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DYNAMICS MODELING AND ROBUST CONTROLLER DESIGN OF THE MULTI-UAV TRANSPORTATION SYSTEM

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Abstract

This paper presents the design process of cooperative transportation system with multiple unmanned aerial vehicles connected with strings. Moving pendulum dynamics is modeled with Newtonian approach via combination of 3-dimensional Cartesian coordinates and spherical coordinates. The modeled system is controlled linear quadratic Gaussian / loop transfer recovery method. Furthermore, system parameter variations are compensated with parameter-robust linear quadratic Gaussian method. The numerical result of the both controls is compared to show improvements in parameter robustness.

1 Introduction

Recently, unmanned aerial vehicles (UAVs) are receiving attentions for their significance in both military and civilian uses. Their major purposes include rescue, reconnaissance, and transportation missions [1]. Among all the capabilities, this paper focuses specifically on the transportation mission, which is in high demand for future technology.

As UAVs are vulnerable to heavy loads, cooperation of multiple UAVs is essential. Though solid connection between the payload and each UAV is easy to control, gripping methods are problematic. Consequently, this paper deals with wire-linked transportation system. The most challenging point for using strings must be the fluctuating motion of the payload. In order to apply the system to practice, delicate control of load is necessary.

Mathematical modeling of moving pendulum motion is first required to be derived prior to controller design. Most precedent researches are limited to experiments [2, 3] or statistic stability analysis [4, 5]. Others suggest modeling with Kane method [6, 7, 8] or Udwadia-Kalaba equation (UKE) [9, 10], while Kane method leaves the problem of constraint equation and UKE computes the constraint force later with unconstraint states. Also, even though PID controller can be designed for non-minimal state-space representation, linear feedback control methods require controllability and thus minimal system. To reduce computational cost and simplify the model into minimal representation, Newtonian method with a combination of Cartesian coordinates and spherical coordinates is introduced.

Classical optimal control methods are applied to the consequent dynamics. As transportation system requires observer-based control, linear quadratic Gaussian (LQG) method needs to be implemented. Furthermore, loop transfer recovery (LTR) [11, 12] method is augmented for compensation, while parameter-robust LQG (PRLQG) [13, 14] is another option for the variation of load mass.

This paper is composed as follows. The first part deals with derivation of pendulum dynamics, and the second shows the design techniques of robust controller. Then, simulation setups and results for several scenarios are displayed. Finally, a discussion on stability or performance issues and conclusion is followed.

2 Moving Pendulum Dynamics Modeling

2.1 General Equation of Moving Pendulum

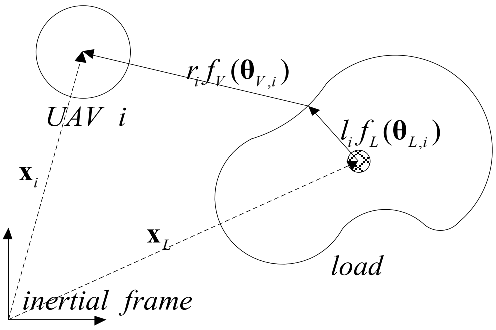


Fig. 1 UAVs and a Mass System

The 3-dimensional transportation system of a slung load with unspecified number of UAVs is presented with vector representations as in Fig.1. Subscripts with V and L represent vehicle and load respectively, and all the position and angle vectors are defined with respect to inertial frame. Assuming the strings are connected close to center of gravity of each UAV, effect of moments of UAVs is negligible to the pendulum motion. Also, vibration of the wires is not considered so that the wires can be approximated as hinge rods between the payload and each UAV. Given the assumptions above, Newtonian approach leads to the equation (1).

$$\begin{cases} m_V \ddot{\mathbf{x}}_i = F_{V,i} - \sum T_i f_V(\boldsymbol{\theta}_{V,i}) \\ m_L \ddot{\mathbf{x}}_L = F_L \;\; + \sum T_i f_V(\boldsymbol{\theta}_{V,i}) \\ I_L \ddot{\boldsymbol{\theta}}_L \;\; = \sum l_i T_i (f_L(\boldsymbol{\theta}_{L,i}) \times f_V(\boldsymbol{\theta}_{V,i})) \end{cases} \tag{1}$$

