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THE PROCEDURE OF LINER IMPEDANCE EDUCTION BY FINITE ELEMENT METHOD

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Abstract

Review of existing methods of liner impedance eduction was carried out and as most universal one the finite element method was selected. The computational program of liner impedance eduction by finite element method has been developed. The verification of computing results was made on the basis of experimental data obtained in duct with flow facility. Acceptable agreement of computed results with experimental ones was noted. It was found that the objective function in search domain has only one minimum, which simplifies defining impedance. It was established that reduction of microphones number in the experiment slightly effects on calculating values of liner impedance.

1 Introduction

Development of an effective noise reduction system is an important subject area in aircraft engine creation. The ICAO Assembly (September, 2010) approved the target specification of noise and emission reduction according to which, for example, in the medium term (by 2018) an achievable progress of noise reduction in the class of regional aircrafts is to be 13.0 EPNdB, for short and medium-haul aircrafts is to be 21.0 EPNdB. The problem of noise reduction is being solved actively by the countries of the world “aviation club”. Russian designers have faced the problem of effective noise reduction system development while creating the PD-14 engine for airliner of the new generation MS-21. By now various ways potentially providing necessary levels of noise reduction have been offered, but their level is not sufficiently high. Research and development of new technologies depend on successful solution of fundamental problems in the fields of aeroacoustics and gasdynamics directly connected with the turbulence phenomenon and improvement of computational experiments of noise formation and propagation.

To solve these problems Perm National Research Polytechnic University (PNRPU) and Aviadvigatel OJSC (Perm, Russia) have created the Acoustic Research Center. Material and technical base of Acoustic Research Center is a duct with flow facility (Fig. 1) created in accordance with the Central Aerohydrodynamic Institute (Zhukovsky, Russian Federation) and allowing to research acoustic properties of various liners in the air flow.

Similar facilities already exist in such departments and research centers as NASA Langley Research Center (USA) [1], Boeing Wichita Noise Lab (USA) [2], United Technologies Research Center (USA) [3], Department of Aeronautical and Vehicle Engineering (Sweden) [4], Institute of Propulsion Technology, Engine Acoustics (Germany), Aerodynamics and Energetics Modeling Department (France) [5].

Primary tasks when creating the duct with flow facility were:

- review of existing methods of the liner impedance eduction;
- selection of the most universal method;
- computational program development on the basis of the method chosen;
- verification of computational results.

Review of existing methods of the liner impedance eduction on the basis of acoustic pressure measurements in the duct with flow facility reveals that the most widely spread methods among them are those based on the analytical solution of an initial mathematical model. The analytical solutions contain values

Fig. 1. Duct with flow facility in the PNRPU Acoustic Research Center

of acoustic pressure amplitudes of direct and reflected waves being defined commonly on the basis of measurements by the 2- [6-8], 3- and 4-microphone [9] methods. These methods have a number of weaknesses, some of them are mentioned in [10]. Also a final form of the analytical solution significantly depends on the duct geometry and microphone positions. Thus, the field of application of the analytical solutions is quite limited.

Among numerical approaches the finite element method is commonly used [11-12]. It has a number of advantages compared to the analytical solutions:

- flexibility (in case of changing positions and number of microphones the computational procedure does not change, the method suits for the duct of any configuration);

- automatic account of high-order modes in calculation;

- account of transversal modes propagation beyond the liner edges.

Thus, to solve the problem of the liner impedance eduction in the duct with flow facility the finite element approach has been chosen.

2 Mathematical Model and Solution

The mathematical model describing propagation of acoustic disturbances in the duct with gas flow is based on the convected Helmholtz equation. For the two-dimensional case it has the following form:

$$(1-M_0^2)\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}-2ikM_0\frac{\partial p}{\partial x}+k^2 p=0 \qquad (1)$$

where p is acoustic pressure; $k=\omega/c$ is wave number; c is sound velocity; ω is angular frequency; M is average Mach number of nonperturbed gas flow; i is imaginary unit; x, y are longitudinal and transverse coordinates of the duct.

Boundary conditions to solve the equation are the following:

- at the duct inlet $\quad p=p_1^{e} \qquad (2)$

- at the duct outlet $\quad p=p_n^{e} \qquad (3)$

- on the rigid wall $\quad \dfrac{\partial p}{\partial y}=0 \qquad (4)$

- on the liner wall

$$\frac{\partial p}{\partial y}=\frac{ikp}{Z}+\frac{2M_0}{Z}\frac{\partial p}{\partial x}+\frac{M_0^2}{ikZ}\frac{\partial^2 p}{\partial x^2} \qquad (5)$$

where p_1^e, p_n^e are acoustic pressure measured by microphones at inlet and outlet of duct; Z is nondimensional impedance.

