# Heavy spectra under operational loads can reduce life but not fatigue scatter factor

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# Background

With regard to fatigue scatter factor in fatigue test, both the strength specifications of aircraft (GJB67.6A-2008<sup>[1]</sup> of china and JSSG-2006 <sup>[2]</sup> of USA) stipulates that test life under mean spectrum is 4 times of service life (3 times of crack initiation life and 1 time of crack growth life) and test life under heavy spectrum is 2 times of service life, i.e. test life under heavy spectrum can reduce 2 times of fatigue scatter factor. If they are applied rashly without proof and verification by research and test, significant risk will result in service life of aircraft. In this regard, author of this paper go on the following study of both stage.

## **1. Theoretical curve of fatigue scatter factor** (study of first stage)

1.1 Theoretical curve of fatigue scatter factor  $(L_f)$  given by testing and computation

To find the effects of heavy spectrum on fatigue scatter factor  $L_f$ , author of this paper, the standard deviation S of the fatigue test life of 139 steel alloy specimens and 267 aluminum alloy specimens is used and obtain the theoretical curves of  $G_{\text{max}}/G_{\text{b}}$  (maximum stress/strength limit) to  $L_f$  by computation, call it Zhang Fuze theoretical curve of fatigue scatter factor in sphere of learning, as shown in Fig. 1<sup>[3]</sup>.



Fig.1 Zhang Fuze theoretical curve of fatigue scatter factors

## 1.2 Analysis of characteristics for Zhang Fuze theoretical curve of Fig.1

(1) There is a minimum value for the fatigue scatter factor  $L_f$ , i.e.  $L_{f\min} = 1$  when the maximum stress  $6_{\max}$  equals the strength limit  $6_b$  ( $6_{\max} = 6_b$ ).

- 2 Under the same stress, scatter factor  $L_f$  is different for different material.
- ③  $L_f \approx 2$  when 6 max/ 6 b =0.5~0.6 range.

# 1.3 Findings from theoretical curve of Fig. 1

Fig. 1 shows that the fatigue scatter factor  $L_f$  increases as stress  $\sigma_{\text{max}}$  decreases. This indicates that  $L_f$  is the low numerical value for high stress and  $L_f$  is the high numerical value for medium and low stress. According to statistics, the frequency of medium and low stress accounts for 99.7% under operational load spectrum. It can be inferred that heavy spectrum under operational loads does not reduce the fatigue scatter factor  $L_f$ .

**2. Effect of Heavy Spectrum under Operational Loads on Life and Fatigue Scatter Factor** (study of secondary stage)

## 2.1 Heavy spectrum under operational loads can reduce life

Take the analog equation of life [4-5], as equation (1)

$$\lambda' \sum_{i=1}^{k'} \left( \Delta \mathbf{S}_i' \right)^m n'_i = \lambda \sum_{i=1}^{k} \left( \Delta \mathbf{S}_i \right)^m n_i \tag{1}$$

Where

 $\Delta S_i$  is the amplitude of *i* stress in the mean damage spectrum;

 $n_i$  is the frequency of *i* stress in the mean damage spectrum;

 $\lambda$  is the number of cycles in the mean damage spectrum;

 $\Delta S'_i$  is the amplitude of *i* stress in the heavy spectrum;

 $n'_i$  is the frequency of *i* stress in the heavy spectrum;

 $\lambda$  ' is the number of cycles in the heavy spectrum.

For ease of discussion, the following assumptions are made :

1) The life of specimen of heavy spectrum is not known life ( $\lambda'$ ), and the life of specimen of mean damage spectrum is the known life ( $\lambda$ ).

2)  $\lambda \sum_{i=1}^{k} (\Delta S_i)^m n_i$  is the total damage value of test given under mean damage

spectrum and  $\sum_{i=1}^{k} (\Delta S_i)^m n_i$  is the damage value of a block under the mean damage spectrum, which is a known value.

3) 
$$\left[\sum_{i=1}^{k'} (\Delta S'_i)^m n'_i\right]'$$
 is the damage value of a block under the heavy spectrum,

which is not known value.

Due to heavy spectrum in reference 1 and 2, it's a fatigue damage value is 40% more than that of mean spectrum in the same operation load range, so that obtain equation (2).

$$\left[\sum_{i=1}^{k'} (\Delta S'_i)^m n'_i\right] > \sum_{i=1}^{k} (\Delta S_i)^m n_i$$
(2)

Comparison of equation (1) and (2) results in equation (3)

$$\lambda' < \lambda \tag{3}$$

As shown in equation (3), , the  $\lambda'$  (i.e. lift) of heavy spectrum of operational loads is less than the  $\lambda$ (i.e. lift) of mean spectrum of operational loads, which leads to the conclusion that heavy spectrum of operational loads can reduce life.

