n - \Phi)^{-1}Gw$, $\bar{c}_w := H_c(I_n - \Phi)^{-1}Gw + Lw$.
Consider the following set recursions:

$$\mathcal{C}_0 := \mathcal{C} \sim L_d\mathcal{D},\, \mathcal{C}_k := \mathcal{C}_{k-1} \sim H_c\Phi^{k-1}G_d,\, \dots,$$
$$\mathcal{C}_\infty := \bigcap_{k=0}^{\infty} \mathcal{C}_k,$$

where $\mathcal{A} \sim \mathcal{E}$ is defined as $\{a : a + e \in \mathcal{A}, \forall e \in \mathcal{E}\}$. It can be shown that the sets $\mathcal{C}_k$ are non-conservative restrictions of $\mathcal{C}$ such that $\bar{c}(t) \in \mathcal{C}_\infty, \forall t \in \mathbb{Z}_+$, implies that $c(t) \in \mathcal{C}, \forall t \in \mathbb{Z}_+$. Thus, one can consider only disturbance-free evolutions of the system and adopt a "worst-case" approach. By introducing the following sets

$$\mathcal{C}^\delta := \mathcal{C}_\infty \sim \mathcal{B}_\delta,\; \mathcal{W}^\delta := \left\{ w \in \mathbb{R}^m : \bar{c}_w \in \mathcal{C}^\delta \right\}$$

where $\mathcal{B}_\delta$ is a ball of radius δ centered at the origin, we shall assume that there exists a vanishing $\delta > 0$ such that $\mathcal{W}^\delta$ is non-empty. In particular, $\mathcal{W}^\delta$ is the closed and convex set of all commands whose corresponding steady-state solutions satisfy the constraints with a tolerance margin δ.

The CG alghorithm provides at each time step a constant virtual command $g(\cdot) \equiv w$, with $w \in \mathcal{W}^\delta$, such that the corresponding disturbance-free evolution fulfils the constraints over a semi-infinite horizon and its "distance" from the constant reference is minimal. In this respect consider the set $\mathcal{V}(x) = \left\{ w \in \mathcal{W}^\delta : \bar{c}(k,x,w) \in \mathcal{C}_k,\, \forall k \in \mathbb{Z}_+ \right\}$, where $\bar{c}(k,x,w) = H_c \left(\Phi^k x + \sum_{i=0}^{k-1} \Phi^{k-i-1} Gw \right) + Lw$ is the constrained output vector at time k from the initial condition x under the constant command $g(\cdot) \equiv w$. The CG output is chosen according to the solution of the following constrained optimization problem

$$g(t) = \arg \min_{w \in \mathcal{V}(x(t))} \|w - r(t)\|_\Psi \tag{31}$$

with $\|w\|_\Psi := w^T \Psi w$, $\Psi = \Psi^T > 0$ being a suitable weighting matrix. In this section a supervisory based CG framework capable to deal with the plant structure modifications that could take place during the on-line operations is introduced. In particular the basic CG scheme is generalized in such way that as the properties of basic CG are preserved. To this end, a suitable supervisory unit is designed for orchestrating the switching amongst the CG candidates during the on-line operations. The overall technique is termed Hybrid CG (HCG) control scheme.

Consider the discrete-time nonlinear system model

$$x_p(t+1) = f(x_p(t), u(t)) \qquad (32)$$

where $x_p(t) \in X \subseteq \mathbb{R}^n$ and $u(t) \in U \subseteq \mathbb{R}^m$ are the system state and control vectors, respectively, X, U being convex and compact sets. Assume that $f(x,u)$ is continuously differentiable in its arguments and that the plant (32) could operate in N pre-specified working regions, characterized by N equilibrium points, denoted as $(x_{p_i}^{eq}, u_i^{eq})$, $i = 1,\ldots,N$. Suppose that for each equilibrium couple $(x_{p_i}^{eq}, u_i^{eq})$a linearized model from (32) can be derived [11]

$$\delta x_p(t+1) = \quad A_i \delta x_p(t) + B_i \delta u(t)$$

where $z = [\delta x_p^T, \delta u^T]^T$ with $\delta x_p = x_p - x_{p_i}^{eq}$, $\delta u = u - u_i^{eq}$ and A_i, B_i the Jacobian matrices of the linearized systems on the state and input