Application of Galerkin's finite element method to equation (1) gives

$$\int_V [N]^T \left\{ (1-M_0^2)\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right\} dV - \\ - \int_V [N]^T \left\{ 2ikM_0 \frac{\partial p}{\partial x} + k^2 p \right\} dV = 0 \tag{6}$$

where $[N]$ is shape function of finite element. To solve it the finite element is used with shape functions formed on the basis of Lagrange polynomial; it ensures continuity of the required function in nodes after integration, but not its derivatives. Therefore in equation (6) it is necessary to reduce the second order of derivatives to the first one. Applying Green's theorem yields

$$\Bigg[(1-M_0^2)\int_A \frac{\partial [N]^T}{\partial x}\frac{\partial [N]}{\partial x} dA + \int_A \frac{\partial [N]^T}{\partial y}\frac{\partial [N]}{\partial y} dA + \\ + 2ikM_0 \int_A [N]^T \frac{\partial [N]}{\partial x} dA - k^2 \int_A [N]^T [N] dA - \\ - (1-M_0^2) l_x \int_L [N]^T \frac{\partial [N]}{\partial x} dL - \\ - l_y \int_L [N]^T \frac{\partial [N]}{\partial y} dL \Bigg] \{P\} = 0 \tag{7}$$

where l_x, l_y are directional cosines; A is area of finite element. Application of the same conversions to expression (5) yields

$$\int_L [N]^T \frac{\partial [N]}{\partial y} dL = \frac{ik}{Z} \int_L [N]^T [N] dL + \\ + \frac{2M_0}{Z} \int_L [N]^T \frac{\partial [N]}{\partial x} dL - \frac{M_0^2}{ikZ} \int_L \frac{\partial [N]^T}{\partial x} \frac{\partial [N]}{\partial x} dL \tag{8}$$

In equation (7) contour integrals are 0 for all walls except the liner, where expression (8) is used. Application of equation (7) for all finite elements and taking into account the boundary conditions (2-4) results in the system of linear equations

$$[K]\{P\} = \{F\} \tag{9}$$

where $[K]$ is complex coefficient global matrix; $\{F\}$ is global vector formed by boundary conditions; $\{P\}$ is vector of acoustic pressures in finite element nodes. Solution of system (9) provides all values of acoustic pressures in finite element nodes.

3 Liner Impedance Eduction Procedure

Procedure for determining the liner impedance can be divided into two stages: initial estimate search and solution adjustment. The first stage (Fig. 2) consists of the following steps:

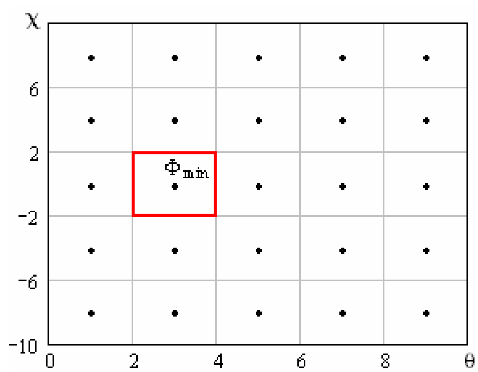

Fig. 2. Initialization for the method of alternating variables.

1) The searching range is assigned. For the most liners

$$0 \le \theta \le 10, \qquad -10 \le \chi \le 10$$

Here θ, χ are real and image part of impedance.

2) All searching range is meshed.

3) System (9) is solved for every center of the cell accepted as the value of impedance $z = \theta + i\chi$.

4) The objective function is defined by the expression

$$\Phi = \sum_{j=1}^{n} \left| p_j^c - p_j^e \right| \tag{10}$$

5) The value of impedance corresponding to the minimum of Φ is accepted as an initial point for the method of alternating variables.

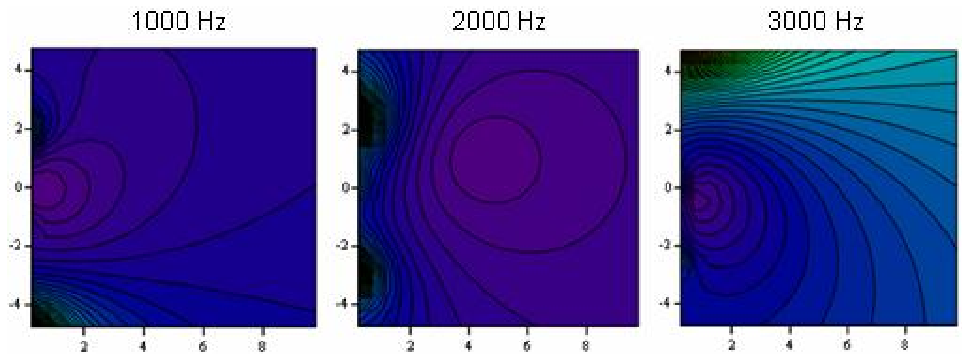

Fig. 3.1 Objective function (M=0.000).