Reduction on the number of states and arrangement using matrix inversion results in the general equation of motion as equation (2),

$$\begin{bmatrix} \ddot{\mathbf{x}}_L \\ \ddot{\boldsymbol{\theta}}_L \\ \ddot{\boldsymbol{\theta}}_{V,i} \\ T_i \end{bmatrix} = \begin{bmatrix} m_V & m_V l_i f_L'(\boldsymbol{\theta}_{L,i}) & m_V r_i f_V'(\boldsymbol{\theta}_{V,i}) & f_V(\boldsymbol{\theta}_{V,i}) \\ m_L & 0 & 0 & -f_V(\boldsymbol{\theta}_{V,i}) \\ 0 & I_L & 0 & -l_i f_L(\boldsymbol{\theta}_{L,i}) \times f_V(\boldsymbol{\theta}_{V,i}) \end{bmatrix}^{-1} \left(\begin{bmatrix} F_{V,i} \\ F_L \\ 0 \end{bmatrix} - m_V \begin{bmatrix} l_i g(\boldsymbol{\theta}_{L,i}) + r_i g(\boldsymbol{\theta}_{V,i}) \\ 0 \\ 0 \end{bmatrix} \right) \tag{2}$$

where function f is defined with difference according to the existence of volume or space dimension, and function g is defined to eliminate second derivatives as equation (3).

$$g(\boldsymbol{\theta}) = \frac{d^2 f(\boldsymbol{\theta})}{dt^2} - \ddot{\boldsymbol{\theta}} f'(\boldsymbol{\theta}) \tag{3}$$

2.2 Modeling of One UAV and a Point Mass

For instance, transportation system with one UAV and a point mass is considered. As $\boldsymbol{\theta}_L$ is not defined for this case, states are given as $\mathbf{x}_L^T = [x \;\; y \;\; z]$ and $\boldsymbol{\theta}_V^T = [\theta \;\; \phi]$. Transformation of angular states of the vehicle into displacement is conducted as

$$f_V(\boldsymbol{\theta}) = \begin{bmatrix} \cos\theta\cos\phi \\ \sin\theta\cos\phi \\ \sin\phi \end{bmatrix} \tag{4}$$

2.3 Modeling of Two UAVs and a Bar Mass

Another example is given with two UAVs and a bar load. Angles of load $\boldsymbol{\theta}_L^T = [\theta \;\; \phi \;\; \psi]$ are augmented to the state vector in section 2.2. Transformation of angular states of the load into displacement vector is additionally conducted with equation (5).

$$f_L(\boldsymbol{\theta}) = \begin{bmatrix} \cos\psi\cos\theta \\ -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi \\ \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \end{bmatrix} \tag{5}$$

2.4 Verification of Modeling with Lagrangian Method

Equation of motion can be derived by Lagrangian method alternatively. Lagrangian method is useful to model simple pendulum motion, but complicated to compute large systems or to generalize the equation of motion. For the system in section 2.2, Lagrangian can be computed as

$$\begin{aligned} L &= m_V g z + m_L g\left(z + l\cos\theta\right) + \frac{1}{2} m_V (\dot{x}^2 + \dot{z}^2) \\ &+ \frac{1}{2} m_L \left[\left(l\dot{\theta}\cos\theta - \dot{x}\right)^2 + \left(l\dot{\theta}\sin\theta - \dot{z}\right)^2 \right] \end{aligned} \tag{6}$$

Implementation of Lagrangian equation leads to the same equation with combination of equation (2) and (4).