4.1 Time-varying set-points

Suppose that references are allowed to belong to a finite levels set (see [12] for details)

$$r \in \mathcal{R} := \{r_1,\ldots,r_q\},\ r_i \in \mathbb{R}^m, i = 1,\ldots,q, \qquad (33)$$

and for each $i-th$ linearized model a single primal controller/reference governor unit CG_i is derived with $\mathcal{W}_i^\delta$, $i \in \mathcal{N} := \{1,2,\ldots,N\}$ the set of all commands whose corresponding steady-state solutions satisfy the constraint with margin δ. In order to ensure that each set point inside $\mathcal{R}$ can be tracked let suppose that $\mathcal{R} \subset \bigcup_{i=1}^{N} \mathcal{W}_i^\delta$ and $\forall i \in \mathcal{N}$ there exists at least $j \neq i \in \mathcal{N}$ such that

$$Int\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\} \neq \emptyset \qquad (34)$$

where $Int\{\cdot\}$ denotes the set interior operator. Finally let consider the output admissible set for the generic CG_i

$$Z_i^\delta := \{[r^T, x^T]^T \in \mathbb{R}^m \times \mathbb{R}^n | c_i(k,x,r) \in \mathcal{C}, \forall k \in \mathbb{Z}_+\}$$

and let $\mathcal{X}_i^\delta$, $i \in \mathcal{N}$ be the set of all states which can be steered to feasible equilibrium points without constraint violation

$$\mathcal{X}_j^\delta := \left\{ x \in \mathbb{R}^n | \begin{bmatrix} w \\ x \end{bmatrix} \in Z_i^\delta \text{ for at least one } w \in \mathbb{R}^m \right\}$$

In view of (34) , the following condition holds

$$Int\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\} \neq \emptyset,\ \ i,j \in \mathcal{N}$$

A convenient transition reference $\hat{r} \in Int\{\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta\}$, with $\hat{x} \in Int\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\}$ the equilibrium steady-state corresponding to $\hat{r}$, can be defined such that $[\hat{r}^T, \hat{x}^T]^T \in Z_i^\delta \cap Z_j^\delta$. Assume that CG_i unit is in use at $t = \bar{t}$, $r(\bar{t}) \in \mathcal{W}_i^\delta$, $r(\bar{t}+1) \in \mathcal{W}_j^\delta$ and the condition $\mathcal{W}_i^\delta \cap \mathcal{W}_j^\delta \neq \emptyset$ holds true, an HCG scheme can be adopted according to the following switching logic:

Switching procedure -

1) If the distance between the equilibrium x_i^{eq} and the actual state $x(t)$ is minimal, the supervisor solve and apply

$$g(\bar{t}+k) := \arg \min_{w \in \mathcal{V}_i(x(\bar{t}+k))} \|w - r(\bar{t})\|_\Psi,$$

$$k = 1,\ldots,\bar{k}$$

2) At $t = \bar{t} + \bar{k}$ as soon as $x(t) \in Int\{\mathcal{X}_i^\delta \cap \mathcal{X}_j^\delta\}$ and

$$j := \arg\min_k \|x_k^{eq} - x(t)\| \qquad (35)$$

supervisor switchs to CG_j and solve

$$g(\bar{t}+k) := \arg \min_{w \in \mathcal{V}_j(x(t))} \|w - r(\bar{t}+1)\|_\Psi,$$

$$t \geq \bar{t} + \bar{k} + 1$$

4.2 Time-varying constraints

Consider L different constraint scenarios, denoted by $\mathcal{C}_j$, $j \in \mathcal{J} := \{1,2,\ldots,L\}$, and introduce the following sets doubly indexed w.r.t. to the current couple equilibrium/constraints scenario:

$$\mathcal{W}_{(\bullet,\ j)}^\delta := \{w \in \mathbb{R}^m : \bar{c}_w \in \mathcal{C}_j^\delta\}, \forall j \in \mathcal{J} \qquad (36)$$

where $\mathcal{W}^{\delta}_{(\bullet,\ j)}$ (the bullet denotes a fixed equilibrium configuration) is the set of all commands w whose steady-state evolutions of c satisfy the $j-th$ constraint configuration $\mathcal{C}_j$ with a tolerance margin δ . Assume $\mathcal{W}^{\delta}_{(\bullet,\ j)} \neq \{\}, \forall j \in \mathcal{J}$ and $\mathcal{C}^{\delta}_j \neq \{\}$, moreover $\mathcal{W}^{\delta}_{(\bullet,\ j)}$ satisfies the *set overlapping* property : let $(j_1, j_2) \in \mathcal{J}$, then

$$\mathcal{C}^{\delta}_{j_1} \bigcap \mathcal{C}^{\delta}_{j_2} \neq \emptyset \Leftrightarrow \mathcal{W}^{\delta}_{(\bullet,\ j_1)} \bigcap \mathcal{W}^{\delta}_{(\bullet,\ j_2)} \neq \emptyset \quad (37)$$