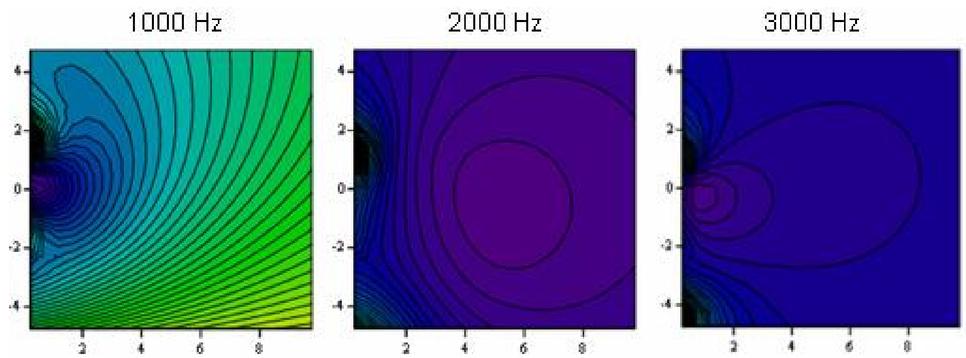

Fig. 3.2 Objective function (M=0.172).

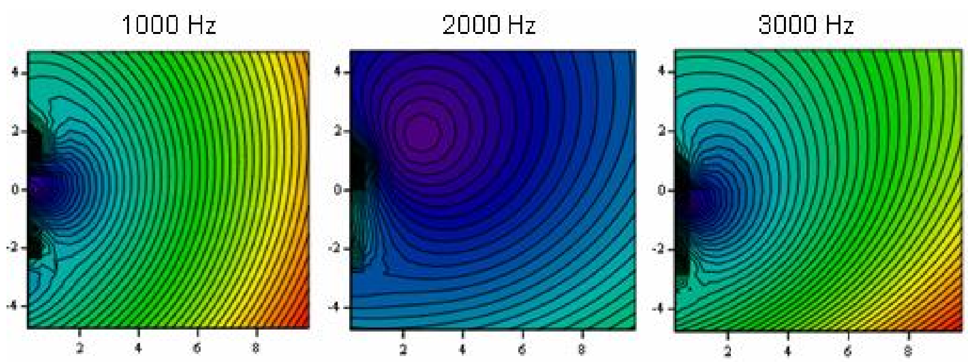

Fig. 3.3 Objective function (M=0.255).

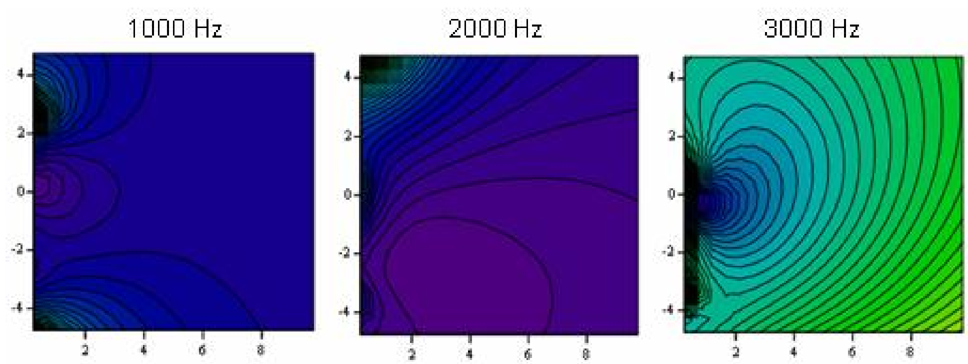

Fig. 3.4 Objective function (M=0.400).

Then the second stage comes. Impedance value *z* is defined more precisely by the fast variant of the method of alternating variables. From the point found at the previous stage movement starts along a real axis; the step of descending is equal to 0.1. The movement continues while the objective function is decreasing. If the decrease stops, the movement starts along an imaginary axis in the direction of decreasing of the objective function. If the objective function does not decrease along both axes, the step of descending is divided into two and again the movement starts along the real axis. The search continues until the descending step is less than 0.001. Thus the impedance corresponding to the objective function minimum is accepted as the liner impedance required.

The objective function being obtained under expression (10) has the only minimum (Fig. 3.1-3.4). Thus it ensures reliable identification of the impedance at any step reduction coefficient and any initial point of the given search range. In the process the first stage can be omitted, however an improper initial point increases considerably the time for calculations by the method of alternating variables.

4 Results

The suggested procedure of the liner impedance calculation shows acceptable agreement of obtained acoustic pressures with experimental data of paper [12]. Some impedance values obtained by computation of the objective function on 31 microphones and used for computation of acoustic fields are indicated in Table 1. The objective function values corresponding to the obtained impedances are indicated in Table 2. The amplitudes of calculated acoustic pressures on the duct upper wall as well as experimental acoustic pressures, and also acoustic fields in the duct are shown in Fig. 4-6.