3 Controller Design

3.1 LQG/LTR Method

LQR/LQG method is an optimal control method for linearized system dynamics. Structure of LQR/LQG controller is given as follows:

$$\dot{\hat{x}}(t) = (A - BK - K_f C)\hat{x}(t) + K_f y \qquad (7)$$

where controller gain $K = R^{-1}B^T P$ is chosen from algebraic Riccati equation (ARE) as equation (8) and filter gain $K_f = P_f C^T V^{-1}$ is defined by equation (9), which is another form of ARE, with the design parameters R, Q, W and V.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (8)$$

$$AP_f + P_f A^T - P_f C^T V^{-1} CP_f + FWF^T = 0 \quad (9)$$

As LQG method reduces the stability-robustness and performance specifications, LTR method is augmented to recover the robustness. The controller gain $K = (1/\rho)B^T P$ and the filter gain $K_f = (1/\mu)P_f C^T$ are defined through modified AREs, which are presented in equation (10) and (11), respectively.

$$A^T P + PA - (1/\rho)PBB^T P + Q = 0 \quad (10)$$

$$AP_f + P_f A^T - (1/\mu)P_f C^T CP_f + LL^T = 0 \,(11)$$

where the positive scalar parameter ρ and μ are set close to zero checking the bandwidth, and $L = \begin{bmatrix} L_L & L_H \end{bmatrix}^T$ is set to influence low and high frequency behaviors of singular values.

3.2 PRLQG Method

As LQG/LTR may not be appropriate for unmodeled dynamics caused by a parameter variation, PRLQG is employed. In this method, a parameter variation ΔA is represented as an internal feedback loop (IFL) by decomposing into equation (12).

$$\Delta A = -ML(\varepsilon)N \qquad (12)$$

For computing K and K_f, the weighting matrices are given as equation (13) and (14) respectively.

$$Q_c = w_{cn}^2 N^T N + w_{cc}^2 C^T C \qquad (13)$$

$$Q_f = w_{fm}^2 MM^T + w_{fb}^2 BB^T \qquad (14)$$

Increase in w_{cc} and w_{fb} reflects LTR method while large w_{cn} or w_{fm} corresponds to PRLQG method.

4 Simulation

4.1 Simulation Settings

With the modeled dynamics in section 2.2 and 2.3, control techniques of section 3 are implemented for simulation. Specifications of both scenarios are given in Table 1.

Table 1. System specification

		Case 1	Case 2
UAV	Number	1	2
	Mass (kg)	1.145	1.145
	Size (m)	ϕ 0.25	ϕ 0.25
Payload	Type	Point mass	Bar
	Mass (kg)	0.1, 0.3, 0.9	0.1, 0.3, 0.9
	Size (m)	None	0.2 x 0.2 x 1.0
String	Mass (kg)	None	None
	Size (m)	1	1

Whereas geometric specifications of overall transportation system are determined in Table 1, dynamic properties of UAVs are not included as control inputs are given as forces and moments.

Design parameters for the specified controller are set as follows: Q is 10 and 1, and FWF^T is 0.1 and 1 for non-derivative terms and first derivative terms respectively. Diagonal terms of R for all the states are set 10, and those of LL^T are 0.01. These rules are applied for further expansion of transportation system, which is major advantage of using LQG/LTR or PRLQG method.

Command is given as feedforward structure in case 1, but output feedback structure is utilized for case 2. As the number of inputs and outputs needs to be coincident for using feedforward command as shown in equation (15), feedforward structure cannot be implemented for some cases. Therefore, it is estimated that the first case would show more stable and faster response.

$$\bar{N} = N_u + K_C N_x \;\; where \begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (15)$$

4.2 Numerical Results of Point Mass Transportation System

Verification of one point mass transportation system modeling is conducted with ease by moving the UAV in natural frequency of pendulum motion. Intuitively, pendulum motion of the payload increases and the sine trajectory of UAV must be enlarged by the coupling effect. Given the constant input to z-axis and sinusoidal signal to y-axis with natural frequency $\sqrt{g/l}$, pendulum trajectory in Fig. 2 shows resonance effect. Initial velocity to x-axis with no input applied shows constant behavior to x-axis. Therefore, it can be implied that coupling effect of moving pendulum is well-modeled.