Definition 1 *The state $x \in \mathbb{R}^n$ is $\mathcal{C}^{\delta}_j$-admissible, $j \in \mathcal{J}$, if there exists $w \in \mathcal{W}^{\delta}_{(\bullet,\ j)}$ such that $c(k,x,w) \in \mathcal{C}^{\delta}_j, \forall k \in \mathbb{Z}_+$. The pair (x,w) is said $\mathcal{C}^{\delta}_j$-executable.*

Definition 2 *Let $x \in \mathbb{R}^n$ be a state $\mathcal{C}^{\delta}_{j^-}$-admissible, $j^- \in \mathcal{J}$, and $\mathcal{C}^{\delta}_{j^+}$, $j^+ \neq j^-$, a constraint configuration to be fulfilled at future time instants. The state x is switching-$\mathcal{C}^{\delta}_{j^-}$-admissible if there exists $w \in \mathcal{W}^{\delta}_{(\bullet,\ j^-)}$ such that $c(k,x,w) \in \mathcal{C}^{\delta}_{j^+}, \forall k \in \mathbb{Z}_+$. The pair (x,w) is said switching-$\mathcal{C}^{\delta}_{j^-}$-executable and the constraint configuration $\mathcal{C}^{\delta}_{j^+}$ switchable.*

Moreover $\mathcal{V}_{(\bullet,j)}(x) := \{w \in \mathcal{W}^{\delta}_{(\bullet,\ j)} : c(k,x,w) \in \mathcal{C}^{\delta}_j, \forall k \in \mathbb{Z}_+\}$, $\forall i \in \mathcal{I}$ represent the sets of all constant virtual sequences in $\mathcal{W}^{\delta}_{(\bullet,\ j)}$ whose c-evolutions, starting from a $\mathcal{C}^{\delta}_j$-*admissible* state x, satisfy the prescribed constraint configuration $\mathcal{C}^{\delta}_j$ also during transients. As a consequence, for a fixed $j \in \mathcal{J}$, $\mathcal{V}_{(\bullet,\ j)}(x) \subset \mathcal{W}^{\delta}_{(\bullet,\ j)}$. Then, whenever the supervisory unit selects the CG candidate with respect to the j-th constraints configuration $(CG_{(\bullet,\ j)})$, a command $g(t)$ is computed as a solution of the following constrained optimization problem

$$g(t) = \arg \min_{w \in \mathcal{V}_{(\bullet,\ j)}(x(t))} \|w - r(t)\|_{\Psi} \qquad (38)$$

An admissible HCG strategy can then be developed if at each switching instant $\bar{t}$, chosen by the supervisory unit, the current state $x(\bar{t})$ is *switching-admissible*. The following sets

$$\begin{aligned} &\mathcal{X}^{\delta}_{(\bullet,\ j)} := \{x \in \mathbb{R}^n : c(k,x,w) \in \mathcal{C}^{\delta}_j, \\ &\text{for at least one } w \in \mathcal{W}^{\delta}_{(\bullet,\ j)}, \forall k \in \mathbb{Z}_+\}, \forall j \in \mathcal{J} \end{aligned} \qquad (39)$$

are finally introduced to characterize all the states $\mathcal{C}^{\delta}_j$-*admissible* (each state $x \in \mathcal{X}^{\delta}_{(\bullet,\ j)}$ can be steered to an equilibrium point without constraint violation).

5 Simulations

In this Section the effectiveness of the proposed strategies has been verified by means of numerical simulations involving a detailed full nonlinear model of HAPD aircarft including 25 symmetrical and 25 asymmetrical aeroelastic modes, sensors and actuators dynamics.