The figures show that accuracy of liner impedance eduction becomes worse at high Mach numbers of flow. It is mainly caused by the fact that when increasing the flow velocity its profile is distorted greatly.

Table 1. Impendence values (31 microphones)

M	1000 Hz		2000 Hz		3000 Hz	
	θ	χ	θ	χ	θ	χ
0.000	0.472	0.025	4.149	1.350	0.674	-0.370
0.079	0.425	0.059	3.459	1.823	0.612	-0.320
0.172	0.389	0.143	4.996	0.187	0.685	-0.218
0.225	0.358	0.180	4.791	0.027	0.630	-0.222
0.335	0.290	0.282	4.881	-1.399	0.598	-0.206
0.400	0.240	0.362	2.939	-2.001	0.566	-0.192

Table 2. Objective function values, Pa (31 microphones)

M	1000 Hz	2000 Hz	3000 Hz
0.000	13.224	49.733	50.392
0.079	21.665	71.729	86.916
0.172	58.791	82.160	117.571
0.225	90.793	94.031	116.858
0.335	120.292	110.294	92.731
0.400	226.398	115.878	112.572

The analysis of impedance determination accuracy depending on the number and arrangement of microphones (Fig. 7) has been conducted. Results of the liner impedance calculations at different microphone number are shown in Fig. 8. Objective function values are presented in tables 3.1-3.3. In the table cells for 19, 11, 7 microphones the upper value is an objective function obtained when searching the impedance under corresponding microphone number. The lower value is an objective function obtained on 31 microphones with the impedance calculated previously on corresponding number of microphones.

One can see that the impedance obtained at different number of microphones gives close objective functions on 31 microphones. This indicates the possibility to use the finite element approach for simulation of optimal microphone quantity in terms of the equipment-cost-to-solution-quality principle.

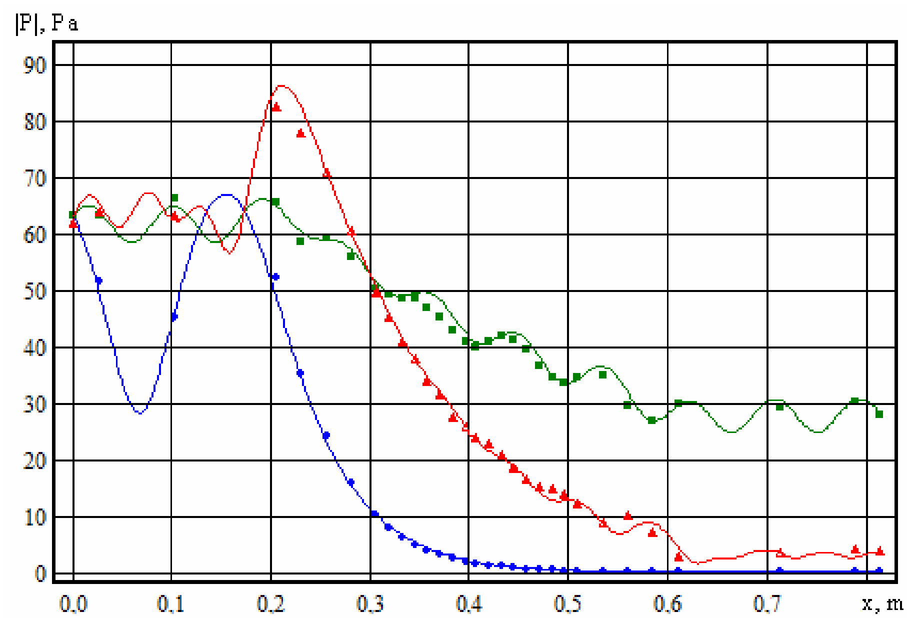

Fig. 4.1. Acoustic pressures on microphones (M = 0.000, 31 microphones)
1000 Hz, 2000 Hz, 3000 Hz.