# 2.2 Heavy spectrum under operational loads can not reduce the fatigue scatter factor

# 1) Two examples of measured mean load spectrum and damage calculation.

As shown in Tables 1 and 2. [Note:  $n_y$  is the service overload value (1-8g);  $n_b$  is

the design overload value (12g);  $n_i$  is the overload frequency number]

$n_y/g$	$n_y / n_b$	n <sub>i</sub>	Damage value	Damage rate
1.5	0.125	2924	14803	3.2%
1.7	0.142	1317	11000	2.4%
2.1	0.175	1273	24757	5.5%
3.0	0.250	682	55242	12%
4.0	0.333	597	152832	34%
4.8	0.400	208	110415	24%
5.2	0.433	87	63611	14%
5.7	0.475	14	14778 3.3%	
6.5	0.541	3	5355 1.2%	
Σ		7105	452794	100%

 Table 1
 The practical mean spectrum of a certain training plane<sup>[6]</sup> and damage calculation

$n_y/g$	$n_y / n_b$	$n_i$	Damage value	Damage rate	
2	0.17	1968	31488	1.7%	
3	0.25	7156	579636	31.7%	
4	0.33	2302	589312	32.2%	
5	0.42	584	365000	19.9%	
6	0.50	138	178848	9.9%	
7	0.58	36	86436	4.7%	
Σ		12184	1830720	100%	

 Table 2
 The practical mean spectrum of a certain fighter plane<sup>[7]</sup> and damage calculation

### 2) Analysis of measured mean spectrum for table 1 and table 2

a. Low and medium overload accounts for the majority, i.e. all are low and medium load except (6.5g) in Table 1 and (7g) in Table 2.

b. The frequency of low and medium overload accounts for the absolute majority. It amounts to 99.96% in Table 1 and 99.7% in Table 2.

c. The damage value of low and medium overload accounts for the absolute majority. It amounts to 98.8% in Table 1 and 95.3% in Table 2.

In summary, the value, frequency and damage value of low and medium load all account for the absolute majority for the practical mean spectrum under the operational loads.

# 3) Heavy spectrum under operational loads can not reduce the value of fatigue scatter factor

In terms of damage, the fatigue damage of the heavy spectrum shall be 40% more than the measured mean spectrum. According to analysis of the spectra in Tables 1 and 2 and measured spectra from home and abroad, the maximum load of the operational load spectrum of the fighter aircraft is within 67% of the design load, with a contribution of approximate 5% to the fatigue damage. Medium load makes the most contribution to the damage value of the spectrum. In aggregate, low and medium load makes a contribution of over 95% to the fatigue damage.

As the heavy spectrum and the measured mean spectrum are representative of the actual operations condition of the fleet, their maximum, minimum and medium load values are all within the operational load range. Then for the heavy spectrum of high damage in reference [1] and [2], its damage value can not be increased by increasing the load. It can be increased only by increasing the frequency of load of all levels. As mentioned above, medium load contributes the most to the damage of a spectrum while the contribution of maximum and minimum load is minimal. In order that the distribution of damage value of the heavy spectrum follows the pattern of operations

condition, it is essential to significantly increase the frequency of medium load when increasing the damage value of the spectrum, so that the damage value of its medium load amounts to 90% of the total damage. With the addition of the damage of low load, the damage of low and medium load makes up 95%. It is inferred from the theoretical curve in Figure 1 that as the high damage value of the heavy spectrum is obtained by increasing the frequency of the low and medium load , it still maintains the attribute of the fatigue scatter factor of low and medium load and can not be reduced. Therefore, the fatigue scatter factor of the life tested under the heavy spectrum will not be reduced, i.e. heavy spectrum under operational loads can not reduce the fatigue scatter factor.

## **3. Verification by test** (study of secondary stage)

In this paper, the valid test data of 5 groups are used to verify the conclusion of this paper that heavy spectrum under operational loads can reduce the life but not the fatigue scatter factor.

#### **3.1 Test results**

See Table 1.

Nu. of 5 types specimen	heavy spectra			mean spectra		
	mean life/h	standard deviation S	effective specimen	mean life/h	standard deviation S	effective specimen
1	9732	0.0346	5	20247	0.0202	7
2	6807	0.1195	7	13637	0.1148	7
3	11125	0.0452	5	24449	0.1256	6
4	6757	0.0531	7	15930	0.0153	7
5	7525	0.0712	7	19317	0.0453	7
mean	8569	0.06472		18716	0.06424	

 Table 1
 Testing data of 5 types specimen under mean spectra and heavy spectra<sup>[8]</sup>

#### **3.2 Analysis of test results**

1) The damage value of heavy spectrum is 40% higher than that of mean spectrum, which complies with the requirement of specifications of reference [1] and [2].

2) The specimens are widely representative, with various materials and structures.

3) The valid specimens (5-7) meets the requirements.

4) The test shows good regularity, i.e. the damage of heavy spectrum is 40% higher than that of mean spectrum, and the test life of mean spectrum (18716) is twice as much as that of heavy spectrum (8569). This complies with the law that high damage results in low life and verifies the conclusion of this paper that heavy spectrum of high damage under operational loads can reduce the life.

5) The test shows that the average standard deviation (S) of life of 5 group test specimens under both spectra is the same (S=0.06424 for mean spectrum and S=0.06472 for heavy spectrum). Therefrom, it is concluded that the 2 different damage spectra have the same fatigue scatter factor. (See equation (4) of fatigue scatter factor, if standard deviation S is same, so that fatigue scatter factor  $L_f$  is same under  $U_p$ ,  $U_r$  and n of both spectra to take same value ). This verifies the conclusion of this paper that heavy spectrum of high damage under operational loads can not reduce the fatigue scatter factor.

$$L_f = 10^{\left(\frac{U_r}{\sqrt{n}} - U_p\right)S} \tag{4}$$

## 4. Conclusions of both stage study

The given convulsion 1 is the Zhang Fuze theoretical curve of fatigue scatter factor of Fig.1. It is very importance theoretical curve for to determine the fatigue scatter factor in international fatigue territory.

The given convulsion 2 are heavy spectrum of high damage under operational loads can reduce the test life but not the fatigue scatter factor. This conclusion is in conflict with both the aircraft strength specifications of china and USA. The these research results are published for operational safety of aircraft.

## References

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