5.1 MPC Control Allocation Scheme Numerical Results

A Norm-Bounded Differential Inclusion (NLDI) representation of the HAPD nonlinear model has been firstly recast into a Polytopic Linear Differential Inclusion (PLDI) representation by using 9 design models shown in Fig. 3 characterizing different flight conditions

- the true air speed belongs to $[17, 23]$ *m/s*;
- the altitude varies between 300 *m* and 700 *m*.

Then, PLDI has been outer approximated as the NLDI by applying the optimization procedure described in [9]. The NLDI representation is obtained under the following assumptions: i) actuator and sensor dynamics have been considered negligible; ii) aero-elatisc dynamics have been assumed to be instantaneous by resorting to residual stiffness techniques. The proposed NB-MPC strategy has been applied with a prediction horizon N=1 and has been evaluated considering the following roll test manoeuvre :

Doublet on roll-rate demand (p_{ref})**-** *At $t = 1s$ p_{ref} is set to $7\,deg/s$ for a duration of $2s$. Then, at $t = 5s$ the reference on p is assumed to be* *equal to $-7\,deg/s$ for the same time interval (see dashed-line in Fig.4).* □

Fig.4 shows the time responses for a set of 4 starting levelled forward flight testing conditions shown in Fig. 3. The proposed NB-MPC seems to be able to track required reference signal taking into account all prescribed constraints on control surfaces in the operating envelope considered. Moreover, it is interesting to underline how the strategy is capable to take advantage from the aircraft redundant actuation capability: in fact the actuation deficit on the ailerons is compensated by some of the available elevators, see Fig. 5. As a consequence, this allow to have sufficient authority to achieve an almost exact tracking on the p reference signal shown in Fig. 4 without any constraint violations.

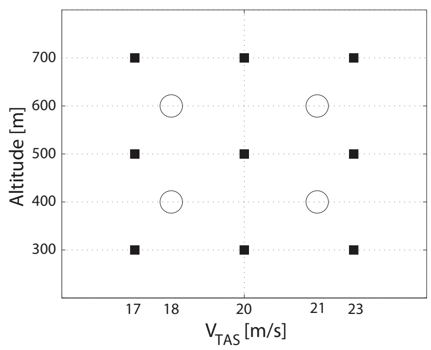

Fig. 3 V_{TAS}-Altitude flight envelope. Squares: design points used to generate the PLDI; Circles: testing points

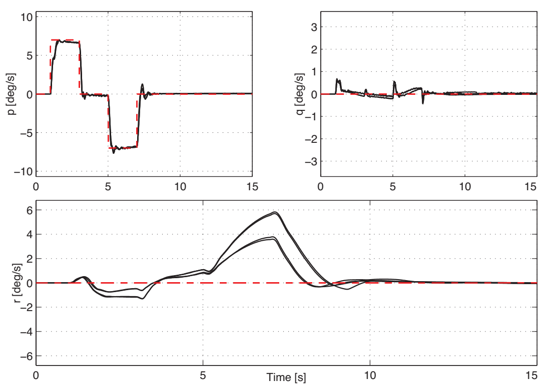

Fig. 4 Response of the HAPD in terms of angular rates. Dashed lines are the reference signals.

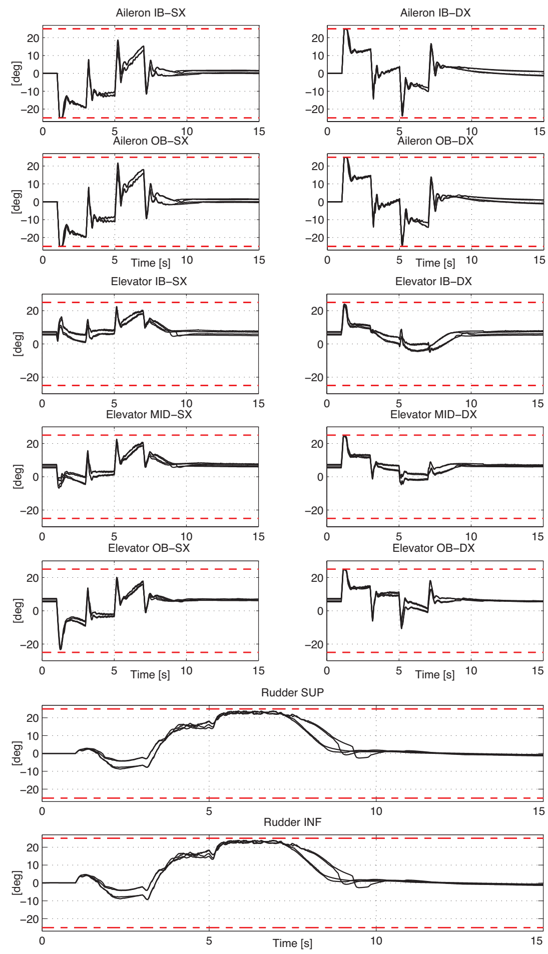

Fig. 5 Control effort. Dashed lines indicate the deflection limit for each control surface.