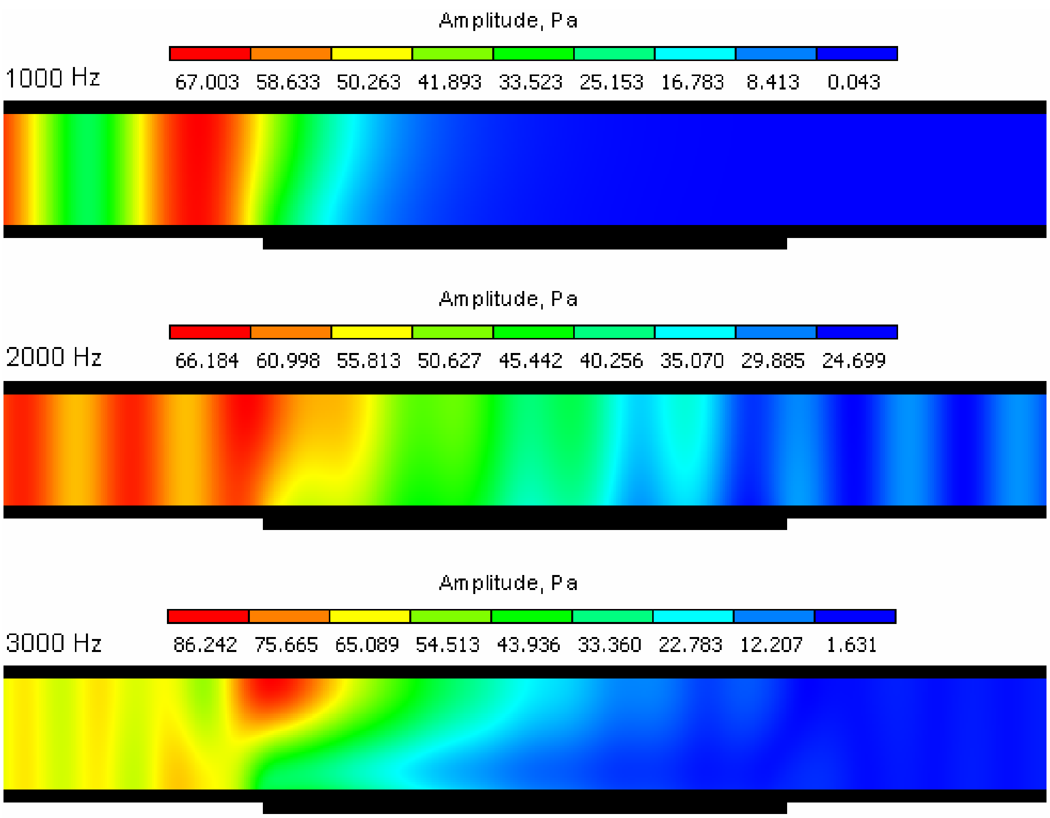

Fig. 4.2. Acoustic pressures in the duct with flow
(M = 0.000, 31 microphones).

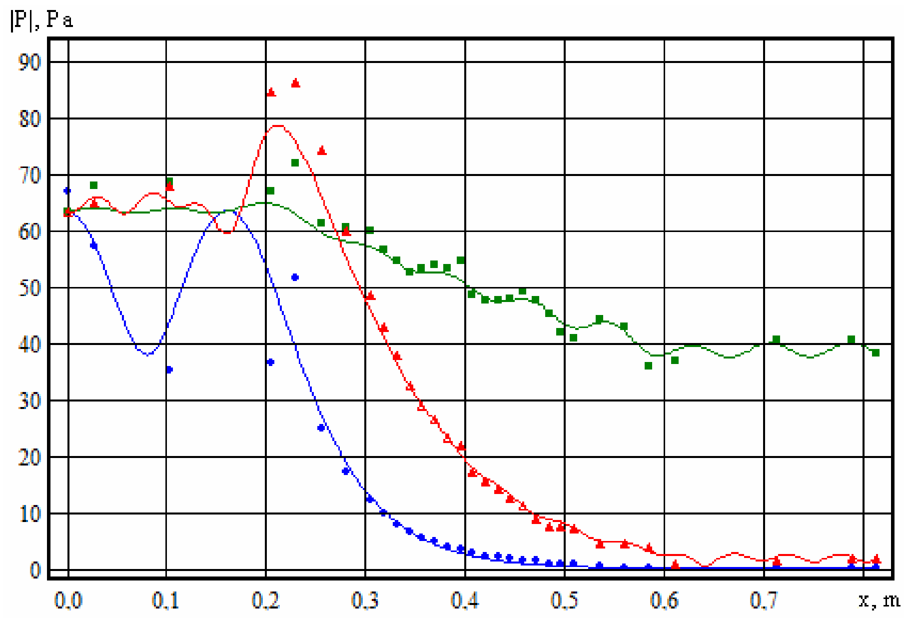

Fig. 5.1. Acoustic pressures on microphones (M = 0.225, 31 microphones)
1000 Hz, 2000 Hz, 3000 Hz.