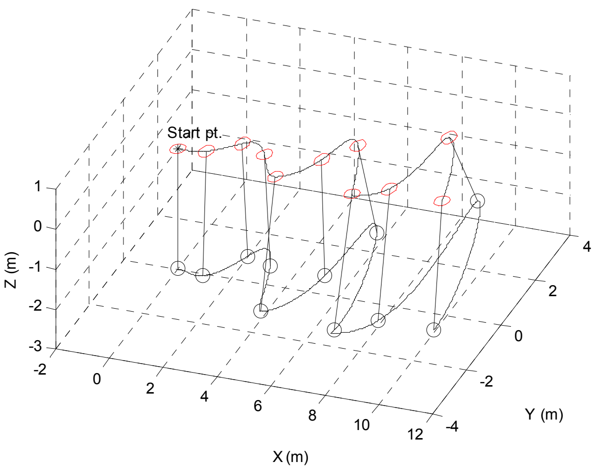
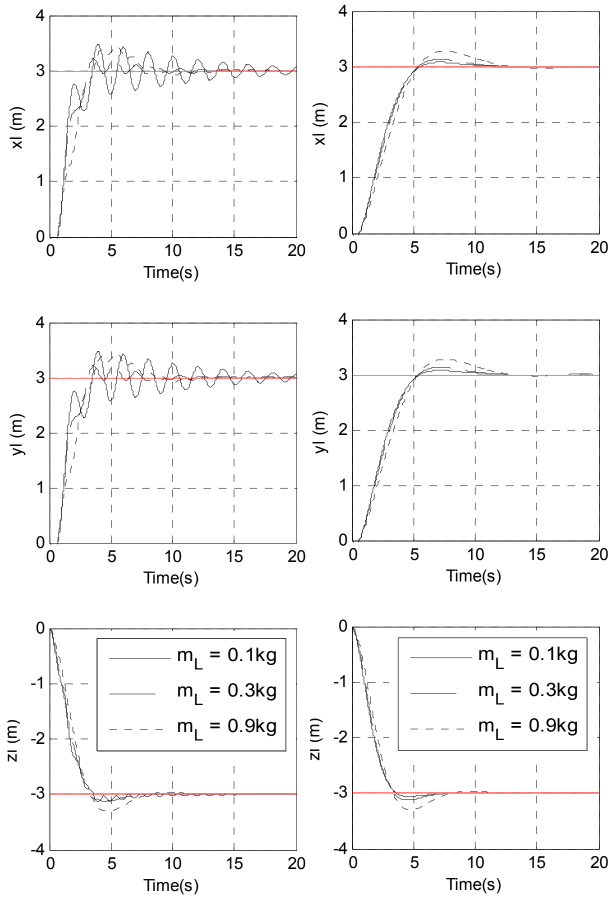


Fig. 2. Point mass system modeling

With optimal control design methods, step response of payload position is evaluated in Fig. 3. Given a single position command from the origin to (3, 3, -3) m, the payload's position response of LQG/LTR and PRLQG clearly differs from each other. While response of LQG/LTR is fast and fluctuating, the system controlled by PRLQG has longer settling time but indifferent stable response for varying parameter, as expected in previous sections. Also, it can be inferred that there is no big difference with linearized system with original nonlinear system. Compared to z-axis performance, x- and y-axis response has more district changes, as perturbation of system matrix A lies in the string motion influencing mainly on x-y motion of the load.



(a) LQG/LTR (b) PRLQG

Fig. 3 Step response

Trajectory following performance of PRLQG controller is shown in Fig. 4. The payload is moving along the waypoints (2,0,-2), (0,2,-2), (-2,0,-2), and (0,-2,-2) m, and command for a new waypoint is given when the load arrives within the tolerance 0.1m. The result shows that the trajectory of payload has negligible changes with parameter variation, and thus the controller is successfully designed.