5.2 HCG Numerical Results

Also the benefits of the proposed **HCG** reconfiguration strategy have been verified by means of numerical simulation involving the full nonlinear HAPD aircraft model. In this case longitudinal dynamics have been excited by means of the following pitch angle manoeuvre:

Doublet on the pitch angle demand (θ_{ref})- *At steady state wing levelled forward flight conditions at altitude $h_0 = 500m$ and true air speed $V_0 = 20m/s$, at $t = 1\,s$ a pitch angle command of $20\,deg$ is first given for a duration of $2\,s$; then a doublet reference command of $10\,deg$ is given within the time interval $[5, 11]\,s$ (see dahsed line in Fig.7).*

For **HCG** design purposes, the same assumptions i) and ii) of Section 5.1 have been made.

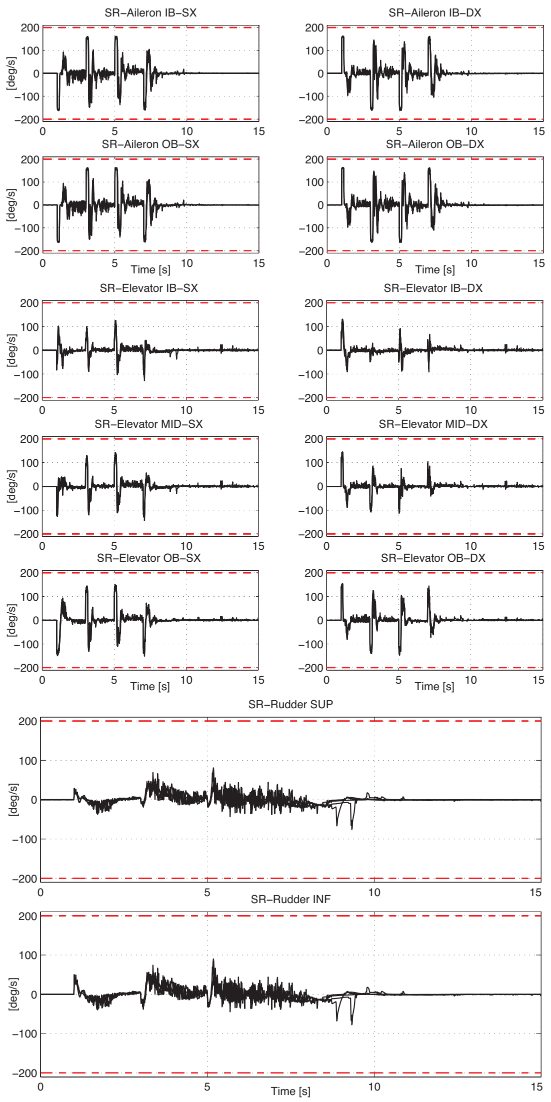

Fig. 6 Slew-Rate Control effort. Dashed lines indicate the prescribed constraints in terms of maximum allowable Slew-Rate for each control surface.

Moreover two levelled forward flight conditions, corresponding to two equilibrium conditions of the 6DoF nonlinear aircraft model (1)-(8), are considered:

C1. (Altitude $=500m$, $V_0 = 17m/s$) $\rightarrow$

$$\begin{cases} x_1^{eq} = & [17, 2.4, 0, 0, 0, 0, 0, 2.4]^T, \\ u_1^{eq} = & [4.7, 4.7, 4.7, 4.7, 4.7, 4.7, 0, 0, 0, 0, 0, 0]^T \end{cases}$$