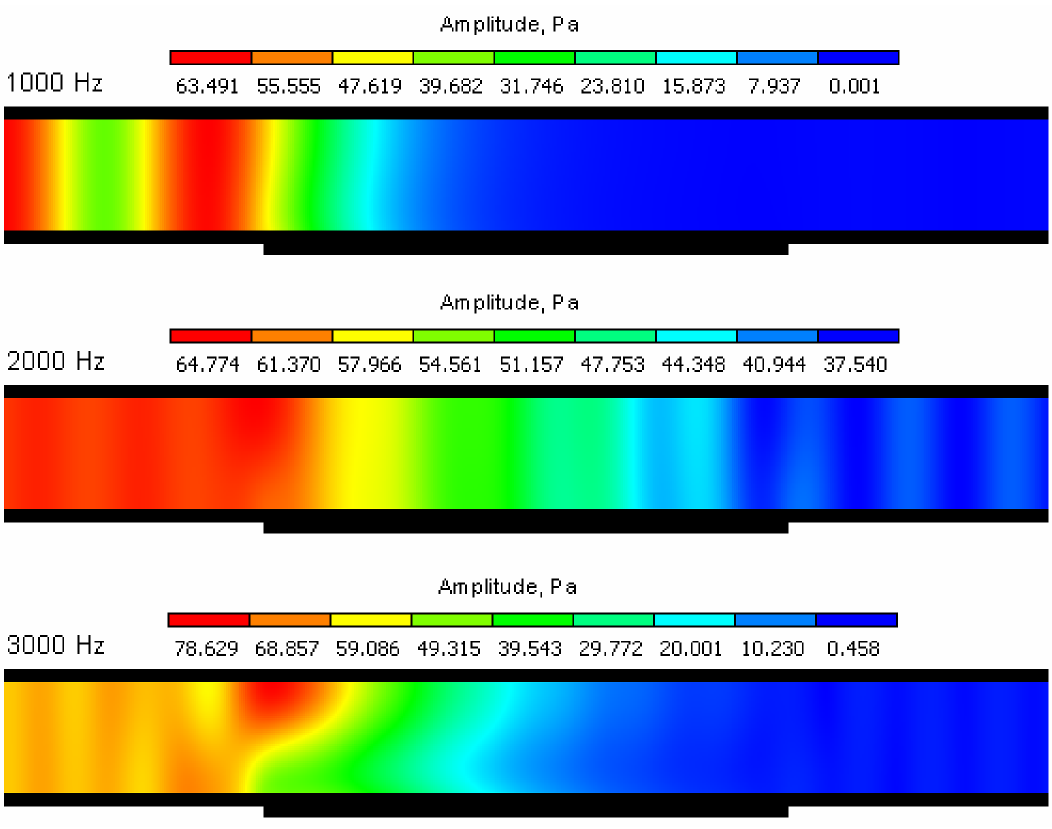

Fig. 5.2. Acoustic pressures in the duct with flow
(M = 0.225, 31 microphones).

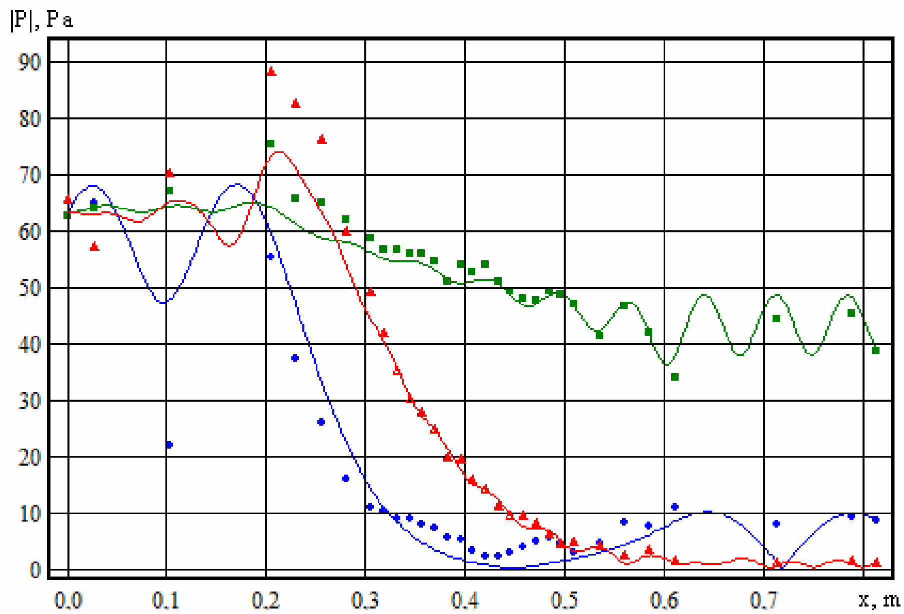

Fig. 6.1. Acoustic pressures on microphones (M = 0.400, 31 microphones)
1000 Hz, 2000 Hz, 3000 Hz.