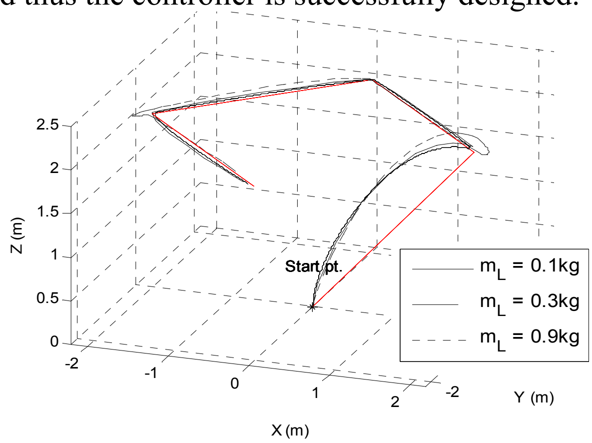
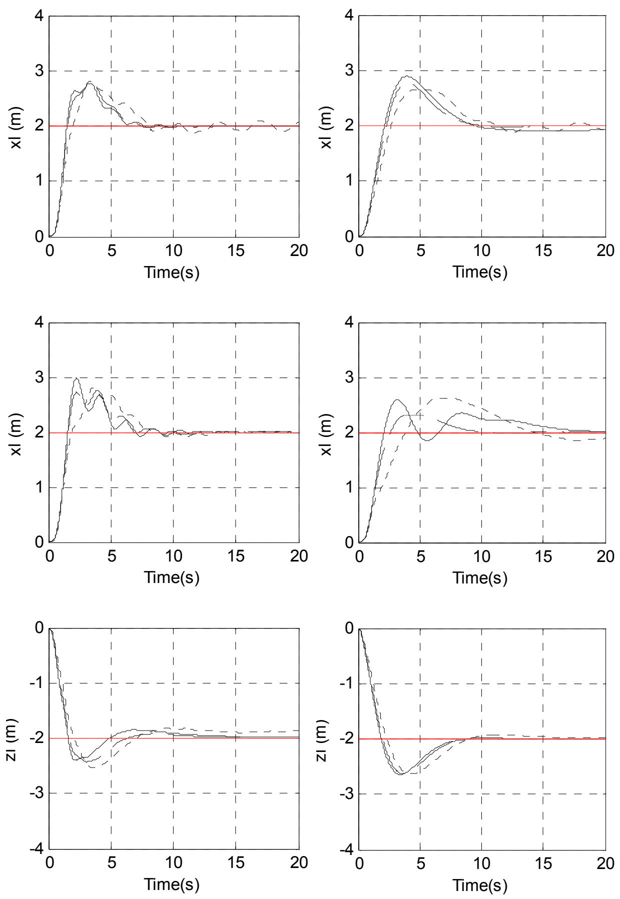


Fig. 4 Trajectory following performance

4.3 Numerical Results of Bar Mass Transportation System

Step response of two UAVs and one bar mass transportation system is shown in Fig. 5. The command is given from the origin to (2, 2, 2) m. The bar lies in x-axis and the two UAVs are stabilized with string inclination 10 degrees; initial forces along x-axis are existent. The overall formulation does not change during the response. As predicted in section 4.1, the system shows increased overshoot and settling time compared with the case in section 4.2. Where the first case showed indifferent response in x- and y-axis response, the bar shaped payload yields different frequencies along x- and y-axis. Hence, it is difficult to achieve parameter robustness along both axes using PRLQG, but at least one axis shows improvements in parameter robustness and increase in settling time.



(a) LQG/LTR (b) PRLQG

Fig. 5 Step response

5 Conclusion

In this paper, cooperative transportation system with multiple UAVs and slung load is modeled with Newtonian method. Generated dynamics is controlled with different optimal control techniques: LQG/LTR and PRLQG. While previous studies did not focus on controller or utilized simple PID controllers, applying analytic control technique, LQG/LTR, made the determination procedure of controller gains easy for extended system. Also, implementation of PRLQG method, which is not well-known compared to classic controls like LQG/LTR, resulted in improvements in robustness against transporting varying mass of payload. Computer simulation showed simple verification of modeling and analysis of improvements in parameter-robustness trading increase in settling time off. Further studies are expected to include consideration of wire collapse and collision, and the experimental procedures and results.

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