C2. (Altitude $=500m$, $V_0 = 23m/s$) $\rightarrow$

$$\begin{cases} x_2^{eq} = & [23, -1.6, 0, 0, 0, 0, 0, -1.6]^T, \\ u_2^{eq} = & [7.6, 7.6, 7.6, 7.6, 7.6, 7.6, 0, 0, 0, 0, 0, 0]^T \end{cases}$$

where $x(t) = \{ V(t), \alpha(t), \beta(t), p(t), q(t), r(t), \phi(t), \theta(t) \}$ and $u(t) = \{ Elevator\ IB - DX(t),\ Elevator\ IB - SX(t),\ Elevator\ MID - DX(t),\ Elevator\ MID - SX(t),\ Elevator\ OB - DX(t),\ Elevator\ OB - SX(t),\ Aileron\ IB - DX(t),\ Aileron\ IB - SX(t),\ Aileron\ OB - DX(t),\ Aileron\ OB - SX(t),\ Rudder\ SUP(t),\ Rudder\ INF(t) \}$.

The linearized models obtained in correspondence of the above equilibrium conditions have been discretized using forward Euler differences with a sampling time $T_s = 0.01\,s$ and used for the implementation of the proposed predictive strategy.

The main numerical results are collected in the Figs. 7-9. As highlighted in Fig. 7, the HCG device outperforms the single $CG_{(1,1)}$ action when the tracking capabilities on the pitch angle are considered. This is clearly achieved by means of the CG switchings. In fact the $CG_{(1,1)} \rightarrow CG_{(2,1)}$ switching occurring at $t = 1.3\,s$ and at $t = 9.4\,s$, have the merit to enforce the elevators control action (see Fig. 8) and, as a consequence, the overall tracking performance.

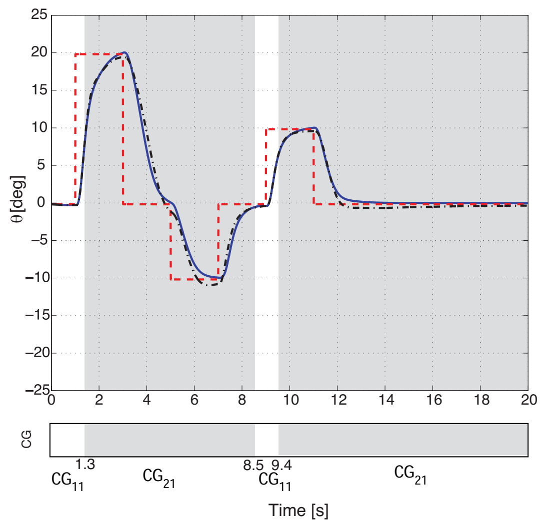

Fig. 7 **HCG** (continuous line); $CG_{(1,1)}$ (dashed-dotted line). The dashed line is the reference signal θ_{ref}.

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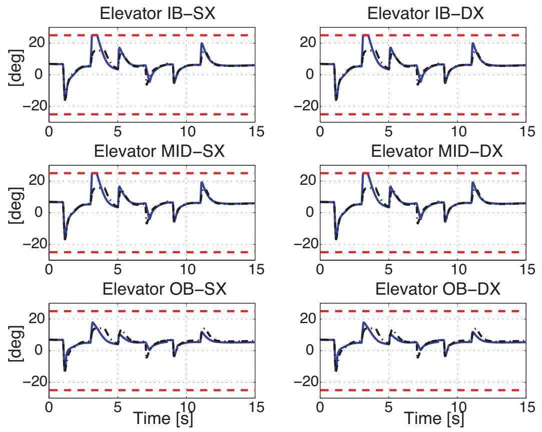

Fig. 8 Elevators control effort. The dashed lines represent the prescribed constraints.

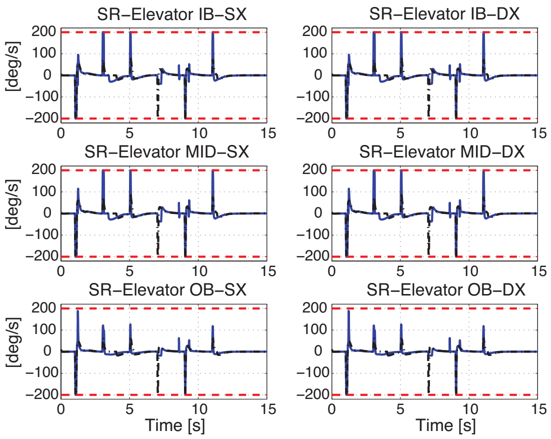

Fig. 9 Rates of variation on elevator deflections. The dashed lines represent the prescribed constraints.

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