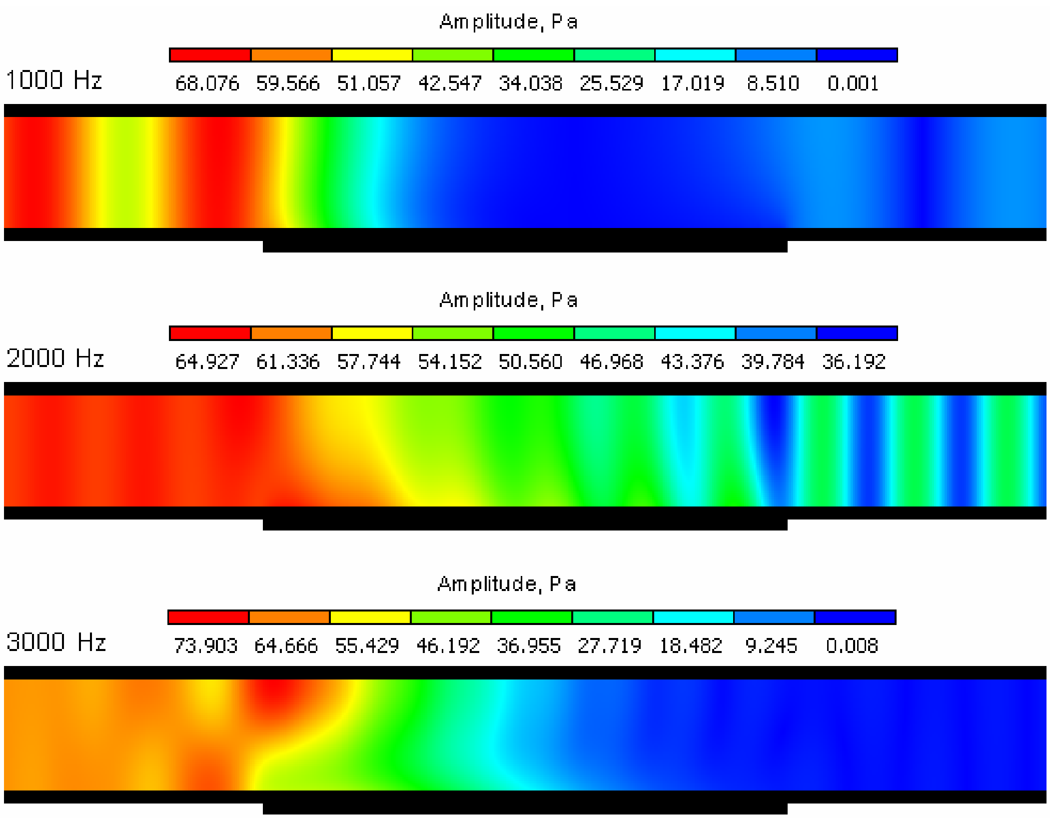

Fig. 6.2. Acoustic pressures in the duct with flow
(M = 0.400, 31 microphones).

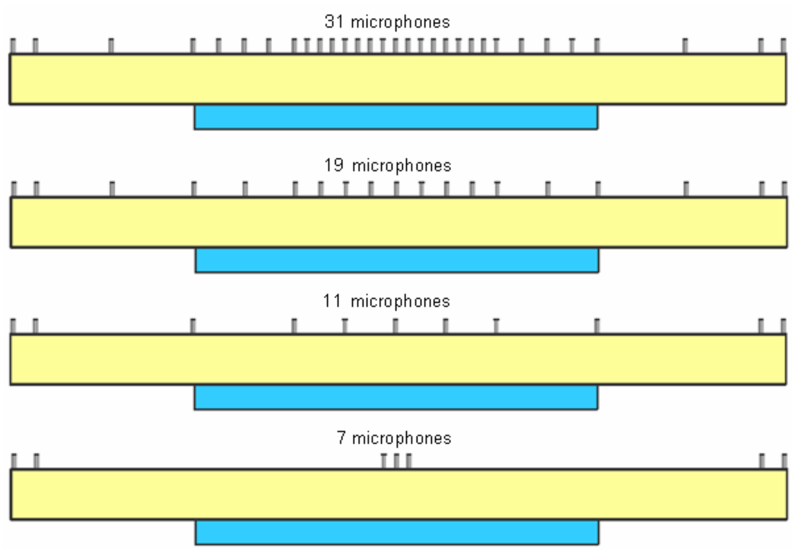

Fig. 7. Arrangement of microphones.

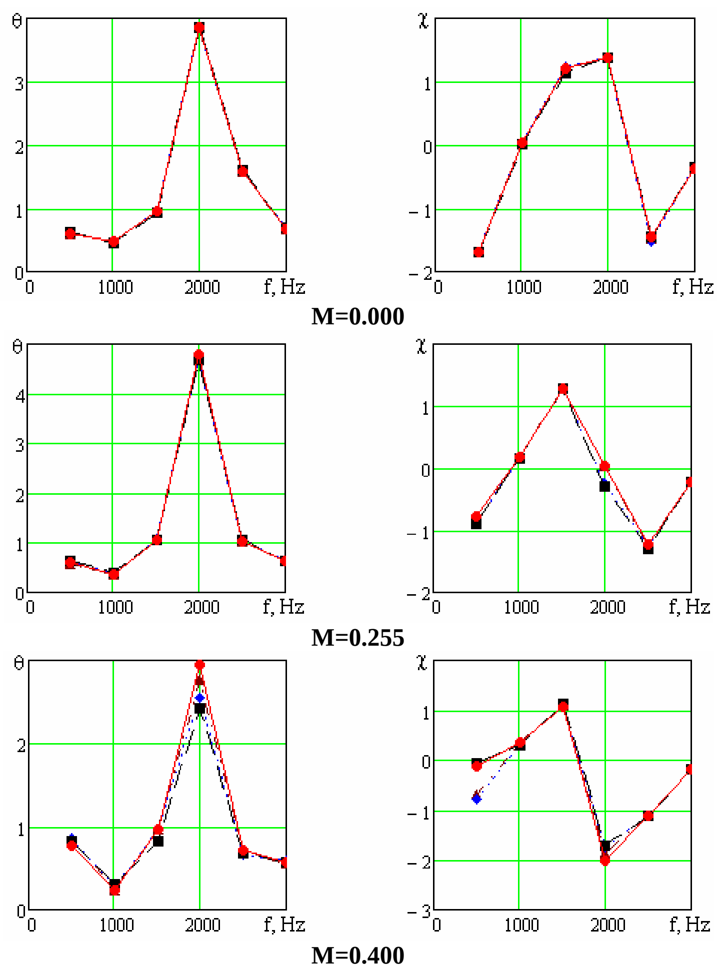

Fig. 8. Liner impedance at different number of microphones
●●● 31 microphones, ▲▲▲ 19 microphones, ◆◆◆ 11 microphones, ■■■ 7 microphones.

Table 3.1 Objective function (1000 Hz)

M	Microphone number			
	31	19	11	7
0.000	13.22	9.95 13.31	6.01 13.23	2.51 16.39
0.079	21.67	16.45 21.78	8.15 21.93	4.03 26.30
0.172	58.79	43.72 59.62	20.48 59.62	9.97 65.21
0.225	90.79	67.35 90.92	39.88 91.12	12.78 96.39
0.335	120.29	82.46 120.29	38.57 124.57	15.12 141.28
0.400	226.40	157.02 226.90	84.08 233.38	25.25 238.17

Table 3.2 Objective function (2000 Hz)

M	Microphone number			
	31	19	11	7
0.000	49.73	28.16 51.24	14.33 50.06	6.33 49.76
0.079	71.73	42.83 71.92	21.55 72.76	8.20 75.76
0.172	82.16	48.32 83.23	26.57 84.23	13.38 85.42
0.225	94.03	53.45 94.15	33.71 101.71	13.79 104.18
0.335	110.29	72.95 114.47	41.49 120.54	18.49 116.17
0.400	115.88	79.50 119.42	44.50 129.76	23.22 138.02

Table 3.3 Objective function (3000 Hz)

M	Microphone number			
	31	19	11	7
0.000	50.39	31.46 50.41	17.84 53.41	7.11 55.49
0.079	86.92	56.19 86.96	30.63 88.51	11.39 89.59
0.172	117.57	73.46 118.02	44.57 118.05	14.17 117.70
0.225	116.86	73.28 116.94	55.93 116.91	11.74 119.21
0.335	92.73	64.40 93.04	41.58 93.07	14.63 93.37
0.400	112.57	76.13 113.54	48.14 120.52	15.35 120.29

5 Conclusion

The research conducted has lead to the following conclusions.

1) Review of existing scientific publications on the problem of liner impedance eduction in the duct with flow facility showed that the most universal method is the finite element method (its realization does not depend on the number and arrangement of microphones; it automatically takes into account the high-order modes and transverse modes beyond the liner edges; it is suitable for a duct of any configuration).

2) Investigation of objective function behavior in impedance search revealed that the objective function is unimodal. Thus, the search procedure can be started in any point of a set area with any initial step and any step reduction factor.

3) Comparison of the calculation results of acoustic pressures at microphones with the experiment results revealed that increment in flow velocity decreases accuracy of impedances. It is mainly caused by the fact that when increasing the flow velocity its profile is distorted greatly (especially in a narrow duct). Thus, for precise calculation of acoustic pressures it is necessary to take into account derivatives of steady flow parameters in the mathematical model.

4) Calculations on the basis of different number of microphones show that impedance slightly changes at a certain decreases of the microphone number. Thus, the given type of calculation can be used to determine optimal microphone number in terms of the equipment-cost-to-solution-quality relation.

Acknowledgment

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